Array Directivity and Aperture Efficiency

For high gain apertures where the far-field pattern may be expressed as a Fourier transform of an aperture illumination $u(\xi,\eta)$, it is possible to show the beam coupling factor τ is also given by [4]

$$\tau = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_3(\xi,\eta)\cdot u_4{}^*(\xi,\eta)\, d\xi\, d\eta. \tag{41}$$

Where u_3 and u_4 are the aperture illuminations for the patterns f_3 and f_4, respectively, ξ and η are the aperture coordinates, and the illuminations are normalized such that

$$\iint_{-\infty}^{\infty} |u(\xi,\eta)|^2\, d\xi\, d\eta = 1. \tag{42}$$

It is further possible to show that for broadside excitation ($\delta = 0$), (34) is equivalent to

$$D = \frac{4\pi}{\lambda^2}\frac{|\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_{\text{tot}}(\xi,\eta)\, d\xi\, d\eta|^2}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |u_{\text{tot}}(\xi,\eta)|^2\, d\xi\, d\eta}. \tag{43}$$

Where $u_{\text{tot}}(\xi,\eta) = u_3(\xi,\eta) + u_4(\xi,\eta)$. This is the standard equation for the directivity of an aperture antenna. Therefore, the beam coupling factor τ in (34) reconciles array gain and area, resolving the element gain paradox discussed by Hannan [5].

Discussion and Conclusions

Throughout this paper it has been assumed that the embedded element pattern is "given" and, therefore, τ is a known constant as provided by (10). In reality, the element patterns and τ will change as a function of S_{11} and S_{12}. A good illustration of this follows directly from (19): for closely spaced antennas with $\tau > 0$, if the antennas are tuned such that $S_{11} = 0$, then $|S_{31}|^2 = 0$ and there is zero far-field power flow! Since this is obviously not realistic, what must happen is that a perfect passive match must cause the element patterns to change by squinting substantially off boresight in opposite directions, such that τ becomes zero. But the fact that the element pattern can change does not invalidate the analysis, which essentially shows a quantitative interrelationship between any element pattern, mutual coupling, and passive reflection. Even in the example just given, the theory provides useful information; it says that a "perfect" passive match will inevitably result in a *severe* distortion of the element patterns.

A second example of the interdependence of the embedded element pattern and the coefficients S_{11} and S_{12} is provided by the Hansen–Woodyard excitation example given previously. Over a narrow frequency band and for a fixed scan at endfire it will always be possible to add a matching section to tune out the active reflected wave and achieve a perfect match in the active mode. The matching section evidently must change the embedded element pattern such that τ becomes equal to zero and D_{el} is increased by the excess gain over the assumed isotropic element gain. Again in this case the theory still provides useful information: 1) the optimum mutual coupling as defined by (31) is still optimum in the sense that the reflected wave that must be tuned out is minimized: 2) it provides a quantitative value of the minimum VSWR which will occur between the matching device and the element (24:1 in the numerical example given previously); 3) the quantitative value of the minimum reflected wave plus a knowledge of the location of the matching section can be used to estimate the bandwidth of the system.

In an array design problem it is typically known what embedded element pattern is desired, and the theory provides quantitative results on what values of coupling and passive match are consistent with the desired pattern and element spacing. The examples of Fig. 2 show that τ is not highly sensitive to changes in element pattern beamwidth, and that the dominant variation is due to element spacing. It may be speculated that a major change in τ implies a major change in the element pattern. As a specific example, if it is desired to have an element pattern with a 75° half-power beamwidth, and the element spacing becomes 0.4λ at the low end of the frequency band, Fig. 2 indicates a value of $\tau = 0.57$ for this case. The theory says that it is possible to achieve a zero active reflection for this case *only* if the passive reflection coefficient and the mutual coupling are exactly equal to 0.43 in magnitude (passive VSWR of 2.5:1) and opposite in phase. It may be concluded that any deviation from these values will either cause a nonzero active reflection coefficient or an element pattern different from the desired pattern.

