A Novel Impulsive Noise Cancellation Based on Successive Approximations

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***Abstract*— In this paper we will propose a new method to recover bandlimited (in the DFT sense) signals corrupted by impulsive noise. The new method uses adaptive thresholds in conjunction with soft-decision and successive iterative approximations to find the position and values of all the spikes that the redundancy of the bandlimited signal allows theoretically. Computer simulations confirm the robustness of the proposed algorithms. The problem we have solved is related to error correction codes in real/complex Galois fields; it also has potential applications in Radar CFAR detection, and Sparse Component Analysis (SCA), and OFDM clipping noise removal.**

***Index Terms*— Real GF codes(FFT codes), SCA, Radar (CFAR)**

I. Introduction

IMPULSIVE noise is a common phenomenon occurring in channels that include switching, manual interruptions and lightning. In such channels several samples of the signal (sparse or burst) are lost. To recover the original signal, redundancy is introduced at the transmitter, for example by inserting zeros in the DFT domain or by band-limiting the original signal without padding zeros. The receiver would detect locations of the corrupted samples caused by the impulsive noise and use the redundancy to reconstruct the signal. To be specific, we use the term impulsive noise, for N samples which only M of them are random variables with Gaussian p.d.f and other samples are zero.

Many real signals such as speech and image signals are practically band-limited, i.e., the high frequency DFT (or DCT) coefficients are almost zero. We can take advantage of this redundancy for error concealment and impulsive noise removal. Alternatively, if the signal is not band-limited, we can pad zeros in the DFT domain and then take its inverse transform[1]. This implies that we can detect and correct corrupted samples up to half the number of inserted zeros. This idea is also used in OFDM for clipping noise removal[2], [3], [4], [5], [6], [7], [8].

By definition, in an erasure channel, the locations of losses are known. Error recovery in an erasure channel have been studied extensively by [9], [10], [11], [12], [13] and chapters 5 ,17 and 18 of [1]. The problem of impulsive noise, where locations of errors are not known, have been studied in several papers using DFT codes [10], [2], [14], [15]. In [2], a decoding technique for DFT-based error control codes, based on error locator polynomial is devised. DFT codes are essentially equivalent to Reed-Solomon(RS) codes in real/complex fields. Thus it has all the properties of RS codes such as Maximum Distance Separable (MDS).

Although our application in this paper is primarily related to real field error correction codes, i.e., removal of the impulsive noise from a band-limited signal, the methods proposed can be potentially applied in equivalent problems such as SCA (Sparse Component Analysis) [16], [17], Radar pulse detection using CFAR[18], and OFDM clipping noise removal.

In SCA the problem is detection of sparse sources from an under-determined observation of unknown combinations of the sources, and in Radar applications, the problem is to determine the reflected pulses from the target buried in clutter and additive noise. In the OFDM applications, clipping noise can be removed from the redundancy introduced in the DFT domain. These two applications are the compliment of our application, i.e., real field error correcting codes where impulse removal from a redundant or band-limited signal in the DFT sense is a special case.

In this paper a novel method is used to detect the locations of the impulsive noises and estimate the amplitudes of them. Successive noise detection and amplitude estimation stages are used to improve the quality of signal reconstruction. This method is called Iterative Detection Estimation (IDE)[17] and was used in SCA applications. The successive use of noise detection and the amplitude estimation are portrayed in Fig. 1. The detection block is adapted in successive steps to use the estimated signal in previous estimation step in the detection process. A novel method is introduced to detect the corrupted samples using soft-decision. When better estimates of error is devised, by changing a parameter, the soft-decision tends to hard-decision. In the estimation block we use an iterative method used in erasure problems, because by detecting the position of impulses, we have an erasure problem that can be solved by iteration introduced in [9], [10], [11], [12], [13]. This method efficiently uses nearly all the redundancy introduced in the signal, thus its reconstruction capacity (the number of corrupted samples that are recovered with desired SNR) tends to the theoretical limit (which is maximum error number is half of the inserted zeros).

In section II a method for impulsive noise detection is introduced. In section III, we will discuss an iterative method which is used in the amplitude estimator block. Finally, in section V the results of computer simulations are discussed.

