Gain and Impedance Variation in Scanned Dipole Arrays*

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Summary—The effects of mutual coupling on the gain and element impedance of large electronically-scanned arrays of dipoles above a ground plane have been analyzed as a function of scan angle. The resulting calculated gain and impedance variations are tabulated for planar arrays with elements located on a square grid with spacings ranging from 0.5 to 0.8 wavelength, for dipole-to-ground plane heights ranging from $\frac{1}{8}$ to $\frac{3}{8}$ wavelength.

The results of the study show that the gain of a dipole array as a function of scan angle depends markedly upon the element-to-element spacing and appears to be quite insensitive to certain other parameters, such as the height of the dipoles above the ground plane. On the other hand, the change in element impedance with scan is considerably affected by some parameters which have little effect on the array gain behavior, and possibilities for minimizing mismatch caused by scanning are pointed out making use of these results.

I. Introduction

WHILE several recent papers [1–4] have been published on the subject of mutual impedance effects in dipole arrays, explicit results have usually been presented only for arrays using half-wavelength spacing between elements and quarter-wavelength spacing from dipole-to-ground plane, or arrays without ground planes.[5] Further, attention has been concentrated almost exclusively on the effect of the mutual coupling on the variation of element driving impedance with scan angle, ignoring explicitly the important question of the variation of array gain with scan angle.

The primary purpose of the investigation reported here was to examine both these effects in planar arrays of regularly-spaced dipoles, over a range of element spacings and a range of dipole-to-ground plane spacings, in order to facilitate more enlightened design of scanning dipole arrays. Secondarily, some information about the effects of array size on mutual coupling phenomena was obtained indirectly by calculating relevant data for two different-sized arrays. Some useful mathematical relationships between the behavior of the array and the properties of a single dipole above a ground plane are also presented.

No explicit consideration is given to edge effects in this paper. Rather, attention is solely directed toward the questions of the effects of mutual impedance on the interior elements of large arrays and the ability to predict these effects by the use of data taken on the center element of a small array. Some examples of edge effects on *linear* dipole arrays can be found elsewhere,[6] and data on the impedance variation of edge elements for a half-wavelength element-to-element spacing, quarter-wavelength ground-plane-to-dipole configuration is given analytically by Carter,[3] and experimentally by Kurtz, *et al.*[1]

This paper represents an abstraction of a more detailed report on the subject.[7] Details of derivations and more detailed results can be found therein.

II. The Mathematical Description of Mutual Impedance Effects

A. The Basic Assumptions of this Analysis

It will be assumed throughout the analysis of this paper that we are concerned with arrays of identical, thin, half-wavelength dipoles, such that the *form* of the current on the dipole is essentially invariant to its surroundings. Under this assumption, the effects of the individual dipoles on the antenna performance are completely specified by the antenna terminal voltages and currents, and one can write the conventional mesh-equation relationships[8] between the terminal voltages and currents in terms of the antenna self- and mutual impedances.

To completely describe the behavior of the array, it is necessary to incorporate into the mesh equations the constraints imposed by the array feed network. For some types of feeds, this may represent an overwhelming task. Fortunately, a case of significant practical interest is that of independently-fed antennas (*e.g.*, a separate transmitter and/or receiver between each antenna and the feed network, or a passive network using highly directional couplers). We will restrict our attention to arrays using such feed arrangements, and consider that each element and its feed network can be

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[1] L. A. Kurtz, *et al.*, "Mutual-coupling effects in scanning dipole arrays," IRE Trans. on Antennas and Propagation, vol. AP-9, pp. 433–443; September, 1961.

[2] S. Edelberg and A. A. Oliner, "Mutual coupling effects in large antenna arrays: part 1—Slot arrays," " IRE Trans. on Antennas and Propagation, vol. AP-8, pp. 286–297; May, 1960. Also, "Part 2—Compensation effects," *ibid.*, pp. 360–367; July, 1960.

[3] P. S. Carter, Jr., "Mutual impedance effects in large beam scanning arrays," IRE Trans. on Antennas and Propagation, vol. AP-8, pp. 276–286; May, 1960.

[4] E. A., Blasi, and R. S. Elliott, "Scanning antenna arrays of discrete elements," IRE Trans. on Antennas and Propagation, vol. AP-7, pp. 435–536; October, 1959.

