Teichmüller Theory and Applications

to Geometry, Topology, and Dynamics

Volume 1: Teichmüller Theory
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We thank Laurent Bartholdi, Hendrik Chaltin, Sa'ar Hersonsky, Wolf Jung, Andrew Marshall, Curt McMullen, Mohan Ramachandran, Samuel Roth, Thomas Schmidt, and Seunghee Ye for their contributions to this list.

Page xix Prerequisites, complex analysis: The new proof of the Koebe 1/4-theorem is in Chapter 3, not Chapter 4.

Page 14 Professor Mohan Ramachandran has pointed out that the argument of equation 1.4.8 is not quite clear: are we working in De Rham cohomology or singular (or Čech) cohomology. In singular or Čech cohomology, the argument is correct, but in de Rham cohomology, which I have been using mainly in this chapter (in equations 1.4.6 and 1.4.7) this requires a bit of further argument, since ρ_n is not smooth and doesn't obviously induce anything on de Rham cohomology spaces.

There are various ways around this. One is not to use de Rham cohomology at all: replace equations 1.4.6 and 1.4.7 by saying that the Poincaré dual of δ (i.e., intersection with δ) is a nonzero singular cohomology class, since δ intersects γ_1 transversely in a single point.

Another is to invoke de Rham's theorem, which is proved in Appendix A7.5. That seems a little heavy-handed, a clear case of using a sledge hammer to kill a fly.

Another is to show that continuous maps between smooth manifolds do induce homo- morphisms on de Rham cohomology. This is of course true by de Rham's theorem, but can be proved much more easily, by approximation (at least on σ-compact manifolds), and using partitions of unity.

(1) First show that on any σ-compact manifold there exist Riemannian metrics that are *controlled at infinity* in the sense that there exists $\rho_0 > 0$ such that any pair of points distance $< \rho_0$ apart are joined by a unique geodesic.
(2) Next show that if X and Y are σ-compact manifolds with Riemannian metrics, then every continuous $f : X \to Y$ can be uniformly approximated by C^∞-maps, and if the metric of Y is controlled at infinity, then any two approximations within $\rho_0/2$ are smoothly homotopic.

Page 16 In the last paragraph before Section 1.7, "Thus our map ... " would be better as "Thus, by the reflection principle, our map ... ." (The reflection principle says that if $U \subset \mathbb{H}$

is open and $f : U \to \mathbb{C}$ is an analytic function such that $\operatorname{Im} f(z) \to 0$ when $\operatorname{Im} z \to 0$, then $f(z) = \overline{f(\bar{z})}$ extends f analytically to $U \cup U^* \cup (\overline{U} \cap \mathbb{R})$. We do not have to assume that f extends continuously to $\overline{U} \cap \mathbb{R}$.)

Page 21 Exercise 1.8.11: Part 1 should read "Two nonidentity Möbius transformations commute if and only if they have the same fixed points, or are commuting involutions each interchanging the fixed points of the other." (As the statement currently stands, f(z) = -z and g(z) = 1/z are a counterexample.)

Page 58 Line immediately after equation 2.4.28, $-\det H$ should be $\det H$. In the same line, add "with the quadratic form $H \mapsto -\det H$":

in these coordinates S with the quadratic form $H \mapsto -\det H$ is $E^{2,1}$ on the nose.

Page 59 Proposition 3.1.2, end of part 3: "or it has a subgroup of index 2" should be "or it has an infinite cyclic subgroup of index 2".

Page 71 Two lines before diagram 3.3.6, $f(0) = 0$ should be $\tilde{f}(0) = 0$.

Page 74 In the 4th line, γ_i andγ_j should be $\widetilde{\gamma}_i$ and$\widetilde{\gamma}_j$

Page 82 We probably should have drawn the curve a_3 with the opposite curvature.

Page 94 Third line of the proof of Proposition 3.8.9: an inequality sign is missing. "The band $0\operatorname{Im} z < \pi h$" should be "The band $0 < \operatorname{Im} z < \pi h$".

Page 104 Second line after equation 3.9.10: "the isometric circles $C(\gamma_n)$ and $C(\gamma_n)$" should be "the isometric circles $C(\gamma_n)$ and $C(\gamma_n^{-1})$".

