Communication

The Active Element Pattern

D. M. Pozar

Abstract—**This review article will discuss the use of the active element pattern for prediction of the scan performance of large phased array antennas. The introduction and application of the concept of the active element pattern goes back at least 30 years [1]–[6], but the subject is generally not covered in modern antenna engineering textbooks or handbooks, and many contemporary workers are unfamiliar with this simple but powerful idea. In addition, early references on this subject do not provide a rigorous discussion or derivation of the active element pattern, relying instead on a more qualitative interpretation. The purpose of this communication is to make the technique of active element patterns more accessible to antenna engineers, and to provide a new derivation of the basic active element pattern relations in terms of scattering parameters.**

I. Basic Features of the Active Element Pattern

First consider an N-element uniform linear array of identical elements, as shown in Fig. 1. (We consider a linear array here for mathematical simplicity, but the same conclusions will also apply to a uniform planar array.) Ordinary array theory ignores mutual coupling effects between array elements, and expresses the pattern radiated by the array in the well-known pattern multiplication form of an element factor times an array factor. Let this element factor be $f_0(\theta)$. This factor is identical to the pattern of a single element taken in isolation from the array, and is the same for any element in the array.

Now consider the same array, with the pattern taken with a feed at a single element in the array, and with all other array elements terminated in matched loads, as shown in Fig. 2. The pattern obtained in this case, $F^e(\theta)$, is called the active element pattern of the array. In general, this pattern will be different from the isolated element pattern $f_0(\theta)$ because adjacent elements will radiate some power due to mutual coupling with the fed element. Also, $F^e(\theta)$ will depend on the position of the fed element in the array so that, for example, edge elements will have different active element patterns than elements near the center of the array. If the array is large, however, most of the elements will see a uniform neighboring environment, and $F^e(\theta)$ can be approximated as equal for all elements in the array.

The utility of the active element pattern comes from the fact that, if all the active element factors can be approximated as equal, then the pattern of the fully excited array of Fig. 1 can be expressed as the product of the active element factor and the array factor, in an analogous fashion to ordinary array theory. In this case, however, all mutual coupling effects are completely accounted for, including the possibility of scan blindnesses. In fact, as will be shown below, the realized gain of the fully excited array at a given scan angle is proportional to the active element pattern gain at this same angle. In addition, the active element pattern at a given angle is also simply related to the active reflection coefficient magnitude of the fully excited array scanned to that same angle.

The importance of these results comes from the fact that the direct measurement of the scanning characteristics of a large phased array antenna is generally very expensive, involving the need for a complete power divider network and a set of phase shifters for each element. However, the measurement of an active element pattern is much simpler, involving only a reasonably large portion of the proposed array with matched loads on all but one of the elements. Measurements of the active element patterns can thus be used to locate and correct array design problems before the full-scale system is fabricated, reducing the risk of a costly design failure. Modern moment method solutions can be used to compute active element patterns [7], but such an effort is probably not as efficacious as the direct solution of the corresponding infinite array problem, which gives more directly the desired scan characteristics of the array under consideration.

Manuscript received December 8, 1993; revised April 6, 1994.
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IEEE Log Number 9403666.

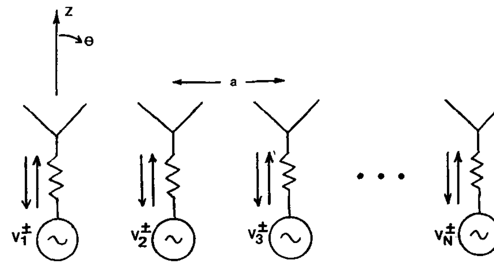

Fig. 1. Geometry of a uniform N-element linear array.

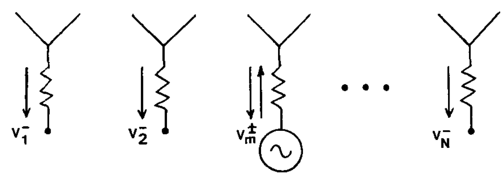

Fig. 2. Defining geometry for the active element pattern of a uniform N-element array.

As an example, Fig. 3 shows the active and isolated E-plane element gain patterns for an infinite planar printed dipole array. The active element pattern shows a scan blindness at an angle of about 46°, while the isolated element pattern shows no such nulls. Due to surface wave loss, the isolated element efficiency is 0.79, which has the effect of lowering its broadside gain. There is no surface wave loss for the infinite array, except at the blindness angle [7].