[5] S. J. Rabinowitz, "The Conductance of a Slot in an Array Antenna," Lincoln Lab., Mass. Inst. Tech., Lexington, Tech. Rept. 192, ASTIA Doc. No. 209 908; December 31, 1958.

[6] J. L. Allen, *et al.*, "Phased Array Radar Studies, 1 July 1959 to 1 July 1960," Lincoln Lab., Mass. Inst. Tech., Lexington, Tech. Rept. 228, ASTIA Doc. No. 249 470, Hayden Library (M.I.T.) No. H335; August, 1960.

[7] J. L. Allen, *et al.*, "Phased Array Radar Studies, 1 July 1960 to 1 July 1961," Lincoln Laboratory, Mass. Inst. Tech., Lexington, Tech. Rept. 236, pt. 3, ch. 1; November, 1961.

[8] J. D. Kraus, "Antennas," McGraw-Hill Book Co., Inc., New York, N. Y., Sect. 11-6; 1950.

represented by an equivalent circuit such as that of Fig. 1. Each antenna is assumed to be driven by a perfect voltage generator of independently-variable open-circuit voltage v and of internal impedance Z_g which will be assumed identical for all elements. For convenience, zero lengths of transmission line are assumed.

This paper concentrates on the effects of mutual coupling in planar arrays in which the elements were placed at the intersections of a rectangular grid, as indicated in Fig. 2, with element-to-element spacings D_x and D_y in the x and y directions, respectively. A double-subscript notation is therefore used, letting the subscript pair mn denote the element located at $x=mD_x$, $y=nD_y$. The antenna currents are thus implicitly related to the generator voltages by

$$v_{mn} = \sum_p \sum_q Z_{mn,pq} I_{pq} \tag{1}$$

where $Z_{mn,pq}$ denotes the mutual impedance between the element located at mD_x, nD_y and the element located at pD_x, qD_y. The impedance of the mnth feed circuit, $Z_{mn,mn}$, is taken as

$$Z_{mn,mn} = Z_g + Z_a$$

where it is assumed that Z_a, the self-impedance of the elements, is identical for all.[9]

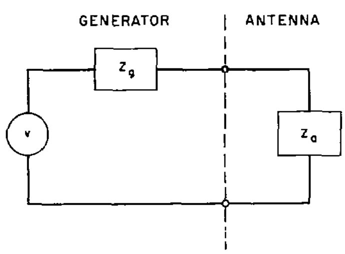

Fig. 1—Assumed equivalent circuit of a typical element.

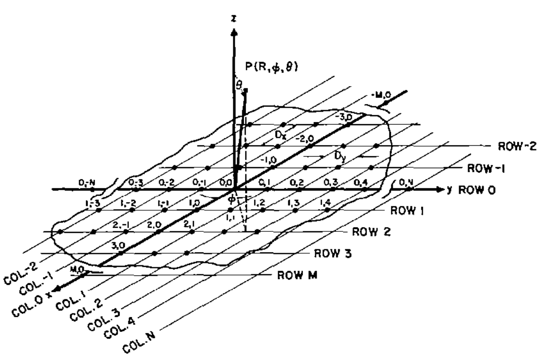

Fig. 2—Generalized planar-array geometry.

[9] Note that the assumption of the invariance of the form of the current on the dipole to its surroundings implies that open-circuiting a dipole effectively removes that dipole from the array, since, if the terminal current is zero, the current on the surface of the dipole is identically zero. Thus, Z_a is both the impedance of one element in the array with all others *open circuited*, or equivalently, the impedance of a single isolated dipole similarly mounted on a ground plane.

The element generator voltages are progressively phased and amplitude tapered so that the open-circuit generator voltage of the mnth element is related to the desired pointing angle by

$$v_{mn}(\tau_0, \mu_0) = a_{mn} e^{-jk[mD_x\tau_0 + nD_y\mu_0]} \tag{2}$$

where the a_{mn} are the real amplitude taper coefficients, and θ_0, ϕ_0 defines the angle at which it is desired to point the beam (in the geometry of Fig. 2) through the direction cosine equivalences

$$\left.\begin{aligned} \tau &= \sin\theta\cos\phi \\ \mu &= \sin\theta\sin\phi \end{aligned}\right\}. \tag{3}$$

Lastly, it is assumed that reciprocity applies throughout, and the entire analysis was carried out from the viewpoint of a transmitting array.

