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A Simple Approximation to the Current on the Surface of an Isolated Thin Cylindrical Center-Fed Dipole Antenna of Arbitrary Length

***Abstract*—The use of a simple trigonometric series to approximate the current distribution on isolated center-fed dipole antennas has been found to give accurate results. Unlike iterative and numerical solutions, the functional form of the approximate distribution permits the simple computation of the axial electric fields and radiating properties of the dipole.**

The basis for many of the recent analyses of thin cylindrical dipole antennas is the integral equation formulated by Erik Hallén [1]

$$\int_{-h}^{h} I_z(z')K(a,z-z')\,dz' = -j(4\pi/\eta)(C_1 \cos kz + (V_0^e/2)\sin k|z|) \quad (1)$$

where η is the intrinsic impedance of the surrounding medium, V_0^e is the generator EMF of the delta function excitation, and $I_z(z')$ is the unknown current distribution. The approximate kernel of the integral equation is given by

$$K(a,z-z') = \frac{\exp\{-jk[(z-z')^2+a^2]^{1/2}\}}{[(z-z')^2+a^2]^{1/2}} \quad (2)$$

and a is the radius of the cylindrical element. Equation (1) is appropriate for the center-driven dipole of infinite conductivity. For the antenna of finite conductivity which is driven off center, the formulation of the integral equation is well known [2].

Many solutions of Hallén's equation have been suggested to determine the approximate current distribution. Among them are iterative methods of Hallén [1] and King [2], numerical techniques of Mei [3] and Hickman [4], and expansion forms of Storm [5] and Duncan and Hinchey [6]. Iterative and numerical solutions provide unattractive forms of solution for the subsequent computation of electromagnetic fields. Duncan and Hinchey note the difficulties of Storm's work, while their own expansion requires high orders of solution and must be extrapolated to the driving point to determine antenna terminal currents. King and Wu [7] have been able to represent the current accurately in a simple trigonometric form, thus permitting the evaluation of associated electromagnetic fields. However, the reduction of the integral equation to an algebraic equation is not simple, and the coefficients of the current expansion must be evaluated by numerical procedures, as is the case in the theory to be presented here.

If the intrinsic impedance of free space is approximated by 120π ohms and the excitation is selected as 60 volts (this causes no loss in generality since the dipole is a linear element), (1) may be reduced to

$$j\int_{-h}^{h} I_z(z')K(a,z-z')\,dz' + C_2 \cos kz = \sin k|z|. \quad (3)$$

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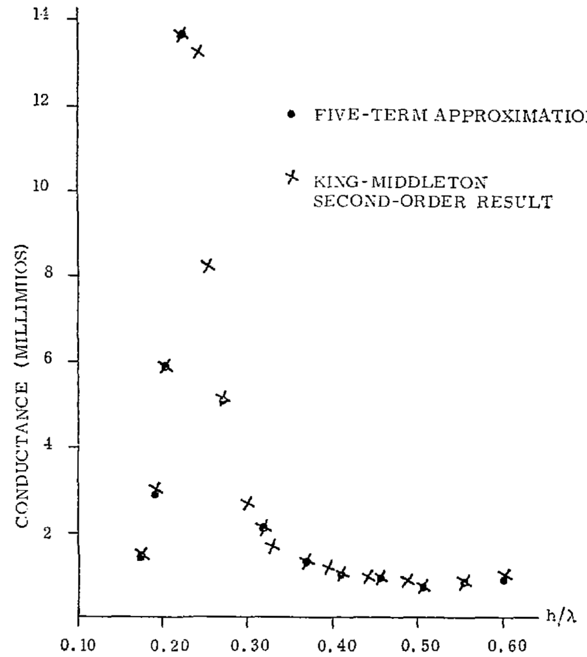

Fig. 1. Variation of conductance of a dipole of variable length Radius = 0.003969λ.

