

University of Tehran
 Department of Electrical and Computer Engineering
Assignment #3 for Robust Control

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Problem 1: Consider the flexible transmission system shown in Fig. 1.

The system consists of a DC motor connected to a flexible mechanical system in a feedback connection with an external controller represented by the PC block. Rotation of the DC motor creates an oscillatory rotation in the load. The objective of the control is to design an external controller (PC) such that load angle ϕ_m tracks its desired value ϕ_{ref} . It is possible to change the load up to 50 (half load) or even 100 (full load) percent. Therefore, the controller must ensure robust stability and robust performance.

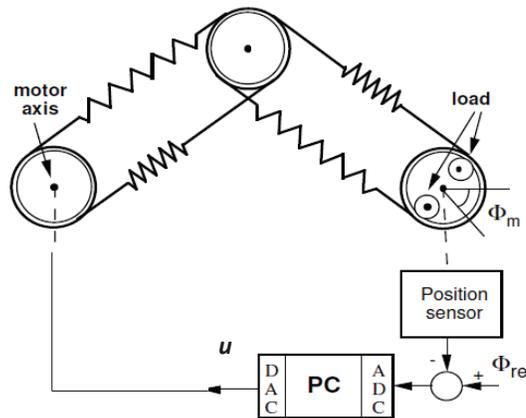


Figure 1: Flexible transmission system

Three discrete-time models for the system (from u to ϕ_m) are given by:

- no load: $G_n(z) = \frac{0.3778z + 0.4619}{z^4 - 1.3943z^3 + 1.608z^2 - 1.3543z + 0.9618}$,
- half load: $G_h(z) = \frac{0.0683z^2 + 0.0575z + 0.1898}{z^4 - 1.9712z^3 + 2.2283z^2 - 1.944z + 0.9808}$,
- full load: $G_f(z) = \frac{0.1155z^3 - 0.0792z^2 + 0.2087z - 0.0918}{z^5 - 2.0873z^4 + 2.3381z^3 - 2.0513z^2 + 0.9797z}$.

where the sampling time is assumed to be 50ms. We consider $G_n(z)$ as the nominal model for controller design.

- a. Investigate frequency response and pole-zeros of each model. Find a continuous-time transfer function for each model. Compare frequency response of the discrete-time models with the continuous-time models. Discuss.
- b. Compute $E(\omega) = \max(|G_n(\omega) - G_h(\omega)|, |G_n(\omega) - G_f(\omega)|)$. What $E(\omega)$ signifies in terms of model uncertainty? Find a low-order stable and minimum-phase transfer function $W(s)$ whose frequency response upper bounds $E(\omega)$ as closely as possible.
- c. The desired performance is to track a unit step by load angle with a settling time no more than 2secs, and negligible overshoot. Find a simple weighting function for this objective.
- d. Transform the system into an LFT form and describe the signals. Note: use our convention that $[p \ w \ u]^T$ is the input of P and $[q \ z \ y]^T$ is its output. Use `P=ltisys(A11,[B1 B2],[C1;C2],[D11,D12;D21,D22])` to construct P .
- e. Using μ synthesis, design a controller that achieves robust performance. Perform a D-K iteration (until μ is minimized) by using `dkit` command in Matlab.

Hint: Download the files `dkit.m` and `initdk.m` and copy them in your current director. By changing the second argument of line 70 in `initdk.m` you can change the number of D-K iterations. Before calling `dkit` command in your program set `DK_DEF_NAME = 'initdk'` to specify the initializing file. After running `dkit`, the final controller is saved in a variable called `k.dk`. Use `Ltiss` to extract the controller.
- f. For each controller obtained in each iteration show μ plots and discuss the robust performance. Write realizations of each controller. In each case plot the resulting output sensitivity function from w to the tracking error. Discuss the differences. Explain what goes on during D-K iteration.
- g. Apply a unit step to ϕ_{ref} and simulate the closed loop system using G_n, G_h, G_f . Discuss the results.

Problem 2: Use LMI toolbox and design an optimal H_∞ observer for the nominal system in Problem 1.

- a. Find a minimal continuous-time realization for the transfer function G_n as (A, B, C, D) . Consider the following standard "certainty equivalent" structure for the observer

$$\dot{\hat{x}} = A\hat{x} + Bu + L(C\hat{x} - y)$$

where $y = \phi_m$. In order to test the observer, using Simulink, construct negative feedback connection of G_n and the designed controller (Ignore all weighting filters). Make sure $A + LC$ has no eigenvalues on the imaginary axis. If this happens try to consider the constraint $Q > \alpha I$ where $\alpha > 0$ and Q is such that $\dot{V} = -\tilde{x}^T Q \tilde{x}$. Justify this approach. Try to achieve convergence time of less than one second for all states.

- b. Implement your observer using u and y . Set observer initial conditions such that initial state estimation error is non-zero. Compare system states with the estimated ones.
- c. Add a sinusoid to motor input u as $d(t) = 0.1 \sin(50t) \cos(10t)$ and compare x and \hat{x} . Increase amplitude of $d(t)$ to make its effect visible in figures if necessary. Note: In simulation consider the perturbation but for implementation of the observer do not use $d(t)$ and consider it unknown. Justify the use of H_∞ observer by simulation. Report optimal L together with your Matlab codes.
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