Despite the number of specializing assumptions that have been made about the type of dipoles, it will be seen that the calculated data checks well in major respects with experimental data taken on dipoles which are quite poor *fits* to the assumed model. Furthermore, the results of the investigation appear to be readily extensible (at least qualitatively) to other array geometries, such as triangular element spacings,[10] subject only to the requirement that the array present a regular environment to the dipoles (equal spacings, identical elements and generators).

B. The Element Gain Function

There are two equally valid approaches to calculating the behavior of an array. Solving (1) for the terminal currents can formally yield both the variation in element driving impedance and array pattern with scan angle. The latter involves also knowing the angular variation of the pattern of an element with all others open-circuited (or, in accordance with our assumptions, the pattern of an *isolated* element similarly mounted above a ground plane). The array pattern $F(\tau, \mu)$ then follows from the so-called *principle of pattern multiplication* as

$$F(\tau, \mu) = f_i(\tau, \mu) \sum_m \sum_n I_{mn} e^{jk[mD_x\tau + nD_y\mu]} \tag{4}$$

where τ and μ are the direction cosines for the array [see (3)], $f_i(\tau, \mu)$ is the pattern[11] of a typical isolated element, and the I_{mn}'s are obtained from solving (1).

While (4) is both correct[12] and useful in some computations, an often advantageous procedure for arriving at the pattern (and subsequently, the gain) of arrays of independently-driven elements consists of using a superposition argument in which mutual coupling effects are viewed as affecting primarily the element patterns, rather than element currents.

[10] E. D. Sharp, "A triangular arrangement of planar-array elements that reduces the number needed," IRE Trans. on Antennas and Propagation, vol. AP-9, pp. 126–129; March, 1961.
[11] Either the E or H field, in agreement with $F(\tau, \mu)$.
[12] To the extent that terminal conditions completely specify the current flowing on the radiator.

If one conceptually places shorting switches around the voltage generators of Fig. 1, any element of the array can be separately energized while all others are terminated in their normal generator impedance. If the radiation pattern of each element is ascertained under these conditions, the total array field will, by superposition, be the sum of each pattern with the proper phase delay as a function of position:

$$F(\tau, \mu) = \sum_m \sum_n f_{mn}(\tau, \mu) i_{mn} e^{jk[mD_x\tau + nD_y\mu]} \tag{5}$$

where $f_{mn}(\tau, \mu)$ is the pattern of the mnth element when all others are passively terminated in Z_g, per ampere of current into the terminals under this condition, and i_{mn} is the current into the terminals of the mnth element, with all others terminated [the value of I_{mn} obtained in (1) when all v's are set to zero except v_{mn}]. We can show[7] that, if the array is large enough so that essentially all element patterns are alike, the gain of an array, phased to point the beam at τ_0, μ_0, is related to the gain of a typical element in the same direction $g_{00}(\tau_0, \mu_0)$, measured with the element embedded in a passively-terminated (in Z_g) array, by

$$G(\tau_0, \mu_0) = g_{00}(\tau_0, \mu_0)\eta N_T \tag{6}$$

where η is the amplitude taper efficiency, and N_T is the total number of array elements:

$$\eta N_T = \frac{\left[\sum_m \sum_n a_{mn}\right]^2}{\sum_m \sum_n [a_{mn}]^2}. \tag{7}$$

[We designate the center element ($m=n=0$) as our *typical* element]. Thus, for a fixed number of elements and a fixed amplitude taper, the array gain is *completely* specified by the *element gain function* $g_{00}(\tau_0, \mu_0)$ of a typical element if the array is large enough so that almost all elements have essentially identical gain functions. The element gain function angular variation is determined solely by the pattern of a single typical element *in the passively terminated array*.[13]

The gain function concept has considerable practical utility and was used extensively in this study. The prime reason for its utility lies in the fact that, while (6) is only accurate for arrays in which almost all gain functions are identical, a typical gain function can be determined (experimentally or analytically) from an array which is only large enough so that the gain function of the *center element* is essentially unaffected by enlarging the array. As justified below, for example, a 5×5 dipole array is often sufficient.

[13] The fact that the angular behavior depends upon the pattern of the element in the array, rather than the pattern of an isolated element has been pointed out by others. For example, Delaney[6] gives experimental support, and it is pointed out in W. E. Rupp, "Coupled energy as a controlling factor in the radiation pattern of broadside arrays," *Abstract 11th Annual Symp. on USAF Antenna Res. and Dev.*, Monticello, Ill.; October, 1961.