Choose the following expansion form for the current:

$$I_z(z) = \sum_{n=1}^{N} B_n \sin(n\pi/2h)(h-|z|). \quad (4)$$

Substitution of this relation for the approximate current distribution into the integral equation (3), leads to the following set of equations:

$$C_2 \cos kz + \sum_{n=1}^{N} B_n j \cdot \int_{-h}^{h} \sin(n\pi/2h)(h-|z'|)K(a,z-z')\,dz' = \sin k|z|. \quad (5)$$

The $N+1$ unknowns in this equation (C_2 and N coefficients B_n) may be determined by selecting $N+1$ evenly spaced points along the half-length of the antenna, and substituting these values of z into (5).

The current distribution and terminal admittance of isolated dipoles have been determined by approximating the current with five terms. The results of such a study are shown in Figs. 1 and 2, which depict the conductance and susceptance of an antenna of radius $a = 0.003969\lambda$ and variable length h. For comparison second-order results from the King–Middleton iteration [2] are included. In general the results are in good agreement. However, in the region $0.255\lambda \leq h \leq 0.414\lambda$, King's susceptance lies below the five-term susceptance, while for $0.446\lambda \leq h \leq 0.621\lambda$, his susceptance lies above the five-term results. Mack's experimental measurements tend to indicate that the five-term results may be the more accurate [8]. Additionally, it has been found that at most five terms are needed in the series form for the current, while excellent results are obtained by using only two terms to approximate the current distribution on the half-wave dipole.

The evaluation of near-zone electric fields is also of considerable interest, but heretofore has also been of considerable difficulty. Brown [9] computed the near-zone electric fields for the classic sinusoidal current distribution $I_m \sin k(h-|z|)$. King and Wu [7], [10] have determined an approximate expression for the axial electric field, but in a form which automatically satisfies the boundary condition $E_z(a,z) = 0$. It is precisely this field, though, that is of paramount interest since it provides a measure of the accuracy of solutions of (1).

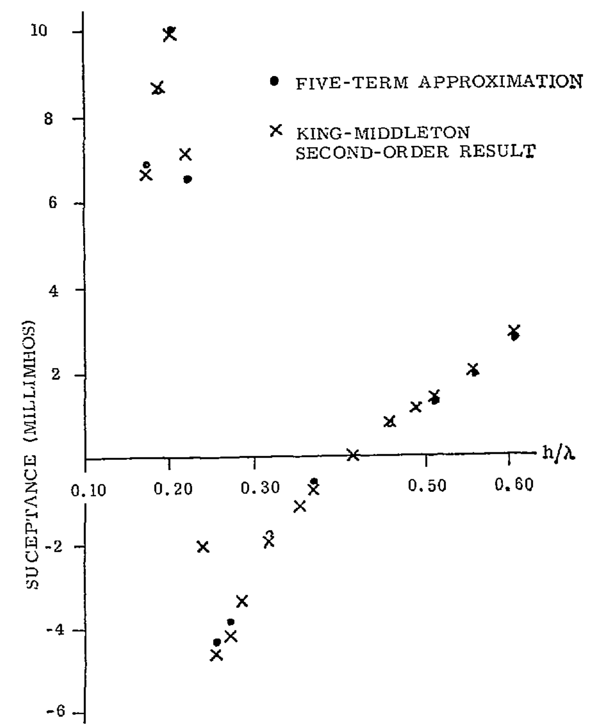

Fig. 2. Variation of susceptance of a dipole of variable length. Radius = 0.003969λ.