Page 137 The last line of the paragraph following equation 4.5.11 is missing a parenthesis: $\eta(|c'| \le \eta(|c|)$ should be $\eta(|c'|) \le \eta(|c|)$

Page 139 In the third line after Exercise 4.5.7, "Let $U_u \subset U$ be a disc of radius r" should be "Let $D_u \subset U$ be a disc of radius r".

Page 140 In the first line of equation 4.5.18, $_U$ should be $_V$, and in the second line, $_V$ should be $_U$.

Page 141 The caption of Figure 4.5.6 is missing an end parenthesis: in the third line, "on $f(S_2$" should be "on $f(S_2)$".

Page 142 In equation 4.5.25 and in the line immediately above, $f(T)$ should be $f(P)$.

Pages 142–143 The part of the proof of Theorem 4.5.4 that follows Lemma 4.5.9 has been rewritten. We give the new version below:

Two more bits of useful plane geometry. Remember that the derivative of an affine map is constant.

Lemma 4.5.9a *If $P \subset \mathbb{R}^2$ is an equilateral triangle and $g : P \to \mathbb{R}^2$ is affine, then*

$$\text{Area } P = \frac{\sqrt{3}}{4}(\operatorname{diam} P)^2 \quad \textit{and} \quad \operatorname{diam} g(P) \ge \frac{\sqrt{3}}{2}\|[Dg]\| \operatorname{diam} P. \qquad 4.5.26$$

Proof The first formula is obvious. For the second, place P with one vertex at the origin, and so that a vector $\mathbf{v}$ with $|[Dg](\mathbf{v})| = \|[Dg]\|\,|\mathbf{v}|$ points into P; scale $\mathbf{v}$ so that it points from the origin to a point of the opposite side of P. Then $|\mathbf{v}| \geq (\sqrt{3}/2)\operatorname{diam} P$, so

$$\operatorname{diam} g(P) \geq |[Dg](\mathbf{v})| = \|[Dg]\|\,|\mathbf{v}| \geq \frac{\sqrt{3}}{2}\|[Dg]\|\operatorname{diam} P. \quad \square$$

Let $f_n : T \to \mathbb{C}$ be the map that is affine on each $T_{i,n}$ and coincides with f on the vertices of the $T_{i,n}$. Thus

$$\begin{aligned}\int_{T_{i,n}} \|[Df_n]\|^2 \; dx\,dy &= \|[Df_n]\|^2 \operatorname{Area}(T_{i,n}) \\ &= \frac{\sqrt{3}}{4}\Big(\|[Df_n]\|(\operatorname{diam} T_{i,n})\Big)^2 \leq \frac{1}{\sqrt{3}}\Big(\operatorname{diam} f_n(T_{i,n})\Big)^2.\end{aligned} \tag{4.5.27}$$

This finally leads to

$$\begin{aligned}\int_T \big\|[Df_n]\big\|^2 dx\,dy &= \sum_i \int_{T_{i,n}} \big\|[Df_n]\big\|^2 dx\,dy \\ &\leq \frac{1}{\sqrt{3}}\sum_i \big(\operatorname{diam} f_n(T_{i,n})\big)^2 \leq \frac{1}{\sqrt{3}}\sum_i \big(\operatorname{diam} f(T_{i,n})\big)^2 \\ &\leq \frac{1}{\sqrt{3}}\frac{4}{\pi}\big(h(3)\big)^2 \sum_i \text{Area } f(T_{i,n}) = \frac{4}{\sqrt{3}\,\pi}\big(h(3)\big)^2 \text{Area } f(T).\end{aligned} \tag{4.5.28}$$

Note that it is essential that we add the areas of the $f(T_{i,n})$, not the areas of the $f_n(T_{i,n})$, because the f_n may well not be homeomorphisms, and the images of the triangles by the f_n may overlap, as shown in Figure 4.5.7, where the two triangles shaded light and dark have images that overlap.

Clearly the f_n converge uniformly to f as $n \to \infty$, so the partial derivatives of f_n converge weakly to the partial derivatives of f. Equation 4.5.28 shows that the partial derivatives of the f_n lie in a fixed ball in $L^2(T)$. So the partials of f must also be in that ball. Thus the distributional derivatives of f are locally in L^2.

