Digital Signal Processing

http://kom.aau.dk/~zt/cources/DSP/

Solutions 2 (MM2)

2.1 $X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}, \ with\ -1 < a < 0$.

a)

$$\mathrm{Re}\{X(e^{j\omega})\} = \frac{1}{2}[X(e^{j\omega}) + X^*(e^{j\omega})]$$

$$= \frac{\frac{1}{2}}{1-ae^{-j\omega}} + \frac{\frac{1}{2}}{1-ae^{j\omega}} = \frac{1-a\cos\omega}{1-2a\cos\omega + a^2}$$

b)

$$\mathrm{Im}\{X(e^{j\omega})\} = \frac{1}{2j}[X(e^{j\omega}) - X^*(e^{j\omega})]$$

$$= \frac{-a\sin\omega}{1-2a\cos\omega + a^2}$$

OR

$$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} = \frac{1}{1-a\cos\omega + j\sin\omega} = \frac{1-a\cos\omega - ja\sin\omega}{1-2a\cos\omega + a^2}$$

c)

$$|X(e^{j\omega})| = \sqrt{X(e^{j\omega})X^*(e^{j\omega})}$$

$$= \sqrt{\frac{1}{1-2a\cos\omega + a^2}}$$

d)

$$\angle X(e^{j\omega}) = \arctan\left(\frac{-a\sin\omega}{1-a\cos\omega}\right)$$

2.2 Let $X(e^{j\omega})$ denote the Fourier transform of $x[n]$.

a)

$$F\{x^*[n]\} = \sum_{n=-\infty}^{\infty} x^*[n]e^{-j\omega n}$$

$$= \left(\sum_{n=-\infty}^{\infty} x[n]e^{j\omega n}\right)^*$$

$$= X^*(e^{-j\omega})$$

b)

$$F\{x^*[-n]\} = \sum_{n=-\infty}^{\infty} x^*[-n]e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x^*[k]e^{j\omega k} = (\sum_{k=-\infty}^{\infty} x[k]e^{-j\omega n})^*$$

$$= X^*(e^{j\omega})$$

2.3 $\left|X(e^{j\omega})\right|_{\omega=0}$

$$\left|X(e^{j\omega})\right|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \,|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n] = 6$$

2.4

$$Y(e^{j\omega}) = \frac{1-e^{-j7\omega}}{1-e^{-j\omega}} = e^{-j3\omega}\frac{\sin\frac{7}{2}\omega}{\sin\frac{\omega}{2}}$$ (Refer to Table 2.3 Item 9)

Since $Y(e^{j\omega})$ contains all frequences, the only possible ω_c is $\omega_c = \pi$

2.5

$$h[n] = \frac{1}{11}\sum_{k=0}^{10}\delta[n-k]$$

$$= \begin{cases} \frac{1}{11}, & 0 \le n \le 10 \\ 0, Otherwise \end{cases}$$

$$H(e^{j\omega}) = \frac{e^{-j5\omega}}{11} \cdot \frac{\sin\frac{11}{2}\omega}{\sin\frac{\omega}{2}}$$