II. Derivation of the Active Element Pattern Relation

Here we provide a derivation of the basic active element array pattern relationships using scattering parameters. For the N-element array of Fig. 1, we assume that the elements can be characterized by a single radiation mode (eg., short dipoles, thin slots, etc.), so that the radiated fields from a single element can be expressed in terms of its terminal voltage V_o as

$$E_o = V_o f_o(\theta)\frac{e^{-jkr}}{r}, \quad H_o = \frac{V_o}{\eta_o} f_o(\theta)\frac{e^{-jkr}}{r}. \tag{1}$$

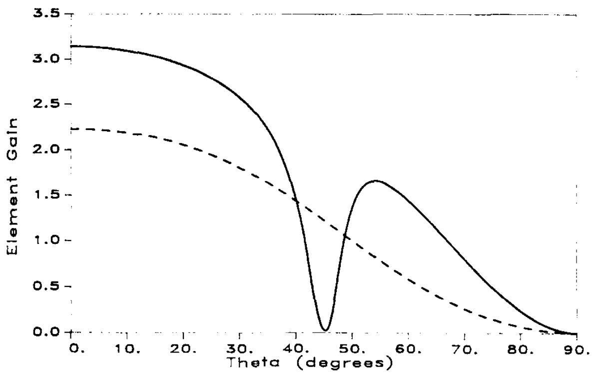

Fig. 3. Active (——) and isolated (- - -) E-plane element gain patterns for an infinite planar dipole array. Substrate thickness is $0.19\lambda_o$, substrate dielectric constant is 2.55, and element spacing is $0.5\lambda_o$ in both planes of the array.

The element feed ports can then be characterized with an $N \times N$ scattering matrix whose elements are

$$S_{mn} = \left.\frac{V_m^-}{V_n^+}\right|_{V_k^+=0 \text{ for } k\neq n} \tag{2}$$

where V_m^+ and V_m^- are the incident and reflected voltage wave amplitudes at the mth element. The incident and reflected voltages are related as

$$V_m^- = \sum_{n=1}^{N} S_{mn} V_n^+ \tag{3}$$

and the total voltage and current at the nth element are

$$V_n = V_n^+ + V_n^-, \tag{4a}$$

$$I_n = I_n^+ + I_n^- = V_n^+ - V_n^- \tag{4b}$$

for a normalized feed line characteristic impedance.

For scanning the array to the angle θ_o we phase the incident excitations as

$$V_n^+ = V_o e^{-jkna\sin\theta} \tag{5}$$

which models the practical case of constant incident power to each element, as in a corporate-fed array. The active reflection coefficient at the mth element is, from (3) and (5),

$$\Gamma_m(\theta_o) = \frac{V_m^-}{V_m^+} = e^{jkma\sin\theta_o}\sum_{n=1}^{N} S_{mn} e^{-jkna\sin\theta_o}. \tag{6}$$

Note that Γ_m depends on the scan angle θ_o unless $S_{mn} = 0$ for $m \neq n$ (no mutual coupling).

The fields radiated by the fully excited array, phased according to (5), are

$$E^a(\theta) = E_o(\theta)\sum_{n=1}^{N} V_n e^{jkna\sin\theta},$$

$$H^a(\theta) = H_o(\theta)\sum_{n=1}^{N} I_n e^{jkna\sin\theta}.$$

Evaluating these at the main beam angle $\theta = \theta_o$ gives

$$E^a(\theta_o) = V_o f_o(\theta_o)\sum_{n=1}^{N}[1 + \Gamma_n(\theta_o)]\frac{e^{-jkr}}{r}, \tag{7a}$$

$$H^a(\theta_o) = \frac{V_o}{\eta_o} f_o(\theta_o)\sum_{n=1}^{N}[1 - \Gamma_n(\theta_o)]\frac{e^{-jkr}}{r}. \tag{7b}$$

The gain of the fully excited array for $\theta = \theta_o$ is then,

$$\begin{aligned} G^a(\theta_o) &= \frac{4\pi r^2}{P_{inc}} Re(E^a H^{a*}) \\ &= \frac{4\pi f_o^2(\theta_o)}{N\eta_o}\left[N^2 - \left|\sum_{n=1}^{N}\Gamma_n(\theta_o)\right|^2\right] \end{aligned} \tag{8}$$

since $P_{inc} = NV_o^2$ for the entire array. Note that this gain definition includes the effect of reflected power, and is usually called the realized gain.