Since f is in $\mathcal{CH}^1$, it satisfies the Jacobian formula (see Proposition 4.2.4, with $g = 1$; we have $\deg f = 1$, since f is an orientation-preserving homeomorphism). Hence

$$\text{Area } f(T) = \int_T f\,dx\,dy. \tag{4.5.29}$$

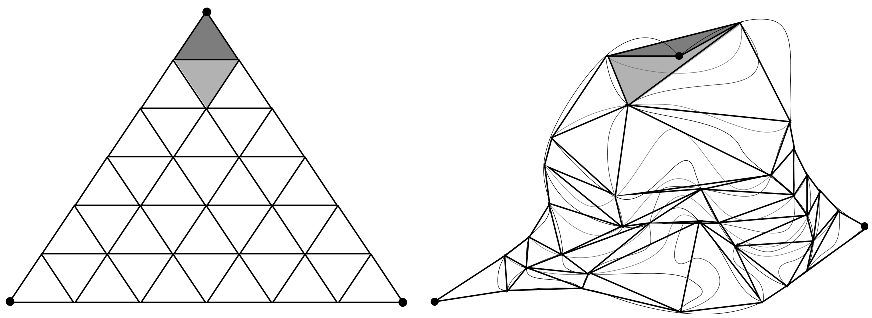


FIGURE 4.5.7 This illustrates the piecewise linear approximation f_n to the map f of Figure 4.5.5. The pale gray curvy lines at right are the original images of the triangulation under f; in bold we see the piecewise linear approximations to those "curvy triangles". The top two triangles at left have images that overlap on the right. Thus we cannot simply add the areas of the piecewise linear triangles.

So by equation 4.5.28, for all sufficiently small triangles $T \subset U$ we have

$$\int_T \big\|[Df]\big\|^2 \, dx\, dy \le \frac{4}{\sqrt{3}\,\pi} \big(h(3)\big)^2 \int_T f \, dx\, dy. \qquad 4.5.30$$

Thus $\big\|[Df]\big\|^2 \le \frac{4}{\sqrt{3}\,\pi}(h(3))^2 f$ locally in L^1. So f is K-quasiconformal, where

$$K = \frac{4}{\sqrt{3}\,\pi} \big(h(3)\big)^2. \qquad 4.5.31$$

(Here we use the second analytic definition of quasiconformal maps, Definition 4.1.5.) Thus we have proved "quasisymmetric $\Longrightarrow$ quasiconformal", completing the proof of Theorem 4.5.4. $\square$

Page 145 In Definition 4.5.13, W is not defined. It should be: " ... if every point of X has a neighborhood W such that for any three distinct points x, y, $z \in W$ ... "

Pages 146–147 This proof has been rewritten:

PROOF The direction "labeled quasisymmetry $\Longrightarrow$ quasisymmetry" is immediate. Every point of U has a neighborhood W such that for every $x, y, z \in W$, equation 4.5.35 holds. Let u, v, w be the permutation of x, y, z such that $f(u), f(v), f(w)$ realizes $(f(x), f(y), f(z))$. Then

$$(f(x), f(y), f(z)) = \left|\frac{f(u) - f(v)}{f(u) - f(w)}\right| \le \eta\left(\left|\frac{u-v}{u-w}\right|\right) \le \eta((x, y, z)).$$

Now "quasisymmetry $\Longrightarrow$ labeled quasisymmetry". By the triangle inequality, if a, b, c are any three distinct complex numbers, we have

$$\left|\frac{b-a}{b-c}\right| \le \left|\frac{c-a}{c-b}\right| + 1, \qquad 4.5.37$$

so that if a triangle has a short side ss, a middle side ms and a long side ls, then ms/ss gives a good estimate of the skew ls/ss. Moreover, if $(a, b, c) < M$, then we can bound the skew in terms of the ratios of any two sides: $M^2 s_1/s_2 \geq (a, b, c)$.

We will examine three cases, depending one whether $[x, z]$, $[x, y]$, or $[y, z]$ is the short side.