Acknowledgment

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Decoupling and Descattering Networks for Antennas

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Abstract—**The possibilities of connecting a lossless network between input ports and antenna ports such that there is no coupling and scattering between the antennas are discussed. A necessary condition for complete decoupling and descattering is power orthogonality between the patterns of the individual antennas. Numerical and experimental results are presented for monopole antennas.**

I. Introduction

Mutual interaction between individual antennas is responsible for many effects, which often are undesirable. Some of the mechanisms are indicated schematically on Fig. 1(a), which

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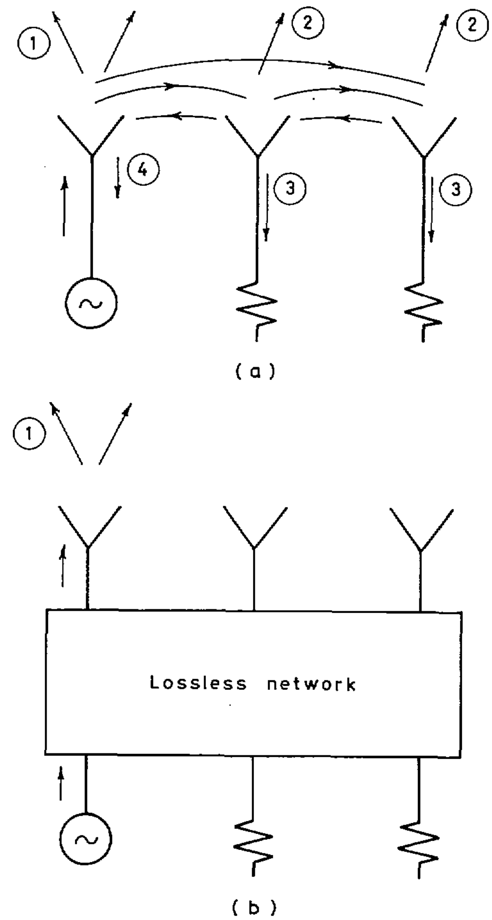

Fig. 1. Schematic representation of interaction between antennas. (a) Without network. (b) With network.

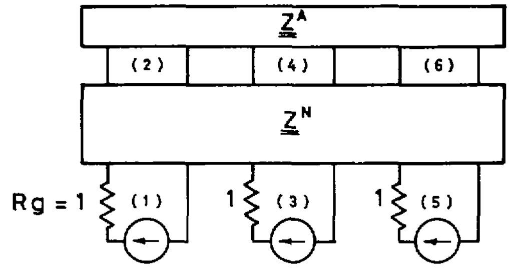

Fig. 2. Network model of interacting antennas and decoupling network N.

shows a three-element array which could be a small phased array (the element pattern) or a feed cluster for a multibeam reflector or lens antenna. One antenna is excited and the others resistively terminated. (1) symbolizes the wanted radiation from the excited element when alone. Due to the interaction between the elements there will be some scattering (2), either induced directly or indirectly through other elements. The scattered field may or may not have the same far-field pattern as (1), but will in any case distort the primary field, maybe both in amplitude, phase and polarization. In the multibeam case, (2) will lead to crosstalk between the beams. (3) represents the power coupled into the loads of adjacent antennas and is usually referred to as the coupling loss. (3) is also related to the active impedance of a phased array. (4) represents the power coupled back into the source, leading to a mismatch.

The object of this paper is to find a $2N$-port lossless network, inserted between the N antenna ports and the N input ports, having the effect of decoupling and descattering the system completely, such that one input port excites only one antenna. The remaining antennas are not radiating at all and no power is lost in the loads (Fig. 1(b)).

Previous efforts in this direction have been directed towards cancelling only the coupling loss for phased arrays, thereby achieving a scan-independent impedance match [1]–[3]. It has in fact been proved that it is theoretically possible to match the active impedance of an element in an infinite phased array by means of an infinite set of connecting circuits [3]. It is important to note that the coupling loss cancellation is a less stringent requirement than the one mentioned above, since contribution (2) is not necessarily zero in the phased-array case.