II. Detection

The received signal contains impulsive noise in addition to the original signal. Detection is performed according to the amplitude of the received signal. Depending on the relative amplitudes of signal and noise, the detector can produce two

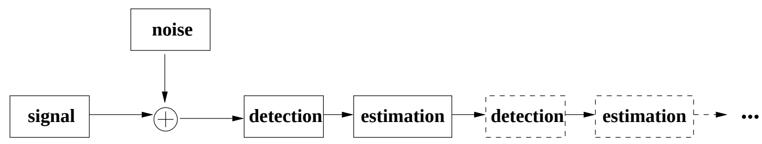

Fig. 1. Overall system diagram

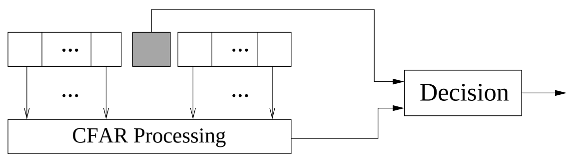

Fig. 2. Typical CFAR detector

kinds of errors. A *missed detection* occurs if a corrupted sample is not detected, while a *false alarm* occurs if a legitimate sample is detected as noise. In the following sections we use the term *mask* for the singal containing locations of impulsive noise because we are going to use it to eliminate the impulses.

A. Adaptive Detection Vs. Non-adaptive Detection

Detection can be either adaptive or non-adaptive. In the non-adaptive method, a fixed threshold is used to compare the amplitude of the sample under scrutiny to determine whether the signal contains a spike at that instant or not. For the adaptive decision, the sample under scrutiny is compared with an adaptive threshold which is determined according to the amplitude of the adjacent samples. When the statistical parameters of the noise is not known, adaptive thresholding techniques based on Neyman-Pearson criterion are used in radar detection to maintain a Constant False Alarm Rate(CFAR)[19]. CFAR is an example of an adaptive detection. In radar applications, the detector should detect the target in the presence of noise and clutter, which is equivalent to the detection of impulsive noise locations between legitimate samples. Thus the techniques used in radar detector can be adopted for the problem at hand.

Figure 2 depicts a simple CFAR detector. The CFAR processing is either amplitude averaging (Cell Averaging-CFAR) or a more complicated combination of adjacent samples (reference cells). In this paper we used a Censored Mean Level (CML) CFAR. In an k^{th} order CML-CFAR of length n, k of the smallest reference cell amplitudes are averaged and the other $n - k$ samples are ignored. The CML-CFAR is used to discard impulsive noises present in the reference cells, which otherwise would increase the adaptive threshold unnecessarily.

B. Hard-decision Vs. Soft-decision

The detection process is not an error-free process in the early stages, specially when the noise level is near the signal level. Thus if the estimator is flexible, instead of hard-decision, a soft-decision method can be used. The soft-decision can tend to hard-decision when better estimates of the original signal and impulsive noise is produced. Using hard-decision within the detection block a two-state mask is generated, i.e, 0 if it detects an impulsive noise and 1 otherwise. The soft-decision block generates a real number between 0 and 1 depending on the certainty of the detector. If the detector generates a number near 0.5, it means it cannot decide whether the sample contains impulsive noise or just pure signal. Simulation results suggest that a function of the form $\phi(x) = e^{-\alpha|x|}$ performs well if it is used to generate mask. x is the difference between the sample under scrutiny and the threshold $\phi(x)$ tends to one if the amplitude of x tends to zero. If α is made larger, the soft-decision tends to hard-decision.

III. Estimation Using an Iterative Method

Estimating corrupted samples is possible because of the redundancy present or introduced in the original signal. In this paper, oversampling is used to introduce redundancy in the signal. This is equivalent to padding zeros in the DFT domain. If N_z zeros are padded in the DFT domain, the receiver should be able to reconstruct N_z lost samples(where the locations of losses is known). In the denoising problem, locations of corrupted samples, i.e., impulsive noise is not known to the receiver, hence the locations of corrupted samples need to be determined before the amplitude reconstruction. This doubles the number of unknown variables for the receiver. Thus for the denoising problem, ideally the receiver should be able to detect and reconstruct $\frac{N_z}{2}$ corrupted samples. Mathematically, the receiver should solve an under-determined system of equations[17].