1. The short side is $[x, z]$. Then

$$\begin{aligned} \left| \frac{f(x) - f(y)}{f(x) - f(z)} \right| &\leq (f(x), f(y), f(z)) \leq h((x, y, z)) \\ &\leq h\left(\left| \frac{x - y}{x - z} \right| + 1 \right). \end{aligned} \tag{4.5.38}$$

2. The short side is $[x, y]$. Then

$$\begin{aligned} \left| \frac{f(x) - f(y)}{f(x) - f(z)} \right| &\leq \frac{1}{(f(x), f(y), f(z)) - 1} \\ &\leq \frac{1}{h^{-1}((x, y, z)) - 1} \leq \frac{1}{h^{-1}\left(\left| \frac{x-z}{x-y} \right| - 1 \right) - 1} \end{aligned} \tag{4.5.39}$$

3. The short side is $[y, z]$. This is more delicate, since the short side does not appear in equation . Lemma 4.5.15 says that when the skew is large, short sides correspond under f.

Lemma 4.5.15 *Suppose that x, y, z is a triangle in U sufficiently small that Definition 4.5.1 applies, and suppose $(f(x), f(y), f(z)) > h(3)$. Then $(x, y, z) > 3$, and the short sides of the triangles x, y, z and $f(x), f(y), f(z)$ have corresponding labels.*

PROOF Since h is monotone increasing and

$$h(3) < (f(x), f(y), f(z)) \leq h((x, y, z)), \tag{4.5.40}$$

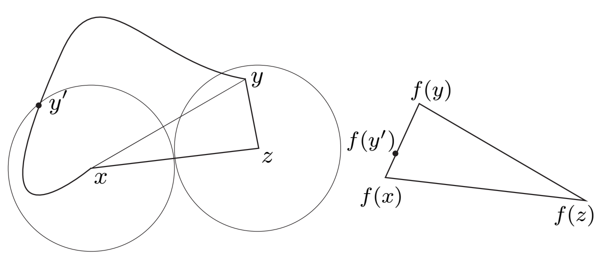


FIGURE 4.5.8 As the point $y(t)$ travels from $f(x)$ to $f(y)$ along the geodesic, the point y' follows some path, which must at some point be half as far from x as z. At that moment, $(x, y', z) \leq 3$.

$3 < (x,y,z)$). Suppose $[y,z]$ is the short side of x,y,z, and suppose by contradiction that $[f(x), f(y)]$ is the short side of $f(x), f(y), f(z)$. Let $y(t)$ travel along the path in U such that $f(y(t))$ travels on the geodesic from $f(x)$ to $f(y)$. Then $y(t)$ starts at x, the center of the circle of radius $d(x,z)/2$ around x and ends up at y, outside this circle. At some point in its travels it must cross the circle, as illustrated in Figure 4.5.8, at some point y'. Then $(x, y', z) < 3$, whereas

$$(f(x), f(y'), f(z)) \geq (f(x), f(y), f(z)) > h(3). \qquad 4.5.41$$

This contradicts

$$(f(x), f(y'), f(z)) \leq h((x, y', z)) < h(3). \qquad 4.5.42$$

□ Lemma

We will subdivide case 3 (where the short side is $[y,z]$) into two subcases.

3′. Assume $(f(x), f(y), f(z)) > h(3)$. Then

$$\frac{2}{3} \leq \left|\frac{f(x)-f(y)}{f(x)-f(z)}\right| \leq \frac{3}{2}, \quad \text{and} \quad \frac{2}{3} \leq \left|\frac{x-y}{x-z}\right| \leq \frac{3}{2}, \qquad 4.5.43$$

so that

$$\left|\frac{f(x)-f(y)}{f(x)-f(z)}\right| \leq \frac{9}{4}\left|\frac{x-y}{x-z}\right|. \qquad 4.5.44$$

3″. Assume $(f(x), f(y), f(z)) \leq h(3)$. Then $(x,y,z) \leq h(h(3))$, so

$$\begin{aligned}\left|\frac{f(x)-f(y)}{f(x)-f(z)}\right| &\leq (f(x), f(y), f(z)) \leq h((x,y,z)) \\ &\leq h\left(h(h(3))^2\left|\frac{x-y}{x-z}\right|\right).\end{aligned} \qquad 4.5.45$$

(The square in $(h(h(3))^2$ comes from the formula $M^2 s_1/s_2 \geq (a,b,c)$ on the previous page.)