The new and important aspects of the networks to be presented are the descattering aspects, while the decoupling problems seem to be solved automatically. There are some necessary conditions which should be satisfied in order that descattering may take place. An obvious one is that the scattering pattern of an element should equal the transmit pattern, since we want to cancel the scattered field from an element over all real angles by exciting the antenna port of that element with a proper wave. This condition is a severe limitation, since only minimum scattering antennas have this property. In practice, this means that the technique is useful for simple wire- and slot-antennas or similar one-mode antennas. It is also for these antennas that a decoupling and descattering network is most needed, though.

II. A Necessary Condition for Descattering and Decoupling

We assume that the interaction between the N identical antennas is described by an $N \times N$ impedance matrix, $\boldsymbol{Z}^A$. We also assume that the antennas are matched when isolated, such that

$$Z_{ii}{}^A = Z_0 = R_g = 1 \tag{1}$$

where Z_0 is the characteristic impedance of the feed lines and R_g is the impedance of the generators, all normalized to unity. Fig. 2 shows the configuration, where $\boldsymbol{Z}^N$ is the impedance matrix for the lossless network; even-numbered ports are connected to the antenna input ports, odd-numbered ports to the excitations.

Writing out the network equations, we find

$$\begin{aligned}
V_1 &= Z_{11}{}^N I_1 + Z_{12}{}^N I_2 + Z_{13}{}^N I_3 + \cdots \\
V_2 &= Z_{12}{}^N I_1 + Z_{22}{}^N I_2 + Z_{23}{}^N I_3 + \cdots \\
&= -Z_{22}{}^A I_2 - Z_{24}{}^A I_4 - \cdots \\
V_3 &= Z_{13}{}^N I_1 + Z_{23}{}^N I_2 + Z_{33}{}^N I_3 + \cdots \\
V_4 &= Z_{14}{}^N I_1 + Z_{24}{}^N I_2 + Z_{34}{}^N I_3 + \cdots \\
&= -Z_{24}{}^A I_2 - Z_{44}{}^A I_4 - \cdots . \\
&\quad \vdots
\end{aligned} \tag{2}$$

In (2) the currents flowing into the network are assumed positive. The equations for the even-numbered ports are now rearranged, such that the antenna self-impedances are isolated

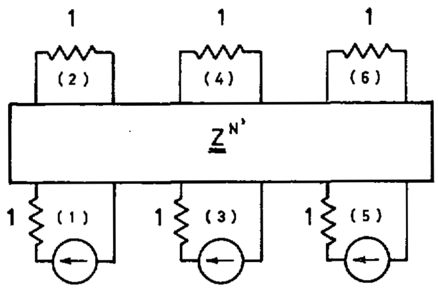

Fig. 3. Network model, where mutual antenna impedances are included in network N'.

on one side, or

$$\begin{aligned} V_1 &= Z_{11}{}^N I_1 + Z_{12}{}^N I_2 + Z_{13}{}^N I_3 + Z_{14}{}^N I_4 + \cdots \\ -Z_{22}{}^A I_2 &= Z_{12}{}^N I_1 + Z_{22}{}^N I_2 + Z_{23}{}^N I_3 \\ &\quad + (Z_{24}{}^N + Z_{24}{}^A) I_4 + \cdots \\ V_3 &= Z_{13}{}^N I_1 + Z_{23}{}^N I_2 + Z_{33}{}^N I_3 + Z_{34}{}^N I_4 + \cdots \\ -Z_{44}{}^A I_4 &= Z_{14}{}^N I_1 + (Z_{24}{}^N + Z_{24}{}^A) I_2 + Z_{34}{}^N I_3 \\ &\quad + Z_{44}{}^N I_4 + \cdots . \end{aligned} \tag{3}$$

Equations (3) describe the situation of Fig. 3, where we have a network N' loaded with the antenna self-impedances. A complete lossless decoupling and descattering will result if N' has the following impedance matrix:

$$\boldsymbol{Z}^N = \begin{Bmatrix} Z_{11} & Z_{12} & 0 & 0 & 0 & 0 & \cdot \\ Z_{12} & Z_{22} & 0 & 0 & 0 & 0 & \cdot \\ 0 & 0 & Z_{33} & Z_{34} & 0 & 0 & \cdot \\ 0 & 0 & Z_{34} & Z_{44} & 0 & 0 & \cdot \\ 0 & 0 & 0 & 0 & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{Bmatrix}. \tag{4}$$

where all the elements are reactive.