Using the equation of continuity and the Helmholtz integral for the scalar potential,

$$-\frac{\partial \phi}{\partial z} = -\frac{1}{j\omega 4\pi\epsilon}\int_{-h}^{h} \frac{\partial I_z(z')}{\partial z'}\,\frac{\partial K(r,z-z')}{\partial z}\,dz'. \tag{6}$$

Noting that $\partial K(r,z-z')/\partial z = -\partial K(r,z-z')/\partial z'$ and integrating (6) by parts leads to

$$\begin{aligned} j\omega 4\pi\epsilon(\partial\phi/\partial z) &= [\partial I_z(z')/\partial z']K(r,z-z')\,|_{-h}^{0} \\ &\quad + [\partial I_z(z')/\partial z']K(r,z-z')\,|_{0}^{h} \\ &\quad - \int_{-h}^{h} \frac{\partial^2 I_z(z')}{\partial z'^2}\,K(r,z-z')\,dz' \end{aligned} \tag{7}$$

where it has been noted that the derivatives of the current may be discontinuous at $z' = 0$. Combining (7) with the Helmholtz integral for the magnetic vector potential, the axial electric field is finally expressed by

$$\begin{aligned} j\omega 4\pi\epsilon E_z(r,z) &= \int_{-h}^{h}\left[\frac{\partial^2 I_z(z')}{\partial z'^2} + k^2 I_z(z')\right]K(r,z-z')\,dz' \\ &\quad - [\partial I_z(z')/\partial z']K(r,z-z')\,|_{-h}^{0} \\ &\quad - [\partial I_z(z')/\partial z']K(r,z-z')\,|_{0}^{h}. \end{aligned} \tag{8}$$

This equation requires that $I_z(\pm h) = 0$ so that there can be no stored charge at the ends of the antenna, and is approximate only inasmuch as the approximate rather than exact kernel is used in the development.

Equation (8) has been used to compute the axial electric field tangential to the cylindrical surface of a half-wave dipole with $h/a = 75$. The current distributions used were the two-term approximation from this trigonometric theory and the two-term form of the current from the King–Wu Theory [11]. The fields are depicted in Fig. 3. Note that the two theories are in good agreement, and both show a rise in $E_z(a,z)$ near $z = 0$. This is a consequence of the delta-function generator. The classic sinusoidal current does not give this rise in field at the driving point.

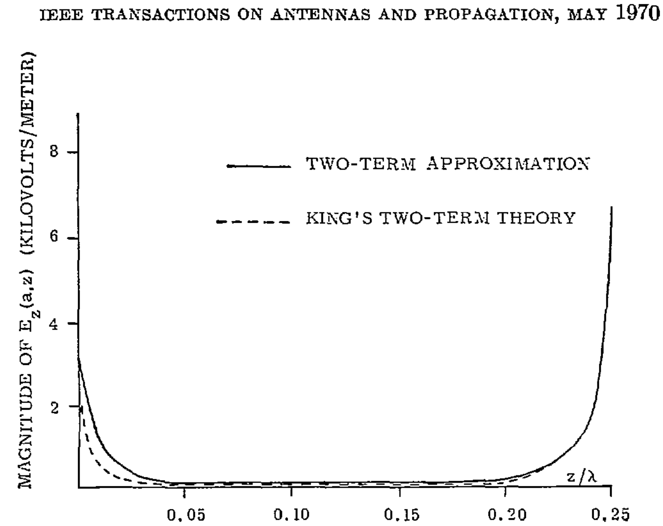

Fig. 3. Axially-directed tangential electric field along a half-wave dipole. $V_0^e = 60$ volts, $h/a = 75$.

A knowledge of the current distribution and electric field permits the evaluation of the time-average Poynting vector along the antenna. Both the two-term trigonometric theory and the two-term form of the current from the King–Wu Theory show that most radiation is from the gap of the antenna.

The general procedure to approximate the antenna current distribution using (4) has been applied to parallel coupled elements with even current distributions, and to isolated elements in dissipative media. The results agree well with computations performed by King. This material will, however, be reported later.

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Radiation from TE_{10} Mode Slots on Circular and Elliptical Cylinders

Wedge diffraction and creeping wave techniques have previously been used for the analysis of the equatorial radiation pattern of an infinite axial TEM mode slot on a circular and elliptical cylinder

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