We now have four different functions of $w := |x-y|/|x-z|$; the first is relevant for $w > 2$ and tends to infinity with w, the second is relevant for $w < 1/2$ and tends to 0 with w, and the other two are relevant for w bounded away from 0 and ∞. It is then easy to construct a monotone increasing homeomorphism $\eta\colon [0,\infty) \to [0,\infty)$ that is larger than all four. □

Page 170 There is an error in the 3rd line of equation 4.8.35; the equation should be

$$f^\mu(z) = \begin{cases} z & \text{if } \operatorname{Im} z \leq 0 \\ \frac{z+\alpha\bar{z}}{1+\alpha} & \text{if } 0 \leq \operatorname{Im} z \leq 1 \\ z + i\frac{1-\alpha}{1+\alpha} & \text{if } \operatorname{Im} z \geq 1 \end{cases}$$

Equation 4.8.36 should be

$$(f^\mu)^{-1}(w) = \begin{cases} w & \text{if } \operatorname{Im} w \le 0 \\ \frac{(1+\alpha)w - \alpha(1+\bar\alpha)\bar w}{1-|\alpha|^2} & \text{if } 0 \le \operatorname{Im} w \le \operatorname{Im} \frac{1-\alpha}{1+\alpha} \\ w - i\frac{1-\alpha}{1+\alpha} & \text{if } \operatorname{Im} w \ge \operatorname{Im} \frac{1-\alpha}{1+\alpha} \end{cases}$$

Page 187 Equation 5.1.12: Replace $\overline{z}\zeta + 1$ in the denominator by $1 - \overline{z}\zeta$.

Page 188 Equation 5.1.14: In the last fraction, the numerator should be $1-|z|^2$, not $1-|\zeta|^2$. We have used that the push forward of measures is the transpose of the pullback of functions.

Page 203 2 lines after Proposition 5.2.12, $|x|$, not $\sup|x|$; 4 lines after Proposition 5.2.12, "or it has a maximum" should be "or $|g|$ has a maximum".

Page 209 Line 10: One reader thought $q(z)dz^2$ is a 2-form. It is not a 2-form, it is a quadratic differential.

Page 214 Perhaps I should have elaborated on the last sentence before Figure 5.3.7:

... for each critical point of q' in Y and each critical trajectory emanating from it, mark the first intersection of that trajectory with J (if it exists), as illustrated in Figure 5.3.7. (We will see right after equation 5.3.12 that it does exist.)

Page 215 Two paragraphs are repeated from page 214.

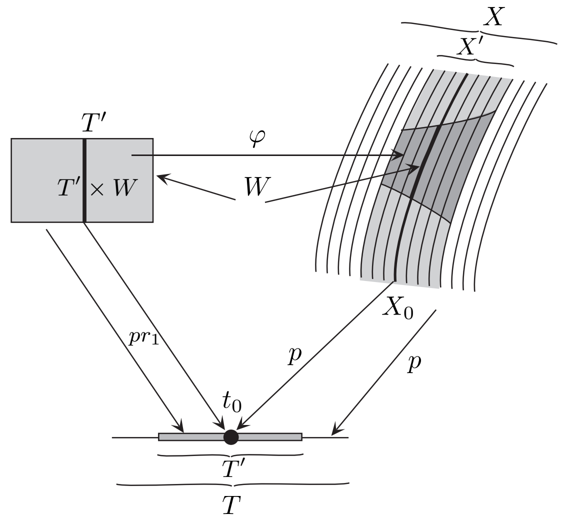
Page 226 A minus sign is missing in line 7: " ... defined over $(\mathbb{P}^1 \times \mathbb{P}^1)\Delta$" should be " ... defined over $(\mathbb{P}^1 \times \mathbb{P}^1) - \Delta$".