It follows from (3) that $Z_{ij}{}^A$ $(i \neq j)$ should also be reactive, which means that we have the following necessary condition for lossless decoupling and descattering. *All the mutual antenna impedances should be reactive.*

This is a more severe constraint than the one about scattering patterns, since the distances between elements have to be fixed such that the mutual impedances are reactive. Furthermore, the condition can only be satisfied exactly at one frequency, although it can be satisfied approximately within a certain band of frequencies. The condition has been derived in detail because we also get the matrix $\boldsymbol{Z}^N$ as an important by-product, but it could have been stated immediately from previous work on multibeam antennas and minimum scattering antennas. Consider the antenna structure in Fig. 1(b) as a multibeam antenna where the individual beam is the radiation pattern of the single antenna. According to Allen [4] and White [5] the beams should be orthogonal over real angles in order to get a lossless network. For minimum scattering antennas the real part of the mutual impedance is proportional to the orthogonality integral [6], thus real $(Z_{ij}{}^A)$ must equal zero.

This means that the distances for which descattering is possible only depend on the power pattern of the element. For rotationally symmetric power patterns of the type

$$P(\theta) = \cos^N(\theta) \tag{5}$$

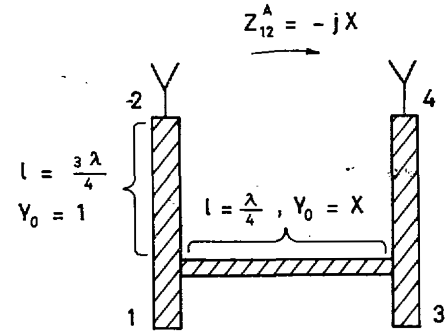

Fig. 4. Transmission line network for two antennas realizing Y^N (eq. (8)).

it may be found [7] that R_{12} equals zero for distances d satisfying

$$J_{(N+1)/2}(k_0 d) = 0 \tag{6}$$

where $J_{(N+1)/2}(x)$ is the Bessel function of order $(N + 1)/2$ and k_0 is the free-space wavenumber. For isotropic antennas ($N = 0$) this leads to $k_0 d = n\pi$, which means that a linear array of isotropic antennas with a spacing $\lambda/2$ may be decoupled and descattered completely.

The example is important, since it shows that for linear arrays it is possible at least in theory to avoid the $\cos\theta$ factor, which is a fundamental limiting factor for the element pattern in planar phased arrays.

III. Realization of Network

To achieve complete decoupling and descattering the impedance matrix $\boldsymbol{Z}^N$ for the network is given by (3) and (4),

$$\boldsymbol{Z}^N = \begin{Bmatrix} Z_{11} & Z_{12} & 0 & 0 & 0 & 0 & \cdot & \cdot \\ Z_{12} & Z_{22} & 0 & -Z_{24}{}^A & 0 & -Z_{26}{}^A & \cdot & \cdot \\ 0 & 0 & Z_{33} & Z_{34} & 0 & 0 & \cdot & \cdot \\ 0 & -Z_{24}{}^A & Z_{34} & Z_{44} & 0 & -Z_{46} & 0 & \cdot \\ 0 & 0 & 0 & 0 & Z_{55} & Z_{56} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{Bmatrix}. \tag{7}$$