If the array is large enough so that virtually all of the Γ_n's are identical, then $\Gamma_n \simeq \Gamma$, and (8) reduces to

$$G^a(\theta_o) = NG_o(\theta_o)[1 - |\Gamma(\theta_o)|^2] \tag{9}$$

where G_o is the gain of a single isolated element:

$$G_o(\theta) = \frac{4\pi r^2 E_o H_o^*}{V_o^2} = \frac{4\pi f_o^2(\theta)}{\eta_o}. \tag{10}$$

Now consider the active element pattern, where the mth element is driven with voltage V_o and all others are terminated. The radiated fields are

$$\begin{aligned} E_m^e(\theta) &= E_o(\theta)\sum_{n=1}^{N} V_n e^{jkna\sin\theta} \\ &= V_o f_o(\theta)[1 + e^{jkma\sin\theta}\Gamma_m(-\theta)]\frac{e^{-jkr}}{r} \end{aligned} \tag{11a}$$

$$\begin{aligned} H_m^e(\theta) &= H_o(\theta)\sum_{n=1}^{N} I_n e^{jkna\sin\theta} \\ &= \frac{V_o f_o(\theta)}{\eta_o}[1 - e^{jkma\sin\theta}\Gamma_m(-\theta)]\frac{e^{-jkr}}{r} \end{aligned} \tag{11b}$$

where we have used the fact that $S_{nm} = S_{mn}$, which is true for an array of reciprocal elements. The gain of the active element pattern is

$$\begin{aligned} G_m^e(\theta) &= \frac{4\pi r^2}{P_{inc}} Re(E_m^e H_m^{e*}) \\ &= G_o(\theta)[1 - |\Gamma_m(-\theta)|^2]. \end{aligned} \tag{12}$$

If the array is large enough that end effects can be ignored, then from symmetry $\Gamma_m(\theta) = \Gamma_m(-\theta)$, and (12) reduces to

$$G_m^e(\theta) = G_o(\theta)[1 - |\Gamma(\theta)|^2]. \tag{13}$$

This is an interesting result, showing that the active element pattern is simply related to the active reflection coefficient magnitude of the fully excited phased array. Furthermore, comparison with (9) shows that

$$G^a(\theta_o) = NG_m^e(\theta_o) \tag{14}$$

if the array is large enough that the individual active element patterns are approximately identical. This result shows that the gain $G^a(\theta_o)$ of the fully excited array at the scan angle θ_o is directly proportional to the active element pattern at that same angle. Therefore, if the measured active element pattern has dips or nulls at particular angles, it can be concluded that the fully excited phased array will have comparable drops in gain when scanned to these same angles.

The extension to the more general planar array case is obtained from the above results by replacing $G_o(\theta)$ with $4\pi ab\cos\theta/\lambda^2$, which is the maximum gain available from a single rectangular unit cell $a \times b$ in a large uniform planar array. Then we have the familiar results that

$$G^e(\theta_o, \phi_o) = \frac{4\pi ab}{\lambda^2}[1 - |\Gamma(\theta_o, \phi_o)|^2]\cos\theta_o, \quad (15)$$

$$G^a(\theta_o, \phi_o) = NG^e(\theta_o, \phi_o) \quad (16)$$

for an array of N elements scanned to the angle θ_o, ϕ_o. As stated at the beginning of this derivation, we have assumed a single-mode relationship between the element radiation field and its terminal voltage. It is important to realize that these modes exist only on the feed lines, and should not be confused with Floquet, surface, space, or other modes that may be relevant above the aperture of the antenna. The preceding derivation implicitly includes these effects in the relation between the active element pattern and the reflection coefficient magnitude of the fully excited array. But (13)–(16) are also valid when multiple feed line modes are present.

Although not explicitly stated, it must be recognized that the active element pattern is useful only in the above sense when the array elements are fed independently, as in the case of a corporate isolated power divider. If a series-type feeding arrangement is used, multiple reflections on the feed line may introduce effects not accounted for in the above analysis, and the active element pattern will not be applicable in the same sense. Also, while the above results demonstrate the close connection between the active element pattern and the impedance mismatch of the fully excited and scanned phased array, it is important to note that the active element pattern can only be related to the magnitude of the reflection coefficient of the full array, and that it is impossible to determine the complex active input impedance of the phased array from its active element pattern.

III. Intuitive Derivation of Active Element Pattern Relations

Of course, the essential active element pattern relations of (15)–(16) can be developed much more simply if one is willing to accept a less rigorous but more intuitive argument. Thus, if we consider a large uniform planar array, the maximum gain available when the array is scanned to the angle θ_o, ϕ_o is

$$G^{max}(\theta_o, \phi_o) = \frac{4\pi ab}{\lambda^2}N\cos\theta_o \quad (17)$$

where N is the total number of elements in the array, and a, b are the element spacings. Then the realized gain of the array accounting for reflection loss, is

$$G^a(\theta_o, \phi_o) = \frac{4\pi ab}{\lambda^2}N[1 - |\Gamma(\theta_o, \phi_o)|^2]\cos\theta_o. \quad (18)$$

Note that this assumes identical reflections at each element. Now, if the active element pattern $G^e(\theta, \phi)$ is defined as the gain pattern of a singly excited element in the array with all other elements match terminated, then by superposition the gain of the fully excited array in the direction of the main beam can be written as

$$G^a(\theta_o, \phi_o) = NG^e(\theta_o, \phi_o)$$

in agreement with (16). Equation (15) then follows by comparison with (18). Observe that this argument makes no direct mention of mutual coupling, edge effects, or feeding methods. The result in (18) is sometimes used to argue that the "ideal" pattern for a phased array element would be $\cos\theta_o$, since in this case $\Gamma(\theta_o, \phi_o) = 0$ for any scan angle. This implies the physically unrealizable result that the array operates with no mutual coupling, however.