Page 236 In subsection "Beltrami forms on quasiconformal surfaces", the second, third, and fourth sentences should be replaced by

It is tempting to define this as the space of almost-complex structures on S of class L^∞, i.e., complex structures on the fibers of the tangent bundle TS (see Appendix A4). This does not work in any natural way: S is not naturally a C^1 manifold, so it doesn't have a tangent bundle TS.

Page 243 Proposition 6.2.7, second line: $p\colon X \to T$, not $xp\colon X \to T$

In Figure 6.2.1, the primes on some of the T are not clear, and there should be no U on the right side.



Page 273 An equal sign is missing in equation 6.6.34, and o should be 0:

$$d(0, \mu) = \ln \frac{1 + \|\mu\|_\infty}{1 - \|\mu\|_\infty}.$$

Page 278 In equation 6.8.12 and also two lines before the equation, $\mathcal{M}^S$ should be $\mathcal{M}(S)$. (Note: when we reprint this book, we will change $\mathcal{M}(S)$ to $\operatorname{Bel}(S)$ and $M(S)$ to $\operatorname{bel}(S)$, to be in keeping with notation to be used in volume 2.)

Page 282 Next to last line of the proof, by "both spaces" we mean the spaces

$$\Big(Q^1\Big(X_{F_{S,s_0}(\tau)} - \widetilde{F}_{S,s_0}(\tau)\Big)\Big) \quad \text{and} \quad \Big(Q^1\Big(X_{F_{S,s_0}(\tau)}\Big)\Big)$$

of quadratic differentials.

Page 293–294 There is some confusion here with Γ and G. In the first line of Section 6.12, Γ should be omitted, to avoid confusion with the Γ of Notation 6.12.1; i.e., replace "groups $\Gamma \subset \operatorname{Aut}(\mathbb{P}^1)$" by "subgroups of $\operatorname{Aut}(\mathbb{P}^1)$".

Also, 6.12.4 could be written, equivalently, as $QF : \mathcal{T}_X \times \mathcal{T}_{X^*} \to \operatorname{Rep}(G)$.

Page 294 Proposition 6.12.4: in the second line of part 2, "a an open subset" should be "an open subset".

Page 306 The comment (after Exercise 7.3.2) that "I don't know how to continue the proof using this approach" reflected my ignorance. Curt McMullen points out that it is known that there is a constant $C(g)$ such that on every Riemann surface of genus g there is a maximal multicurve whose longest curve has length $\leq C(g)$. See Chapter 5 of Peter Buser's book *Geometry and Spectra of Compact Riemann Surfaces*, where C(g) is called "Bers' constant".

Page 309 In Proposition 7.4.4, we should have given the domain and codomain of f: "define the function $f : \mathbb{R} \to \mathbb{C}$ by . . . "

Pages 310–311 A number of changes starting with the text immediately after equation 7.4.15, and continuing through equation 7.4.21. The text should read as follows. Note that in this version there is no equation 7.4.19.

is bounded independently of t. If $\varphi(0) = 0$ this is obvious, so suppose $\varphi(0) \neq 0$, and to lighten notation set $A^2 := t\varphi(0)$, and $R := |1/A|$. Make the change of variables $z = Au$; the integral becomes

$$\int_{\mathbb{D}} \left| \frac{z}{z^2 + t\varphi(0)} \right| |dz^2| = |A| \int_{D_R} \left| \frac{u}{u^2+1} \right| \; |du|^2, \qquad 7.4.15$$

where D_R is the disc of radius R. This integral is well defined for all $A \neq 0$, and tends to 0 as $A \to \infty$. When $A \to 0$, we can break up the integral into the part over D_2, which is some constant C, and the remainder, where $|u^2+1| > |u|^2/2$. Then in polar coordinates,

$$|A| \int_{2<|z|<R} \left| \frac{u}{u^2+1} \right| \; |du|^2 \leq 2\pi|A| \int_2^R \frac{2r}{r^2} r\, dr = 4\pi|A| \left(\frac{1}{|A|} - 2 \right). \qquad 7.4.16$$

Clearly this is bounded as $A \to 0$.

Seeing that the third integral is $O(t)$ is easier:

$$\left| \Big((z^2 + t\varphi(0)) - z^2 \Big) \Big(\psi(z) - \psi(0) \Big) \right| \leq 2|t| \left| \frac{\varphi(0)\psi_1(z)}{z} \right|, \qquad 7.4.17$$

which is integrable.