C. *Some Large Array Approximations*

As pointed out elsewhere,[6,7] the effects of mutual coupling decay rapidly enough with element separation for a dipole array above a ground plane so that one can, in principle, build an array large enough so that any prescribed fraction of the total number of elements will see an environment which is arbitrarily close to the environment that an element would see in an infinite array. For almost all of the elements of such an array, one can derive[7] simple approximate expressions for the element driving impedance and the element gain function, and for the interrelations existing between them.

How large an array is required to justify these assumptions is a question that we will attempt to answer below.

If the array is essentially infinite, all elements will have identical driving impedance and gain functions, and we can confine our attention for convenience to the center element. We further assume that an infinite array has no measurable amplitude taper over any finite portion, and consequently, we can ignore the amplitude taper. In this case, the driving impedance $Z_D(\tau_0, \mu_0)$ for the large array case is simply the sum of the element self-impedance and the *phased* mutual impedances:

$$Z_D(\tau_0, \mu_0) = Z_a + \sum_m \sum_n{}_{m,n\neq 0,0} Z_{00,mn} e^{-jk[mD_x\tau_0 + nD_y\mu_0]} \tag{8}$$

where $m, n \neq 0, 0$ implies the summation excludes the term $Z_{00,00}$.

For deriving an expression for the gain function for a large array, one can revert to (4) and show that the array gain can be related to the gain function of a *matched* isolated element $g_{i_{\max}}(\tau_0, \mu_0)$ by

$$G(\tau_0, \mu_0) = \frac{4R_aR_g}{|Z_g + Z_D(\tau_0, \mu_0)|^2} g_{i_{\max}}(\tau_0, \mu_0)\eta N_T \tag{9}$$

where R_a and R_g are the real parts of the antenna self-impedance and the generator impedance, respectively. Comparison of this result with (6) for the array gain indicates that the relationship between the element gain function, the gain function of an isolated element, and the impedances is

$$\frac{g_{00}(\tau_0, \mu_0)}{g_{i_{\max}}(\tau_0, \mu_0)} = \frac{4R_gR_a}{|Z_g + Z_D(\tau_0, \mu_0)|^2}. \tag{10}$$

In terms of the circuit of Fig. 3, which maximizes the array gain at some arbitrary angle τ_1, μ_1, we have that the gain functions and the element reflection coefficient as a function of scan angle are related by

$$\frac{g_{00}(\tau_0, \mu_0)}{g_{i_{\max 11}}(\tau_0, \mu_0)} = \frac{R_a}{R_D(\tau_1, \mu_1)} |1 - \Gamma(\tau_0, \mu_0)|^2 \tag{11}$$

where $\Gamma(\tau_0, \mu_0)$ is the voltage reflection coefficient seen by the generator of Fig. 3 when the array is phased to point the beam in the τ_0, μ_0 direction.

Eqs. (9)–(11) not only represent useful practical tools for estimating impedance effects from element pattern measurements, but also are useful for analytic purposes. For example, by well-known formulas for $g_i(\tau, \mu)$[14] and a relationship for $R_D(0, 0)$ given by Stark,[15] one can show that[7] the maximum array gain (elements optimally matched at broadside) is given by the familiar expression[16]

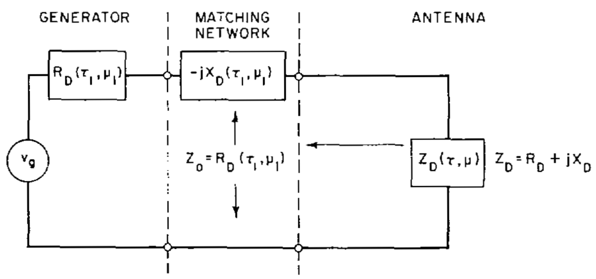

Fig. 3—A circuit for matching the generator resistance R_g to the antenna driving impedance at an angle τ_1, μ_1.

$$G(0, 0) = 4\pi \frac{A}{\lambda^2} \eta \qquad (12)$$

since $N_T D_x D_y = A$, the total area, where η is defined by (7).