For the synthesis of the network it is convenient to work with the admittance matrix, $\boldsymbol{Y}^N$, which may be found directly from (7) in the special case of $Z_{ii} = 0$, $Z_{12} = Z_{34} = Z_{56} = \cdots = j$, $Z_{24}{}^A = jX_{24}{}^A$, $Z_{26}{}^A = jX_{26}{}^A, \cdots$

$$\boldsymbol{Y}^N = \begin{Bmatrix} 0 & -j & -jX_{24}{}^A & 0 & -jX_{26}{}^A & 0 & \cdot \\ -j & 0 & 0 & 0 & 0 & 0 & \cdot \\ -jX_{24}{}^A & 0 & 0 & -j & \cdot & \cdot & \cdot \\ 0 & 0 & -j & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{Bmatrix}. \tag{8}$$

This network may be realized by connecting TEM-lines of length $\lambda/4$ or $3\lambda/4$, depending on the sign of $X_{ij}{}^A$, and characteristic admittance $|X_{ij}|$ between the feeding lines i and j at points $3\lambda/4$ from the antenna port. An example for two antennas is shown in Fig. 4.

IV. Numerical and Experimental Results

The simplest case with a high degree of coupling is an array consisting of monopoles. The monopoles considered have the following data at the resonance frequency:

radius a: $a/\lambda_0 = 0.001$
length L: $L/\lambda_0 = 0.239$
spacing D: $D/\lambda_0 = 0.43$.

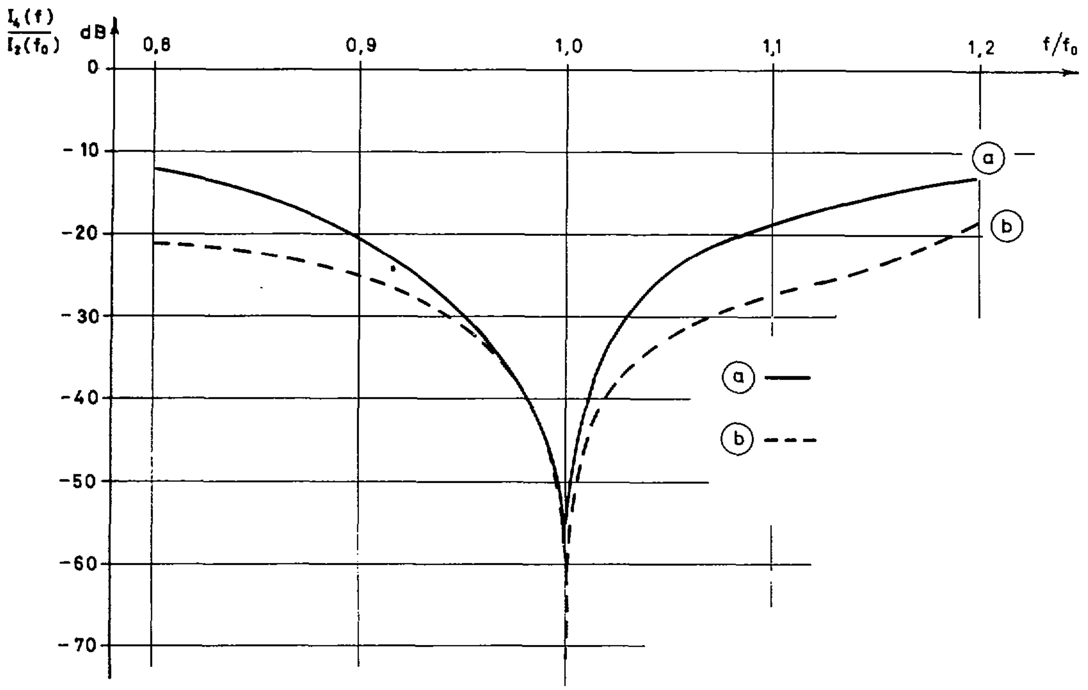

Fig. 5. Numerical results for descattering between two monopoles, $D/\lambda_0 = 0.43$. ⓐ: Network as shown in Fig. 4. ⓑ: Hypothetical case with $X_{12}^A = 0$ and no network.