So it is enough to study the first integral. If $\varphi(0) = 0$, the integral obviously vanishes, so suppose $\varphi(0) \neq 0$.

Again set $A^2 = t\varphi(0)$, make the change of variables $z = Aw$, and set $R = 1/|A|$, to find

$$\begin{aligned} &\int_{\mathbb{D}} \Big((z^2 + t\varphi(0)) - z^2 \Big) \psi(0)\, |dz|^2 \\ &\qquad = \psi(0)\overline{\varphi(0)}\; t \int_{D_R} \big((w^2+1) - w^2 \big)\, |dw|^2. \end{aligned} \qquad 7.4.18$$

The integrand is bounded by 2, so the integral over the unit disc is in $O(t)$, and in finding the contribution of the integral to terms in $t \ln 1/|t|$ we may consider only the integral over $D_R - D_1$. Moreover, the integral is real, since the imaginary part of the integrand has opposite signs at complex conjugate points.

Set $w := re^{i\theta}$. A bit of Euclidean geometry will show that

$$\left| \mathrm{Re}\, \big((w^2+1) - w^2 \big) - \frac{\sin^2 2\theta}{r^2} \right| \leq \frac{4}{r^4}, \qquad 7.4.20$$

and since

$$\int_{D_R - D_1} \frac{1}{r^4}\, r\, dr\, d\theta \leq \pi \qquad 7.4.21$$

it is enough to find the contribution of $\frac{\sin^2 2\theta}{r^2}$ to the coefficient of $t \ln \frac{1}{|t|}$.

Page 327 In equation 7.6.9, $\dfrac{h-y}{y}$ should be $\dfrac{h-y}{h}$ and $\dfrac{h+y}{y}$ should be $\dfrac{h+y}{h}$.

In Figure 7.6.8, the primes aren't very clear. The T_1 and T_2 on the left should each have a single prime; those on the right should have double primes.

Page 342 Period missing at the end of the first sentence. Line 2: isotopy, not homotopy. Beginning of second paragraph: $\mathbb{R}/\mathbb{Z}$, not $\mathbb{R}/Z$. Second paragraph: replace

"Let $\gamma : \mathbb{R}/\mathbb{Z} \to S$ be a smooth parametrized simple closed curve on S."

by

"Let $\gamma : \mathbb{R}/\mathbb{Z} \to S$ be a smooth embedding, so that $\gamma(\mathbb{R}/\mathbb{Z})$ is a parametrized simple closed curve on S."

Page 343 Lemma A2.3: $f \circ \gamma_0 = \gamma_1$, not $f \circ \gamma_1 = \gamma_2$. Proof of Lemma A2.3, 4th line: "may intersect $\gamma_0(S^1)$", not "may intersect γ_0"

Page 344 Figure A2.1: To be consistent with the text, the curve labeled I_1 should be I_0, and I_2 should be I_1. In line 3 of the caption, I_2 should be I_1, and in line 4, γ_2 should be γ_1.

Page 352 Formula A3.11 is missing parentheses; it should be

$$0 \to H_0(\gamma_i) \to H_0(X_{i-1}) \oplus H_0(T_i) \to H_0(X_i) \underset{0}{\to}$$

$$\underset{0}{\to} \underset{\mathbb{Z}}{H_1(\gamma_i)} \to H_1(X_{i-1}) \oplus \underset{\mathbb{Z}^2}{H_1(T_i)} \to H_1(X_i) \to 0.$$

A parenthesis is missing from the last line of the proof, as well; "$\dim H_1(X_i) = \dim(H_1(X_{i-1}) + 1$" should be "$\dim H_1(X_i) = \dim(H_1(X_{i-1})) + 1$".