A. An Iterative Method

Iterative methods introduced in [1], [20], [21], [22] are general approaches to approximate the inverse of a system. The system can be non-linear and/or time varying. If G is the distortion operator representing the system and $y = Gx$ is known, then the objective is to reconstruct x. Thus, synthesizing G^{-1} is aimed. Symbolically, $G^{-1} = \sum_{i=0}^{\infty}(I - G)^i$. Thus the series $\{\sum(I - G)^i\}.y$ approaches x as more iterations are performed. It can be shown that this argument is true only if the norm of the operator $I - G$ is less than 1, i.e., the energy of the signal is greater than the energy of the distortion error caused by the operator G. The following equation depicts the iterative method in a systematic manner:

$$x_{k+1} = x_k + \lambda G(x - x_k) \tag{1}$$

λ is the relaxation parameter that determines the rate of convergence. If λ is larger than a threshold, the iterative method diverges. Figure 3 depicts the approximate inverse system, using the iterative method with λ set to one. The more the number of iterations used, the better the approximation of the inverse system is obtained[1].

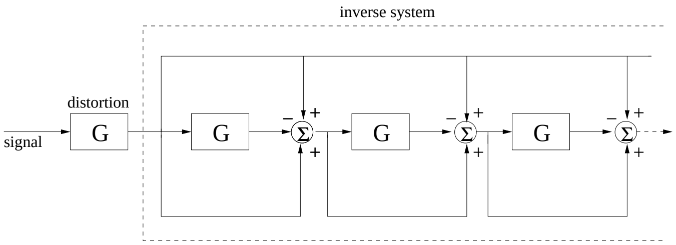

Fig. 3. Block diagram of the iterative method with $\lambda = 1$

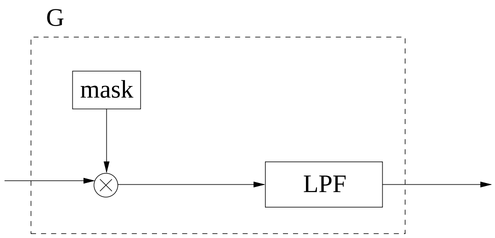

Fig. 4. Distortion block (G)

It has been shown that for small number of iterations, the iterative method results in the pseudo-inverse of the distortion block which is a rough and stable approximate of the inverse system[23]. For larger number of iterations, the inverse system is obtained. If the system is not invertible or ill-conditioned, it is possible that the iterative method diverges or yields a wrong answer. In such situations, a pre-processing may be required.

B. Estimation

In an erasure channel, the location of lost samples are known at the receiver. The iterative method shown in Fig. 3 can be used to recover the missing (erased or corrupted) samples. In the distortion block as depicted in Fig. 4, the mask generated by the detection stage is used to eliminate the corrupted samples. The result is low-pass filtered. We use an FFT-filter as shown in Fig. 5 to eliminate high frequency components of input signal . In the early detection/estimation steps, by adjusting the number of iterations to small numbers, a rough approximation of the signal (resulted from the pseudo inverse of distortion) is obtained. This approximation could be unstable if a large number of iterations is used. When better approximations of the signal is gained, then the number of iterations is set to a larger number, depending on the desired final SNR.

The iterative method is capable of using a mask that is generated with hard-decision or soft-decision detector. If hard-decision detector is used, the amplitude of each sample is compared with a threshold, and will result in 0 if it is larger and 1 otherwise. Using soft-decision, the amplitude of signal

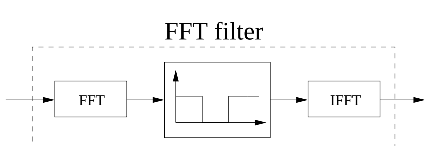

Fig. 5. Lowpass filtering in the FFT domain

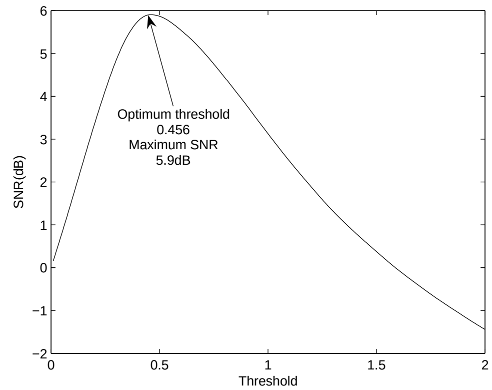

Fig. 6. The optimum threshold for hard-limiter

is subtracted from the threshold and the result is applied to the exponential function mentioned in sectionII.