III. The Computational Program

By use of the foregoing relationships, and expressions for the self- and mutual impedances between dipoles in free space, it is possible to compute gain functions and driving-point impedances for the interior dipole elements of an array. However, even for a relatively small planar array, the calculations require special techniques in programming for a large digital computer. Fortunately, the gain function concept offers the possibility of obtaining meaningful results for large arrays on a modest sized array, and consequently, such computations can be managed in a straightforward manner with a digital computer. A program was therefore written for the IBM 7090 to compute gain functions and driving impedances of the center element of planar arrays up to 63 elements (limited by storage).

The parameters of the program were the following:

1) The number of elements in the array (M and N of Fig. 2).
2) The element-to-element spacings D_x and D_y.
3) The height s of the dipoles above the ground plane.
4) The value of the generator circuit impedance of the equivalent circuit of Fig. 1.

Two different-sized arrays were investigated for estimating the effect of array size: a 7-element (collinear direction) by 9-element (parallel) array as indicated in Fig. 4, and a 5×5 array.

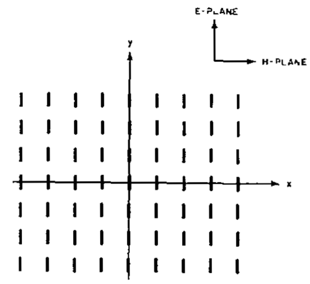

Fig. 4—7×9 element array configuration.

Values of the large array driving-point impedance were computed using the large array approximations, (8) for E-plane, H-plane and diagonal scans. Smith Chart plots were made of this impedance normalized by (see Fig. 3)

$$Z(\tau_0, \mu_0) = \frac{Z_D(\tau_0, \mu_0) - jX_D(0, 0)}{R_D(0, 0)} \qquad (13)$$

and values of VSWR vs scan angle determined. Impedance data was determined for $s/\lambda = 0.125$, 0.187, 0.250, 0.312 and 0.375 and for square-element grid spacings ($D_x = D_y$) of 0.5, 0.6, 0.7 and 0.8 wavelength.

Gain functions were then computed[7] for the values of s and D given above, and normalized by dividing by $4\pi D_x D_y/\lambda^2$. The value of Z_g selected for the impedance matrix was

$$Z_g = Z_D^*(0, 0)$$

(computed as described above), in order to maximize the large-array gain at broadside. Also, to check some experimental results, gain functions were computed for the 5×5 array for another value of Z_g as described in Section VI.

The detailed results (Smith Charts and E, H, and diagonal cuts on the gain function) are given in Appendix A of Allen, *et. al.*[7]

IV. Comparison of the Computed Results with Known Results for Infinite Arrays

In order to estimate the confidence with which one may extrapolate the results for the two small arrays to

[14] Kraus, *op. cit.*, p. 305, (11)–(87), with appropriate changes in notation.

[15] L. Stark, "Radiation Impedance of a Dipole in an Infinite Array," Hughes Aircraft Co., Fullerton, Calif., Formal Tech. Doc., FL60-230; May 1, 1960.

[16] H. A. Wheeler, "The radiation resistance of an antenna in an infinite array or waveguide," Proc. IRE, vol. 36, pp. 478–587; April, 1948.

larger arrays, two theoretical checks were made with known results for large arrays.

First, as pointed out by comparing (6) and (12), the gain function at broadside should be numerically equal to $4\pi D_x D_y/\lambda^2$ for an infinite array, when the generator circuit is matched to the broadside driving impedance of the elements. Secondly, from Stark's results,[15] we have a useful theoretical prediction of the value of $R_D(\tau, \mu)$ for all s and D, and, from Carter's data,[3] a calculated variation of $Z_D(\tau, \mu)$ for $D/\lambda = 0.5$, $s/\lambda = 0.25$ is available. Using these checks, it was found that the error between the computed results and the results for a truly infinite array generally increases with increasing values of s. For s less than a quarter-wavelength, the 7×9 array impedance prediction was found to be correct to within a few per cent, while the error in the 5×5 array prediction, in most cases, reaches a value of 10 to 15 per cent for that spacing. For larger values of s, the 7×9 array still gave usable results in most cases. The decrease in accuracy with s is apparently due to the increase in coupling that results as the elements are raised off the ground screen, increasing the radiation for angles near 90° from array broadside. The gain-function check produced a similar degree of agreement.