Page 354 Equation A4.5: The left side should be $\mu_J(i(a+ib) \otimes x)$, not $\mu_J(i(a+ib) \otimes$

Page 355 In equation A4.10, J is not the J of equation A4.3. The paragraph should be replaced by:

A better way to say this is to consider the open subset

$$\mathcal{K}(TM) \subset \mathrm{Gr}^{\mathbb{C}}(\mathbb{C} \otimes_{\mathbb{R}} TM)$$

of pairs (x, K_x) with $x \in M$ and K_x an n-dimensional $\mathbb{C}$-subspace of $\mathbb{C} \otimes_{\mathbb{R}} T_xM$ such that $K_x \cap \overline{K}_x = \{0\}$. Since the Grassmanians form a C^∞ bundle of complex manifolds, $\mathcal{K}(TM)$ is also a C^∞ bundle of complex manifolds over M. The space $S^k(M, \mathcal{K}(TM))$ of C^k-sections of the bundle $\mathcal{K}(TM)$ over M is the space of almost-complex structures on M of class C^k.

(Note that we have replaced "family" by "bundle" in two places.)

Page 357 Three lines before equation A4.16: $K \subset \mathbb{C} \otimes TM$ should be $K \subset \mathbb{C} \otimes_{\mathbb{R}} TM$.

Page 386 Line 3, missing word: "because *they* are by far . . . ."

Page 392 Third line of Section A7.4: "more real than", not "more real that".

Page 392 End of third paragraph of Section A7.4: $H(X, \mathcal{O}_X^*)$, not $H^(X, \mathcal{O}_X^*)$

Page 397 The proof for proposition A7.5.6 is incorrect. Here is a corrected version: Proof Choose an exhaustion $\mathrm{V}_1 \subset V_2 \subset \cdots \subset U$ of U by open sets such that each V_i is relatively compact in V_{i+1}. Let U_i be the union of V_i and all the compact components of $U - V_i$; then every component of $\overline{\mathbb{C}} - U_i$ contains points that are not in U. Further, choose C^∞ functions h_i on $\mathbb{C}$ that are identically 1 in U_i and identically 0 on $\mathbb{C} - U_{i+1}$.

Given $\alpha \in A^{0,1}(U)$, by equation A7.5.9 we can find $\beta_n \in A^{0,0}(U)$ such that $\overline{\delta}\beta_n = h_n$. Since we can write

$$\beta_n := \beta_0 + (\beta_1 - \beta_0) + \cdots + (\beta_n - \beta_{n-1}), \qquad A7.5.13$$

it is tempting to set $\beta = \beta_0 + \sum_{k=1}^{\infty}(\beta_{k+1} - \beta_k)$; unfortunately, the series does not converge. Instead, note that $\beta_{k+1} - \beta_k$ is analytic on a neighborhood of U_k, and every component of $\overline{\mathbb{C}} - U_k$ contains points not in U. Thus by the Runge approximation theorem, there exist rational functions p_k analytic in U such that

$$\sup_{z \in \overline{U}_k} |\beta_{k+1}(z) - \beta_k(z) - p_k(z)| \leq \frac{1}{2^k} \qquad A7.5.14$$

Now the series

$$\beta := \beta_0 + (\beta_1 - \beta_0 - p_0) + \beta_2 - \beta_1 - p_1) + \cdots \qquad A7.5.15$$

converges uniformly on compact subsets of U, and β satisfies $\bar{\partial}\beta = \alpha$. □

Page 412 Theorem A9.14: Here X is supposed to be a complex manifold of complex dimension n.

Page 416 Two lines after eq. A10.24: replace "which is obviously be the case of the sets U and V above" by "which is the case for locally constant sheaves".

Page 417 Last line: neighborhoods U_i, not U_ij.

Page 419 Corollary A10.2.6: $\mathcal{O}(-\mathrm{div}(s))$ should be $\mathcal{O}(+\mathrm{div}(s))$.

Page 445 Two references were omitted:

[99] A. Weil, *Modules des surfaces de Riemann*, Bourbaki seminar **168** (1957–1958).

[101] S. Wolpert, *On the symplectic geometry of deformations of a hyperbolic surface*, Annals of Mathematics, **117**, no. 3 (1983), 207–234.

There is no reference [100].

Index entries There should be entries for

barycenter, page 185
foliation, 209
H (upper halfplane), 6
Hawaiian earring, 388
mating, 294
Poincaré duality: the italicized entry should be 410, not 409.
pullback of Beltrami forms, 161, 169