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A non-existence theorem for isometric immersions. (English summary)

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The non-embedding theorem by Chern and Kuiper asserts that if an isometric immersion of a compact Riemannian manifold M into $\mathbb{R}^q$ satisfies that for any point $x \in M$ there is a k-dimensional subspace P_x of the tangent space T_xM, for some integer $k \geq 2$, such that the sectional curvature for any plane in P_x is non-positive, then the codimension of the immersion is greater than or equal to k [S. Chern and N. H. Kuiper, Ann. of Math. (2) **56** (1952), 422–430; MR0050962 (14,408e)]. The main result of the article under review consists of a generalization of this theorem for an isometric C^2-immersion of a non-compact manifold M into a Riemannian manifold $\overline{M}^q$. In fact, the author replaces the hypothesis of compactness of M in the statement of the non-embedding theorem by that of having a bounded image of the immersion, and some geometric estimations on the sectional curvatures. Then he states a criterion guaranteeing that the codimension of the immersion is greater than or equal to k. In order to obtain this generalization he uses an auxiliary function: the distance function from a fixed point p on $\overline{M}^q$, whose Hessian is bounded from below by a real-valued function on the tangent bundle of the boundary of a proper ball centered at p. Thus, its bound is applied to control the difference between the sectional curvatures of any plane in P_x considered as a subspace of T_xM and $T_x\overline{M}^q$, respectively. This procedure can be applied because the "weak principle for the Hessian" [S. Pigola, M. Rigoli and A. G. Setti, Mem. Amer. Math. Soc. **174** (2005), no. 822, x+99 pp.; MR2116555 (2006b:53048)] is required to hold on M and the image of the immersion does not intersect the cut locus of p. Further on, the author recovers from this generalization the main results in [L. Jorge and D. Koutroufiotis, Amer. J. Math. **103** (1981), no. 4, 711–725; MR0623135 (83d:53041b)], and also sharpens the results in [A. R. Veeravalli, Bull. Austral. Math. Soc. **62** (2000), no. 1, 165–170; MR1775899 (2001f:53120)].

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References

1. S.S. Chern, N. Kuiper, Some theorems on the Isometric imbedding of compact Riemann manifolds in Euclidean space, Ann. Math. 56 (1952) 422–430. MR0050962 (14,408e)
2. A. Ranjbar-Motlagh, Rigidity of spheres in Riemannian manifolds and a non-embedding theorem, Bol. Soc. Bras. Mat. 32 (2) (2001) 159–171. MR1860867 (2002g:53068)
3. L. Jorge, D. Koutroufiotis, An estimate for the curvature of bounded submanifolds, Amer. J. Math. 103 (4) (1981) 711–725. MR0623135 (83d:53041b)
4. A.R. Veeravalli, A sharp lower bound for the Ricci curvature of bounded hypersurfaces in space forms, Bull. Austral. Math. Soc. 62 (1) (2000) 165–170. MR1775899 (2001f:53120)
5. S. Pigola, M. Rigoli, A.G. Setti, Maximum principles on Riemannian manifolds and applications, Memoirs AMS 174 (822) (2005). MR2116555 (2006b:53048)

6. M.P. do Carmo, Riemannian Geometry, Birkhäuser, Boston, 1993.
7. H. Omori, Isometric immersions of Riemannian manifolds, J. Math. Soc. Japan 19 (2) (1967) 205–214. MR0215259 (35 #6101)
8. R.M. Schoen, S.T. Yau, Lectures on Differential Geometry, Vol. I, International Press, 1994. MR1333601 (97d:53001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.