Based on the above comparisons, it seems reasonable to conclude that the 7×9 array yields quite accurate *large array* data for s up to a quarter-wavelength and reasonably accurate data for greater s. While the information that a 5×5 array yields is probably within the bounds of the usual experimental errors for small s, it is somewhat suspect if the dipoles are mounted more than a quarter-wavelength above ground.

V. Summary and Discussion of the Computed Results

In this section the digested facts gleaned from the computations are described. Unless otherwise stated, the results quoted are those computed for the 7×9 array.

A. The Variation of Broadside Driving Impedance with D and s

Fig. 5 shows the computed driving impedance of the dipoles in the array when it is phased to radiate in the broadside direction. Also shown is the value of the radiation impedance of an isolated dipole above an infinite ground plane. It is apparent that the array environment completely dominates the element, and the impedance of the dipole in the array varies widely from its free-space value.

The resistive component is seen to decrease monotonically with D for a fixed s (except for $s = 0.375\lambda$), as is necessary to cause the broadside element gain function to increase in direct proportion to the area allotted to the element for all s. The discrepancy at $s = 0.375\lambda$ appears to be another manifestation of the slow convergence of mutual effects for large ground-plane spacings.

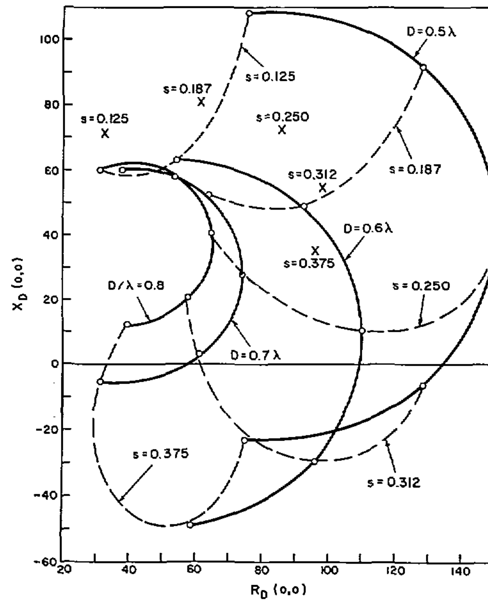

Fig. 5—$Z_D(0, 0)$ for 7×9 array (x = impedance of an isolated element).

B. The Effects of Coupling on the Scan Angle Corresponding to a 3-db Decrease in Gain

The gain-function 3-db points describe the solid angle over which the beam of a large array can be scanned with less than 3-db decrease in array gain.

Indicated in Figs. 6 and 7 are the gain function 3-db H- and E-plane beamwidths, respectively. Although an isolated dipole above ground has a beamwidth quite sensitive to s, the gain-function beamwidths are relatively insensitive to this parameter. For $D > 0.5$ wavelength, the beamwidth corresponds roughly to the included angle $2\theta_{\max}$, over which the array can be scanned without grating lobe formation, given by the well-known relation

$$\frac{D}{\lambda} = \frac{1}{1 + \sin|\theta_{\max}|}.$$

The angle of 3-db decrease and the angle of grating lobe formation occur almost simultaneously in the H plane for most cases. In the E plane, however, if good array pattern control is necessary, the grating lobe formation angle will dictate the maximum usable scan angles, rather than gain considerations.

The gain functions from which this data was plotted were computed under the assumption that the element drives were matched to the element driving impedance with the array phased for broadside radiation, thus maximizing the broadside gain of the array. It is apparent from (10) that the shape of the gain functions will be altered (at the expense of broadside gain) if some other value of generator circuit impedance is chosen. Although a detailed study of this effect was not conducted, patterns were computed for the 5×5 array for a generator impedance matched to the impedance of a single isolated element (see Section VI). It was gen-

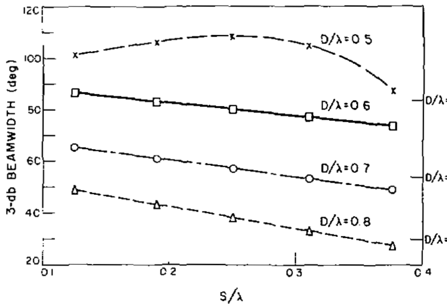

Fig. 6—Half-power H-plane beamwidth of gain function for broadside impedance match vs D/λ and s/λ.

