As the frequency increases, the local-mode solution becomes more complicated because more surface waves then contribute. The plane-wave solution, however, becomes steadily more efficient and tends towards the geometric-optical solution.

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An Alternate Frill Field Formulation

CHALMERS M. BUTLER AND LEONARD L. TSAI

***Abstract*—An alternate form [1] for the near zone E_z fields of an annular ring of magnetic current is derived. The new expression avoids numerical differentiation, offers significant improvements in efficiency, and is particularly useful in the analysis of coaxially driven parallel arrays.**

In [1] the near fields for an annular ring of magnetic current (frill) are obtained by applying numerical differentiation to the electric vector potential F which, for the rotationally symmetric frill, possesses only a nonzero ϕ component F_ϕ. If one proceeds with the vector potential formulation in a direct manner, certain simplifications result in the E_z component of the field so that numerical differentiation can be avoided and one is able to obtain E_z everywhere by evaluating numerically a rather simple integral. Knowledge of E_z is of particular importance in analyses of coaxial-aperture driven arrays of parallel monopoles above a ground plane.

For the magnetic frill current [see Fig. 1]

$$M' = \hat{u}_{\phi'} \frac{-1}{\rho' \ln (b/a)}$$

the electric vector potential is given by

$$F(r) = \frac{\epsilon}{4\pi} \iint_{\text{frill surface}} M'(r') \frac{\exp(-jk\,|r - r'|)}{|r - r'|}\, ds' \tag{1}$$

with the corresponding electric field given by

$$E = -\frac{1}{\epsilon} \operatorname{curl} F. \tag{2}$$

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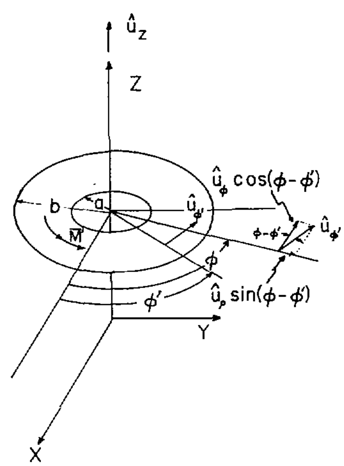

Fig. 1. Frill geometry.

When the magnetic frill is parallel to the xy plane and concentric with the z axis and the geometry of the problem is as depicted in Fig. 1, (1) reduces to

$$F(\rho,\phi,z) = -\frac{\epsilon}{4\pi \ln (b/a)} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} \frac{\hat{u}_{\phi'}(\phi')}{\rho'} \frac{\exp(-jkR)}{R} \rho'\, d\phi'\, d\rho' \tag{3}$$

where

$$R = [\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi') + (z - z')^2]^{1/2}. \tag{4}$$

It is to be noted that the source is in the ϕ' direction and has a projection onto the observation coordinates implied by

$$\hat{u}_{\phi'} = \hat{u}_\rho \sin(\phi - \phi') + \hat{u}_\phi \cos(\phi - \phi') \tag{5}$$

where the terms in (5) are defined in Fig. 1. Thus from (3)

$$F_\phi = -\frac{\epsilon}{4\pi \ln (b/a)} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} \cos(\phi - \phi') \frac{\exp(-jkR)}{R}\, d\phi'\, d\rho' \tag{6}$$

and from (2) the z component of the electric field is seen to be

$$E_z = \left(-\frac{1}{\epsilon} \operatorname{curl} F\right) \cdot \hat{u}_z = -\frac{1}{\epsilon}\left[\frac{\partial F_\phi}{\partial \rho} + \frac{1}{\rho} F_\phi\right] \tag{7}$$

since $F_\rho = 0$ everywhere. Using the form of F_ϕ given in (6), one obtains the derivative needed in (7):

$$\frac{\partial F_\phi}{\partial \rho} = -\frac{\epsilon}{4\pi \ln (b/a)} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} \cos(\phi - \phi') \cdot \frac{\partial}{\partial R}\left[\frac{\exp(-jkR)}{R}\right] \frac{\partial R}{\partial \rho}\, d\phi'\, d\rho'. \tag{8}$$

One integration of (6) by parts with respect to ϕ' and the observation that both $\sin(\phi - \phi')$ and R are periodic in ϕ' with period 2π enable one to write F_ϕ in the following form:

$$F_\phi = -\frac{\epsilon}{4\pi \ln (b/a)} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} \sin(\phi - \phi') \frac{\partial}{\partial R}\left[\frac{\exp(-jkR)}{R}\right] \frac{\partial R}{\partial \phi'} \cdot d\phi'\, d\rho'. \tag{9}$$

TABLE I
COMPARISON OF CALCULATED E_z FOR FRILL WITH $a = 0.003\lambda$ AND $b = 0.005\lambda$ ALONG 45° LINE

