

Assignment #1
Robust Control

Spring 2016
Instructor: M. Namvar

Delivery: 18th of Ordibehesht.

Problem 1: The linearized model of the longitudinal dynamics of an aircraft flying at 0.9 Mach and altitude 8 Km is given by:

$$\dot{x} = Ax + Bu, \quad y_s = Cx + Du$$

with

$$A = \begin{bmatrix} -0.02 & -36.62 & -18.90 & -32.09 & 3.25 & -0.76 \\ 0 & -1.90 & 0.98 & 0 & -0.17 & -0.01 \\ 0.01 & 11.72 & -2.63 & 0 & -31.60 & 22.39 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -30 & 0 \\ 0 & 0 & 0 & 0 & 0 & -30 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 30 & 0 \\ 0 & 30 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

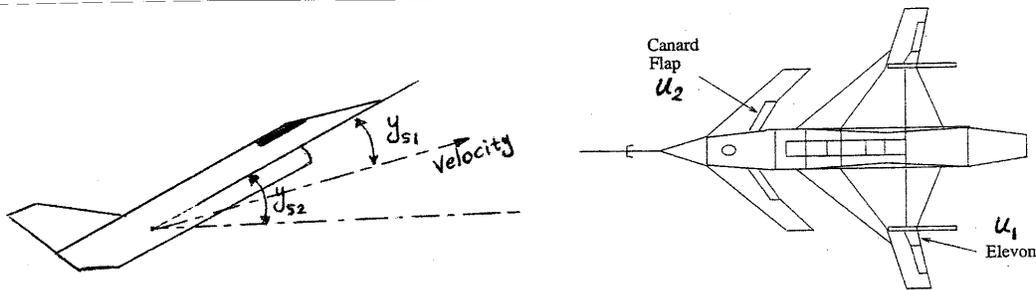


Figure 1:

The control variables u_1 , u_2 are elevon and canard actuator inputs, respectively. The output variables y_{s1} , y_{s2} are angle of attack and attitude angle (pitch angle), respectively. The nominal performance objective is to design a stabilizing controller $C(s)$ which makes the system output y_s track the reference signal $r(t)$ with the minimum possible error.

a) Find poles and transmission zeros of the system and plot them. Is the system stable and/or minimum phase? Investigate system controllability and observability.

b) Consider the filter $F(s) = \frac{s+z}{s+p}I$ with $0.5 < z < 2$ and $0.0005 < p < 0.002$ as weighting function. Find the realization A_F, B_F, C_F, D_F for $F(s)$ and plot $\sigma_i(F(jw))$ in the frequency range $w \in [10^{-4}, 10^3]$. (use this range with 500 resolution points in subsequent plots). Explain what this choice of $F(s)$ signifies in terms of desired performance and robustness. Is it possible to minimize the tracking error without limitation? What are the limitations? Try other filters and discuss their properties.

c) *Nominal performance:* Assume that $\Delta = 0$. Take $w = r$ and choose z as the controlled output. Transform the closed loop system into standard LFT form and determine the realization of the generalized nominal model P in terms of A_F, B_F, C_F, D_F and A, B, C, D . Use `ltisys(.)` to put P in "sys" form. Use `hinfmi(.)` to compute the controller. Use `ltiss(.)` to get the state space realization of $C(s)$.

- d) Plot poles and zeros of the controller and compare them with those of the system. What do you infer? (Investigate pole zero cancelations). Plot singular values of $C(jw)$.
- e) Find state space realization of $S = T_{wy}$ and plot its singular values. Compute $\|T_{wy}\|_\infty$. At which frequencies the tracking error is small/or large Explain. Repeat with different choices of $F(s)$.
- f) Plot closed loop step response together with actuator inputs (plots of y_s and u).
- g) Now, assume that the additive uncertainty $\Delta \neq 0$. Plot $\sigma_i(T_{pq}(jw))$ and determine how much uncertainty the system can tolerate without becoming unstable.
- h) Let $\bar{\sigma}(\Delta(jw)) \leq \gamma(jw)$ where $\gamma(s) = \gamma \frac{s + \alpha}{s + \beta}$ with $0.01 < \gamma < 0.15$, $0.1 < \alpha < 1$, $1 < \beta < 5$. To achieve nominal performance and robust stability at the same time, specify z_1 , z_2 and w . Choose $F(s)$ as performance weighting. Transform the system into the LFT form and solve the simplified H_∞ problem. After running the LMI optimization problem by Matlab, what "the best objective value" signifies? Design the controller for a class of weighting functions.
- i) Compare $\bar{\sigma}(T_{pq})$ and $\bar{\sigma}(T_{wz})$ resulted in part h) and the previous nominal design. Explain thoroughly the differences. Compare step responses, actuator inputs. Try to superimpose the relevant figures in your comparison. How the amplitude of the control input can be kept limited?
- j) If Δ is block-diagonal, discuss robust stability by plotting $\mu_\Delta(M_{11})(jw)$. See page 205 of the textbook for μ plot. Compare with unstructured uncertainty case. How much uncertainty the closed loop system can tolerate in this case?

Problem 2: In Problem 1. assume that $r(t) = w(t)$ is an external disturbance and the objective is to minimize the effect of w on the actuator input u .

- a) Use `hinflmi(.)` and `h2lqg(.)` commands and find $C(s)$ to minimize H_∞ and H_2 norm of T_{wu} . Plot singular values of T_{wu} in terms of frequency in each case and discuss the results (Robustness, noise attenuation, controller degree...). In each case compute H_∞ and H_2 norm of T_{wu} . Why the controller cannot be simply set to zero to make $T_{wu} = 0$. Apply white noise to $w(t)$ and plot $u(t)$ for both controllers. Compare.
- b) Repeat part a) for minimizing $\|WT_{wu}\|$ where W is given in Problem 1.

Hint: Use `[Ak,Bk,Ck,Dk,ACL,BCL,CCL,DCL]=H2LQG(A,B1,B2,C1,C2,D11,D12,D21,D22,'schur')` where `[Ak, Bk, Ck, Dk]` is controller realization.