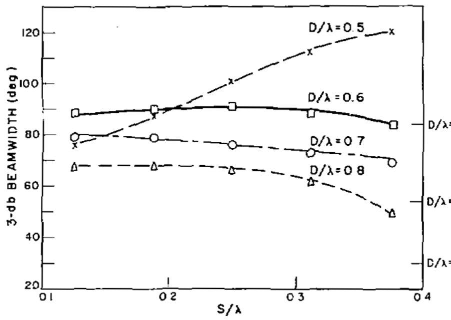

Fig. 7—Half-power E-plane beamwidth of gain function for broadside impedance match vs D/λ and s/λ.

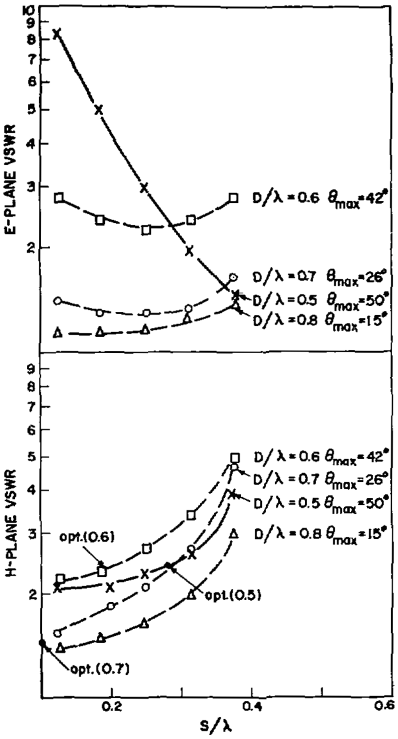

Fig. 8—Maximum VSWR to scan to θ_{max} (impedance match at $\theta=0$).

erally noted that in addition to the expected broadside gain decrease, this mismatch caused an increase in gain for certain other angles (corresponding to angles for which the element drive impedance was approximately equal to the impedance of an isolated element) and a consequent broadening of the gain-function beamwidth. These results indicated that the element generator impedance may offer an interesting tool for accomplishing a certain amount of *tailoring* of the array gain vs angle of scan characteristics.

C. The Maximum VSWR Incurred During Scan

From the computed data for the 7×9 array, the maximum VSWR that would be incurred in scanning to the grating lobe formation angle θ_{max}, defined above, was computed assuming the dipoles were matched when the array was phased for broadside as, for example, by the circuit of Fig. 3. For $D/\lambda=0.5$, since no grating lobes occur for any scan angles, a value of θ_{max} of 50° was arbitrarily chosen.

The resulting VSWR plots for scan in the two principal planes are shown in Fig. 8. Except for $D/\lambda=0.5$, the maximum VSWR is relatively insensitive to s for scan in the E plane but increases with s for scan in the H plane.[17] Consequently, there are values of s which are optimum in the sense of minimum VSWR for a given spacing for specified scan limits in the two principal planes. The values are indicated in the figure for equal scans in the two principal planes. It is seen that, the wider the dipole-to-dipole spacing, the closer the dipoles should be spaced to the ground plane. It is also apparent that for all cases, one can choose s such that the VSWR does not exceed a value of 3:1 and for many cases, it can be held to about 2:1.

[17] The difference in the nature of the E-plane curves for $D/\lambda=0.5$ and those for other spacings is presumably due to the fact that the dipole ends are infinitely close (touching, that is) for half-wavelength spacing.

VI. Some Experimental Results

In order to verify the validity of the foregoing calculated results, and also to test the sensitivity of the effects of mutual coupling on the exact characteristics of the dipoles, some experimental element gain functions were measured.

Crossed dipoles were used to facilitate measurement of E- and H-plane patterns, since only a single-axis antenna mount was available. A typical 900-Mc dipole is shown in Fig. 9. It is seen to be a poor approximation to the mathematical model assumed in the computations; the dipole length is 0.46λ, and the ratio of length to thickness is about 24. The feed structure is a conventional $\lambda/4$ balun, resulting in an appreciable feed structure. The base plates are constructed so that they will fit through square holes in the ground plane, facilitating variation of the height of the element above a ground plane.

The dipole feeds were designed to provide a nominal match at $s=\lambda/4$. Measured VSWR's ranged up to 1.4 for this value of s for all such dipoles, with 1.2 being a typical number. Cross-coupling between orthogonal dipoles was typically -30 db.