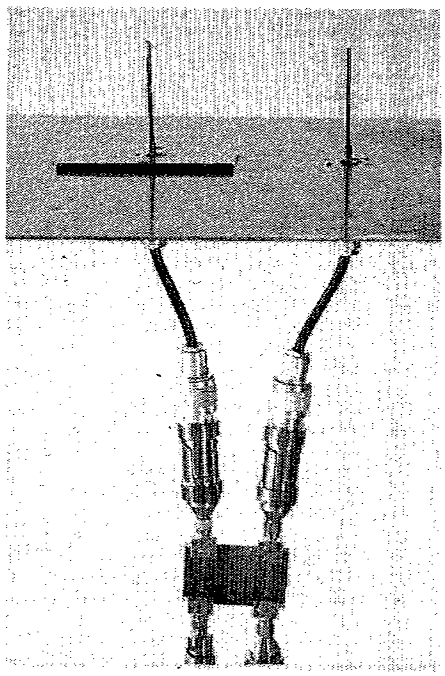

Fig. 6. Experimental setup with two monopoles and stripline network (top plane removed).

The length and spacing are chosen such that the antenna self-impedances are real and the mutual impedances reactive, when two elements are considered.

Fig. 5, curve ⓐ, shows some numerical results for the normalized antenna current of the scattering antenna as a function of frequency. The network is a transmission line network as shown in Fig. 4. The perfect descattering at the resonance frequency is clearly shown, but the bandwidth is rather small, about 8 percent for a descattering less than 30 dB. The limited bandwidth is partly due to the changing self-impedance of the monopoles, and partly due to the nonzero real part of the mutual impedance. Curve ⓑ shows a hypothetical case of broadband antennas, where R_{12} is the only coupling impedance, leading to a slightly larger bandwidth. Curve ⓑ represents a fundamental physical limit due to the nonorthogonality of the radiation patterns.

The experimental network was constructed in stripline, as shown on Fig. 6. Since antenna currents are difficult to measure, the horizontal radiation patterns were measured over the groundplane shown in Fig. 7(a)–(c). In order to avoid finite groundplane effects, the measurements were taken on the groundplane in the near field of the array. The radiation pattern shown in Fig. 7(a) is for the isolated antenna, while that in Fig. 7(b) includes a