C. Burst Error

Based on the characteristics of the channel, impulsive noise may distort consecutive samples. The iterative method may not be able to reconstruct the corrupted due to poor condition number of the matrix G 1. To handle this situation, instead of using DFT which uses $e^{\frac{2\pi}{N}}$ as its kernel, Sorted DFT (SDFT)[US patent no. 6 601 206] [24], [25] can be used which uses $e^{\frac{2\pi p}{N}}$ where p is a prime number. SDFT algorithm permutes the DFT coefficients. Thus SDFT acts as an interleaver and its inverse does the job of de-interleaving.

IV. The Proposed Method

1) Generate thresholds using an adaptive or non-adaptive method. CML-CFAR performed satisfactory in our simulations.
2) Generate a mask using a hard or soft-decision detector. Using the soft decision method introduced in section II performed satisfactory.
3) Use the iterative method introduced in section III to recover the signal. This will result in an estimate of the original signal.
4) Estimate the impulsive noise by subtracting the estimated signal from received signal.
5) Return to the first step. In further IDE steps, use the estimated noise to detect the locations of corrupted samples. If soft-decision is used in the second step, increase α parameter of the exponential function to tend soft-decision to hard-decision. Increase the number of iterations used in the third step to perform better estimations of the original signal.

V. Simulation Results

To prove the effectiveness of the proposed method, different simulations were conducted. In all simulations the signal was a filtered white Gaussian pseudo-random signal. We also tested random signals with uniform distribution with very little difference.

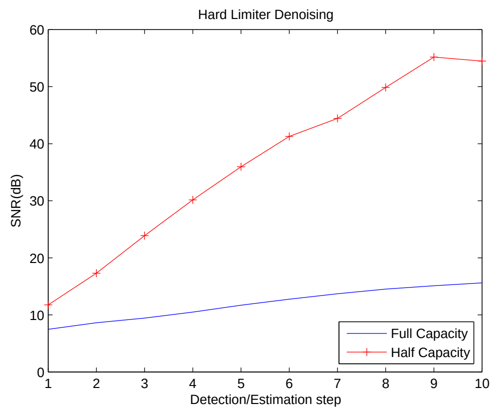

Fig. 7. Multiple Detection-Estimation steps using hard-decision simple detector

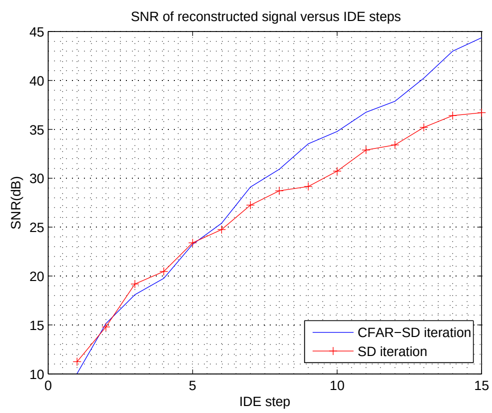

Fig. 8. Comparison of CFAR and simple soft-decision

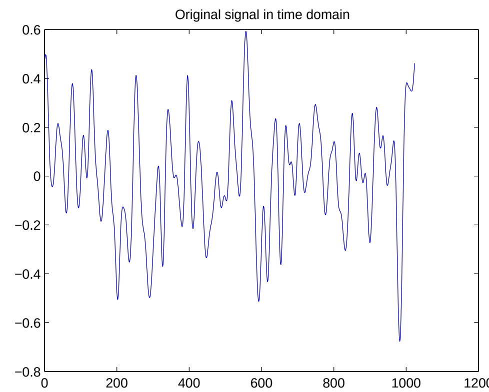

Fig. 9. Original lowpass(oversampled) signal

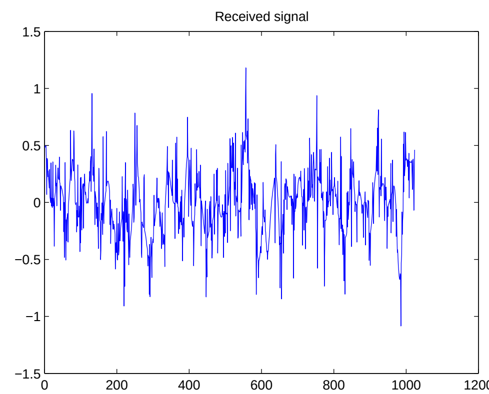

Fig. 10. Noisy signal with %40 impulsive noise

The simplest method to eliminate the added noise is to hard-limit the received signal and then low-pass filter the output of the hard-limiter. It is obvious that this method does not use the redundancy present in the signal efficiently, but it can be viewed as a crude reconstruction algorithm. An optimum threshold exists for the hard-limiter. However, if this process is repeated as in Fig. 1, we have improvement as portrayed in Fig. 7.