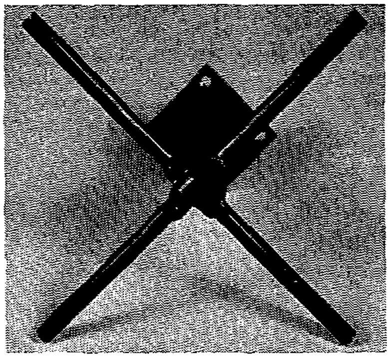

Fig. 9—Typical crossed dipole.

The dipoles were arrayed on a large (16-ft×16-ft) ground plane with interchangeable 6-ft×6-ft center sections. A dipole with a small VSWR when isolated at $s=\lambda/4$ was selected for use as the center (test) element. Another dipole with good match was mounted a quarter-wave above another large ground plane and used as a reference.

All measurements were made on the 600-ft range of the Lincoln Laboratory Ground Reflection Antenna Range.[18]

Rather than attempt to match all dipoles for each D and s, it was decided to leave the dipoles unmatched. It was assumed that the dipole feed system matched a 50-ohm generator to the self-impedance of a thin half-wave dipole at $s=\lambda/4$, for which Carter's formulas yield a value of $85.6+j72.4$ ohms. Since this is equivalent to assuming that the generator impedance was $85.6-j72.4$ ohms, the gain-function computations were rerun for this value of Z_g for a 5×5 array, and experimental element gain functions were measured for $s/\lambda=0.125$, 0.250 and 0.365 for $D/\lambda=0.6$ and 0.8. The pertinent comparisons between the calculated and measured gain functions are indicated in Table I.

TABLE I
COMPARISON OF CALCULATED AND MEASURED GAIN FUNCTIONS

D/λ	s/λ	Broadside Gain (calculated) (db)	Broadside Gain (measured) (db)	Beamwidth* (calculated)		Beamwidth* (measured)	
				H	E	H	E
0.6	.125	6.11	6.0	86	80	88	84
	.250	5.8	5.5	83	97	82	104
	.375	3.58	4.5	89	107	82	116
0.8	.125	8.0	8.0	69	64	72	64
	.250	9.12	9.0	44	63	44	68
	.375	7.8	8.25	34	66	30	68

* 3-db down from gain at angle of maximum gain (not necessarily broadside).

Except for $s=0.375$, the agreement is seen to be very good with regard to gain, and all beamwidths check closely.

If the broadside gain values are adjusted for the assumed mismatch, values within 10 per cent of $4\pi D^2/\lambda^2$ result, except for the case $D/\lambda=0.6$, $s/\lambda=0.375$, for which the result is in error by 15 per cent.

VII. CONCLUSIONS AND OBSERVATIONS

It is apparent from the foregoing results that the array environment, through mutual coupling, dominates the individual dipole element in fixing the array behavior; that is, the properties (gain, beamwidth, impedance) of the individual dipoles are drastically altered when the elements are placed in the array. Roughly speaking, for element-to-element spacings $0.5\leq D/\lambda<1.0$, the effects of the coupling on array gain are:

1) To render the broadside element gain function equal to $4\pi D^2/\lambda$, essentially regardless of the gain of the element when isolated (assuming the elements matched at broadside).
2) To force the beamwidth of the gain function to conform approximately to the included angle over which an array with the given spacing can be scanned without grating lobe formation.

These modifications are accomplished through the mechanism of an element driving impedance which varies with array pointing angle to modify the isolated dipole pattern.

Thus, one might make the observation that mutual coupling is the mechanism through which the behavior of an array with scan angle is found to behave according to logic (more appropriately, perhaps, according to directivity considerations).

Finally, it is seen that there are still degrees of freedom available to aid in reducing the most troublesome aspect of mutual impedance, at least to the transmitter engineer: the VSWR in the feed line. By virtue of (11), it is apparent that, to minimize the VSWR, one should attempt to choose a radiating *element* having a gain pattern that, when isolated, closely approximates the element gain function that directivity considerations indicate will prevail in the array. Such a choice can consist of either selecting an appropriate value of s, as explored above, or more esoteric schemes, such as placing *fences* between the ends of the dipoles, as proposed by Edelberg and Oliner.[19]

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[18] A. Cohen and A. W. Maltese, "The Lincoln Laboratory test range," *Microwave J.*, vol. 4, pp. 57–65; April, 1961.

[19] Edelberg and Oliner, *op. cit.*, pt. 2.