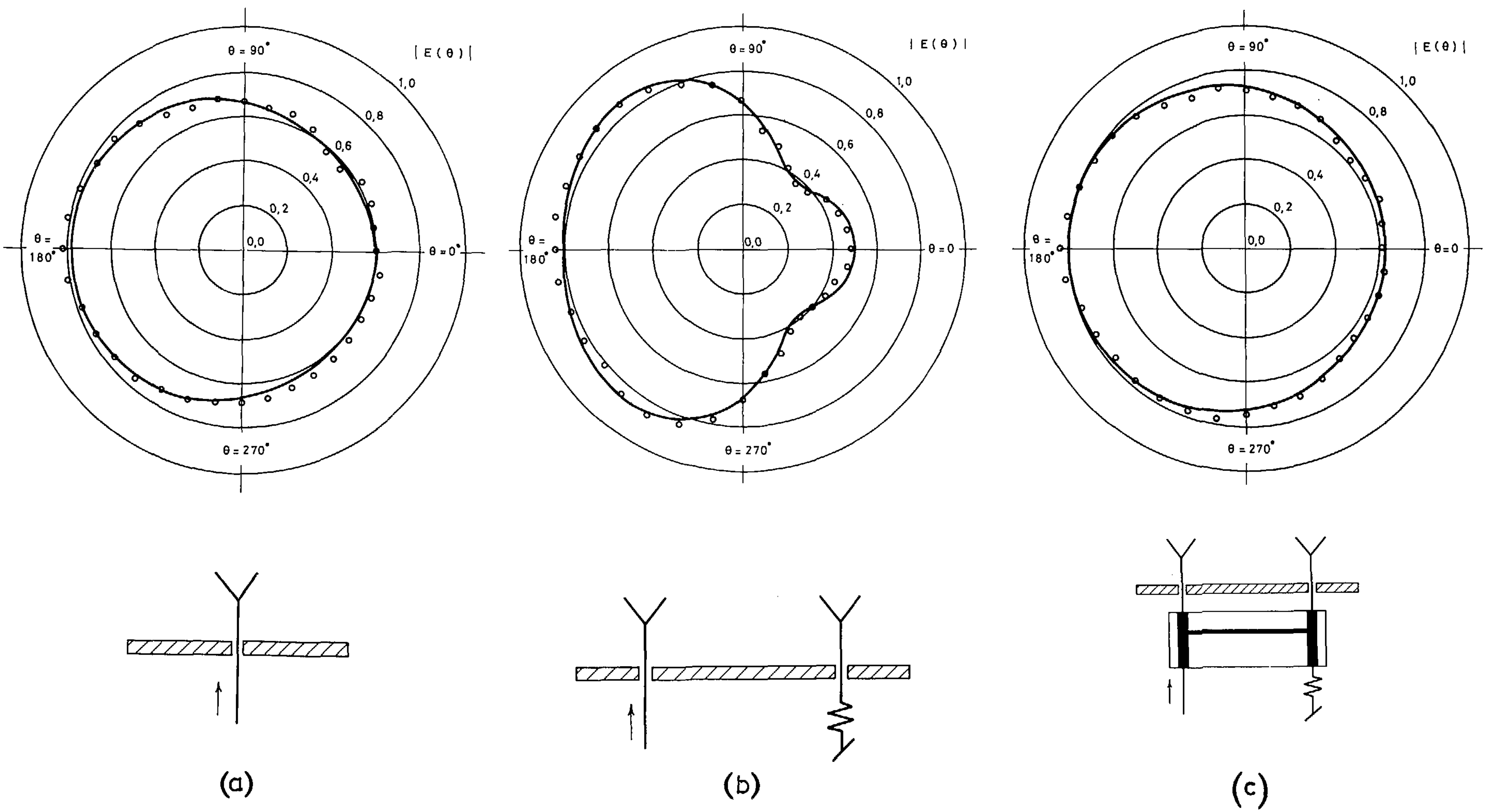

Fig. 7. Horizontal radiation pattern measured over ground-plane at resonance frequency, f_0 = 1.5 GHz. (a) Single monopole. (b) Two monopoles, one resistively terminated. (c) Two monopoles, with network included.

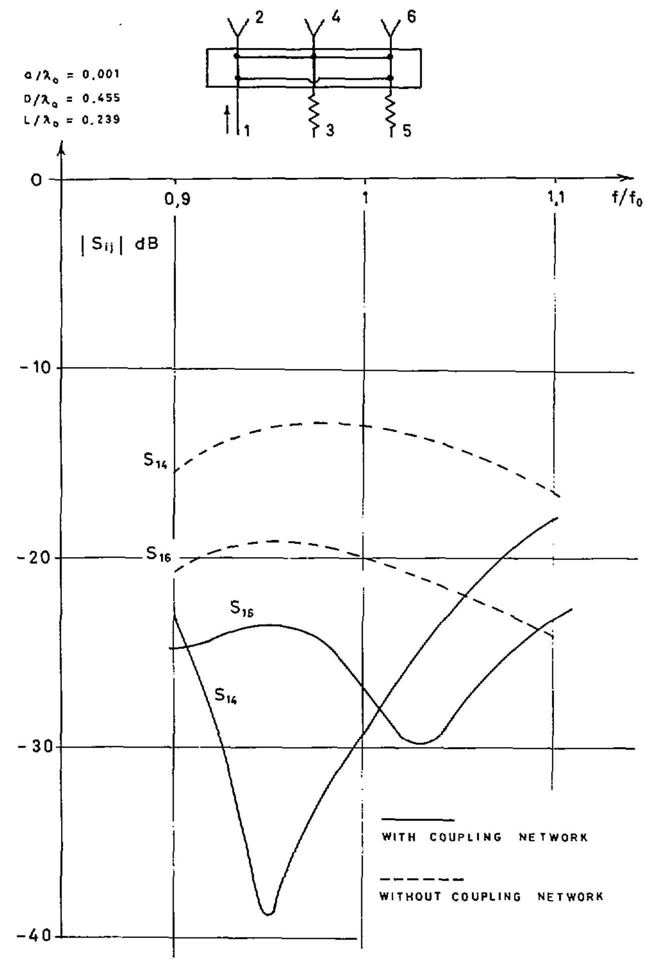

Fig. 8. Numerical results for descattering of three-element linear array of monopoles, with and without network. Network is nonplanar.