On the other hand if we use soft thresholding as explained in section II with repetition of detection and estimation processes, we get much better improvement as shown in Fig. 8, were the number of corrupted samples in received signal is half of the number of inserted zeros in DFT domain. This figure suggests that by adding CFAR to the soft-decision, the recovery is enhanced. It should be mentioned that adding CFAR to the receiver has little computation overhead because its addition results in fewer number of iterations in the amplitude spike noise estimation steps.

Fig. 9 depicts the low-pass Gaussian signal used as the information bearing signal. The redundancy introduced via oversampling was about %80 of the signal length, so the receiver should be able to recover the original signal if %40 of the samples are corrupted by the impulsive noise. If the spike noise amplitudes become very large compared to the amplitude of the signal, the detection of spikes would be a simple task. The main power of the proposed method is its ability to eliminate even minute spike noises introduced by the channel. Fig. 10 depicts the received signal, i.e., noisy signal. Fig. 11 is the denoised signal using CFAR detector after 15 detection/estimation steps.

If the channel impulsive noise exceeds the maximum capacity of the reconstruction algorithm, the SNR of the reconstructed signal is degraded gracefully. Fig. 12 depicts the SNR of the reconstructed signal when the errors introduced by the impulsive channel is %10 more than the theoretical reconstruction capacity. It can be seen that the reconstruction SNR degrades compared to that of Fig. 8 but the algorithm does not diverge.

VI. Conclusion

In this paper a new method for impulsive noise cancellation is proposed. The method could reach the theoretical upper bound of reconstruction capacity (MDS codes for real/complex Galois fields). In the transmitted signal, redundancy can be introduced by padding zeros in the DFT domain. If a discrete signal of length k is transmitted with n samples, then $\frac{n-k}{2}$ errors can be corrected. The proposed method uses several Detection/Estimation steps. The SNR of reconstructed signal can be improved by increasing the number of detection/estimation steps and increasing the number of iterations in

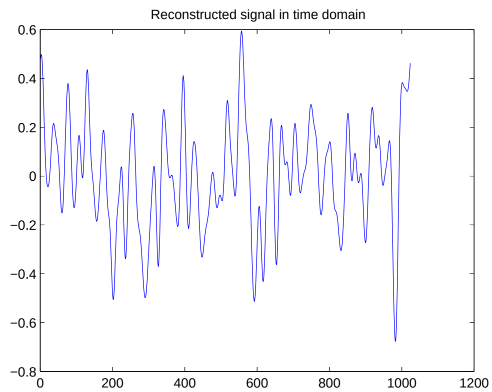

Fig. 11. Reconstructed signal using CFAR-SD detector

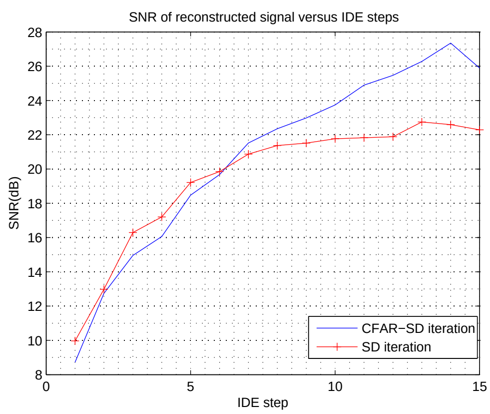

Fig. 12. SNR of the denoised signal when the error rate exceeds the reconstruction capacity by about %10

each estimation block. Soft-decision and adaptive thresholds are used in the detection procedure which results in very good performance. Simulation results show that the SNR of the reconstructed signal degrades gradually if the number of impulsive noise exceeds the theoretical reconstruction capacity. The ideas used in this paper can be used to other real field error correcting codes (DCT, wavelet and random codes), CFAR radar detection, SCA, and OFDM clipping noise removal.

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