$z = \rho$(in λ)	Improved Form (14)	Numerical Differentiation [1, eq. (12)]
0.0005	$0.2047159E02 - j0.1029276E - 03$	$0.2046579E02 - j0.1119673E - 03$
0.0015	$0.1688536E02 - j0.1029251E - 03$	$0.1688470E02 - j0.1044982E - 03$
0.0025	$0.9610034E01 - j0.1030192E - 03$	$0.9614460E01 - j0.1049856E - 03$
0.0035	$0.4387174E01 - j0.1031420E - 03$	$0.4387884E01 - j0.1037690E - 03$
0.0045	$0.2015952E01 - j0.1032633E - 03$	$0.2015988E01 - j0.1039268E - 03$
0.0055	$0.1019608E01 - j0.1032623E - 03$	$0.1019675E01 - j0.1042650E - 03$
0.0065	$0.5716718E00 - j0.1029731E - 03$	$0.5715361E00 - j0.1031028E - 03$
0.0075	$0.3493902E00 - j0.1029350E - 03$	$0.3491055E00 - j0.1038956E - 03$
0.0085	$0.2285606E00 - j0.1027498E - 03$	$0.2286559E00 - j0.1037456E - 03$
0.0095	$0.1576996E00 - j0.1028112E - 03$	$0.1576202E00 - j0.1035746E - 03$

Substitution of (8) and (9) into (7) and replacement of $\partial R/\partial\phi'$ and $\partial R/\partial\rho$ by their respective equivalents,

$$\frac{\partial R}{\partial \phi'} = -\frac{\rho\rho'}{R}\sin(\phi - \phi') \tag{10}$$

and

$$\frac{\partial R}{\partial \rho} = \frac{\rho - \rho'\cos(\phi - \phi')}{R} \tag{11}$$

reduce E_z to

$$E_z = -\frac{1}{4\pi \ln(b/a)} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} \frac{\partial}{\partial R}\left[\frac{\exp(-jkR)}{R}\right] \cdot \left\{\frac{\rho' - \rho\cos(\phi - \phi')}{R}\right\} d\phi'\, d\rho'. \tag{12}$$

In (12), the term in the braces is recognized to be $\partial R/\partial\rho$; hence, E_z simplifies to

$$E_z = \frac{-1}{4\pi \ln(b/a)} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} \frac{\partial}{\partial R}\left[\frac{\exp(-jkR)}{R}\right] \frac{\partial R}{\partial \rho'}\, d\rho'\, d\phi' \tag{13}$$

whose integrand is an exact differential with respect to ρ' and which becomes

$$E_z = \frac{-1}{4\pi \ln(b/a)} \int_{\phi'=0}^{2\pi} \left[\frac{\exp(-jkR)}{R}\right]_{\rho'=a}^{b} d\phi'. \tag{14}$$

Equation (14) is exact and applies for *all observation points* not on the frill proper, but unlike [1, eq. (12)], no numerical differentiation is required. This results in increased efficiency and accuracy since only a simple numerical integration is now needed. Equation (14) reduces readily to the axial form ($\lim \rho \to 0$) given earlier in [1, eq. (25)]. A comparison of the calculated E_z using both (14) and the numerical differentiation form of [1, eq. (12)] is of interest and is presented in Table I. Fields are observed along a 45° line ($\rho = z$) for increasing ρ where one notices that the agreement is quite close. Equation (14) is found to be much more efficient, its evaluation requiring approximately one-tenth the computer time needed for [1, eq. (12)].

The simplification thus achieved for the computation of E_z is perhaps somewhat fortuitous in that the same procedure does not reduce the complexity for calculating E_ρ. Thus the numerical differentiation process given in [1] is still needed for the computation of E_ρ. However, the simpler form for E_z in (14) can certainly be employed to good advantage in a study of ground-plane mounted antennas, particularly, in an analysis of arrays of parallel monopoles such as the Yagi–Uda and log-periodic antennas, where the only knowledge needed of the excitation from the primary frill drive is the z component of electric field E_z on the wire elements.

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The Definition of Cross Polarization

ARTHUR C. LUDWIG

Abstract—**There are at least three different definitions of cross polarization used in the literature. The alternative definitions are discussed with respect to several applications, and the definition which corresponds to one standard measurement practice is proposed as the best choice.**

The use of orthogonal polarization to provide two communications channels for each frequency band has led to interest in the polarization purity of antenna patterns. It is a surprising fact that there is no universally accepted definition of "cross polarization" at the present, and at least three different definitions have been used either explicitly or implicitly in the literature. The *IEEE Standard* [1] definition is "The polarization orthogonal to a reference polarization." For circular polarization this is adequate, but for linear or elliptical polarization the direction of the reference polarization must still be defined.

We will first briefly present the definitions known to the author. Only the case of nominally linear polarization will be considered since the extension to elliptical polarization is straightforward. The three alternative definitions are: 1) in a rectangular coordinate system, one unit vector is taken as the direction of the reference polarization, and another as the direction of cross polarization [2]; 2) in a spherical coordinate system the same thing is done using the unit vectors tangent to a spherical surface [3], [4]; and 3) reference and cross polarization are defined to be what one measures when antenna patterns are taken in the usual manner [2, pp.

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