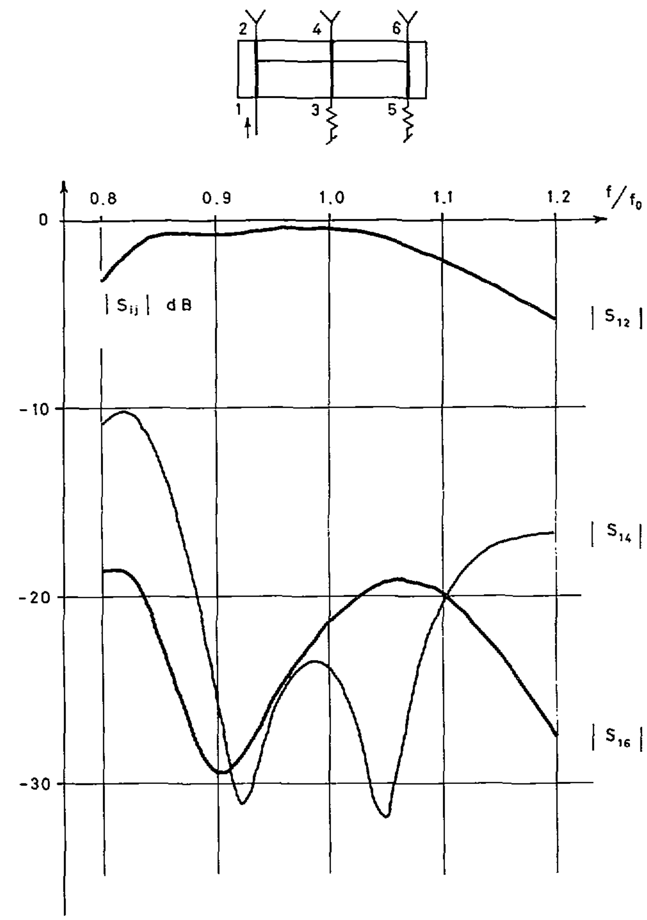

Fig. 9. As in Fig. 8 except that network is planar.

matched, parasitic antenna, but without the network. The pattern in Fig. 7(c) results when the network is included. It is evident that the two antennas are now completely independent. The coupling into the load is very similar to the scattering curves in Fig. 5.

A linear array of three monopoles cannot be descattered completely since a reactive mutual impedance between the two outer elements and adjacent elements cannot be obtained simultaneously. Fig. 8 shows what can be achieved with a nonplanar network and Fig. 9 with a planar network (no direct coupling between the outer lines).

Figs. 8 and 9 are a result of an optimization of the structure, where the deviation of the complete scattering matrix from the ideal scattering matrix at $f = f_0$ is minimized. The scattering coefficients shown are really the antenna currents normalized to the incident current at port 1 under matched conditions.

V. Conclusion

It has been shown theoretically and experimentally that the effects of mutual interaction between two or more antennas may be completely removed by a simple network connecting the lines feeding the antennas. Two conditions must be satisfied: 1) the scattering pattern should equal the transmit pattern for an antenna and 2) the patterns of two antennas should be orthogonal in a multibeam sense, i.e., the mutual impedance should be reactive. The connecting network may consist of transmission lines with a characteristic impedance different from the feeding lines.

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Linearly Polarized Microstrip Antennas

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***Abstract*—An equivalent network for square and rectangular shaped microstrip radiating elements is derived. In order to simplify the problem the radiating element is considered as two slots separated by a transmission line of low characteristic impedance. The slots are characterized**

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