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Simple and Accurate Formula for the Resonant Frequency of the Equilateral Triangular Microstrip Patch Antenna

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***Abstract*—An approximate solution of the capacitance of circular microstrip disk to account for the fringing fields is used for calculation of the resonant frequency formula of the equilateral triangular microstrip patch antenna. The theoretical results are in good agreement with experimental data.**

I. Introduction

The region between the microstrip and the ground plane can be treated as a cavity bounded by magnetic walls along the edges and by electric walls from above and below, then edges extended slightly to account for the fringing fields. These extensions depend on the planar dimensions of the patch, relative dielectric constant, dielectric thickness, and the field distribution at the peripheries. It is difficult to estimate the exact extensions for patches. However, for a given shape, the extension available for the shape closest to the given shape can be used. For example, for a pentagonal patch, Suzuki and Chiba [1] have used the same extension as applicable to a circular disk of the same planar area as that of a given pentagonal patch.

II. Calculations

Several papers [1]–[3] have reported on the resonant frequency of the equilateral triangular microstrip patch. These calculated formulas are based on the effective radius a_e of a circular disk given by [4]

$$a_e = a\left[1 + \frac{2h}{\pi\epsilon_r a}\left\{\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right\}\right]^{\frac{1}{2}} \quad (1)$$

Manuscript received March 20, 1992; revised April 1, 1994.

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IEEE Log Number 9403667.

Comments on Corrections to "An Improved Design Method of Rotman Lens Antennas"

Takashi Katagi, Seiji Mano, and Shin-ichi Sato

For our paper,[1] corrections were suggested in [1] and [2]. In [1], it was suggested that the term $-2g$ is missing on the right side of the expression for b in (4b) of our paper.[1] In [2], it was suggested that the term $2g(a_0-1)/(g-a_0)$ in the same expression for b should be replaced by $2g(g-1)/(g-a_0)$. As pointed out in [1], it is sure that the term $-2g$ is missing on the right side of expression for b in (12) of W. Rotman *et al.*'s paper [3]. By considering the term $-2g$ for b in (12) in [3], it is easily shown that $2g(g-1)/(g-a_0)-2g=2g(a_0-1)/(g-a_0)$. In our paper, $2g(g-1)/(g-a_0)$ is not used, but $2g(a_0-1)/(g-a_0)$ for the expression for b in (4b). So, it is concluded that the expression (4b) of our paper is correct.

Authors' Reply by Pramod K. Singhal and Pramod C. Sharma

Our paper [2], "Correction to 'An Improved Design Method of Rotman Lens Antennas'" is based on [1] being the latest correction reported on in [3]. As pointed out in [2], it needs correction. We agree with the authors' reply that the corrected version in [2] is the same as the original work. In short, the correction [1] was not needed.

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Manuscript received October 24, 1994.
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IEEE Log Number 9411459.

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Comments on "The Active Element Pattern"

R. C. Hansen

A review of "The Active Element Pattern"[1, 2] is always useful, but that Communication is incorrect in saying that the subject is not covered in handbooks. A discussion of active element pattern is in Section 10.3.1 of the widely used *Handbook of Antenna Design*, by Rudge, Milne, Olver, and Knight, Eds. [1]. The key equations (9), (12), and (13) of the Communication by Pozar are, in fact, contained in Ch. 10 of the handbook.

The derivation of (9) is incomplete, as shown in Ch. 10 of the handbook

$$R_a|1-\Gamma(\theta)|^2 = R_g[1-|\Gamma(\theta)|^2]$$

only if $X_g \equiv 0$, *i.e.*, the generator impedance is real. R_a is the scan resistance and R_g is the generator resistance.

Because the adjective "active" carries electron device connotations, more appropriate terms are coming into use. Scan element pattern, and scan impedance are replacing active element pattern and active impedance; see, for example, [2] and [3].

References

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Manuscript received November 28, 1994.
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[1] D. M. Pozar, *IEEE Trans. Antennas Propagat.*, vol. 42, no. 8, pp. 1176–1178, Aug. 1994.

[2] Editor's Note: "The Active Element Pattern" was submitted for review as a Tutorial/Review article. It was handled as such in the review process and was scheduled to be designated as such in the TRANSACTIONS. Due to an oversight in the publication phase, this designation was mistakenly not printed.