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Ranjbar-Motlagh, Alireza (IR-SHAR)
Generalized Stepanov type theorem with applications over metric-measure spaces. (English summary)
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The author defines an L^p-differentiable structure on a metric measure space $X=(X,d,\mu)$ as a countable collection of pairwise disjoint measurable sets X_γ together with Banach space valued maps $\varphi_\gamma\in L^p_{\rm loc}(X_\gamma,B_\gamma)$, such that for every Lipschitz function $g\colon X\to\mathbf{R}$ and every γ there exists g^*_γ in the dual of B_γ satisfying for a.e. $x\in X_\gamma$,

$$\lim_{r\to0}\frac{1}{\mu(B(x,r)\cap X_\gamma)}\times\int_{B(x,r)\cap X_\gamma}\frac{|g(y)-g(x)-g^*_\gamma(\varphi_\gamma(y)-\varphi_\gamma(x))|^p}{r^p}\,d\mu=0.$$

The main result of the paper is the following generalization of the Stepanov theorem: If X is a doubling metric measure space with an L^p-differentiable structure for some $p\ge1$, then every function $f\in L^p_{\rm loc}(X)$ is L^p-differentiable a.e. in the set where

$$\limsup_{r\to0}\frac{1}{\mu(B(x,r))}\int_{B(x,r)}\frac{|f-f(x)|^p}{r^p}\,d\mu<\infty.$$

This is proved by comparing f with Lipschitz functions on sets where the above integral average is bounded by some constant, and by using points of density together with the fact that Lipschitz functions are differentiable a.e.

As an application of the general result, the author then obtains the L^p-differentiability of generalized Sobolev functions and the L^1-differentiability of BV-functions on metric measure spaces. These results recover and extend [J. Björn, Michigan Math. J. **47** (2000), no. 1, 151–161; MR1755262 (2001b:46049)] and [Z. M. Balogh, K. Rogovin and T. Zürcher, J. Geom. Anal. **14** (2004), no. 3, 405–422; MR2077159 (2005d:28008)].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.