Psychology of Communication in Mathematics

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In this paper we try to characterize a successful mathematical communication. In this regard, we utilize the notions of “Parent”, “Child”, “Adult” introduced by T. Harris to recognize different internal mathematical personifications in students and teachers [1]. We introduce “Emotion” and “Reason” as two external personifications, which represent social mathematical interactions. We analyze these educational personifications in two educational systems. The first educational system is a system which is oriented towards problem solving. Then we try to give a model for mathematical creativity and implement the above personifications in that model. As a result a few educational perspectives are introduced. Then we try to analyze these personifications in an educational system which is oriented towards developing mathematical maturity rather than emphasizing on problem solving. We give another model for mathematical creativity with this new perspective and end up with a few different educational perspectives. Along the path, we try to give a model for group thinking and introduce the notion of “atlas of history of concepts”.

In this paper, we try to develop an understanding of internal and external mathematical personifications which are in action when we are engaged in a mathematical conversation. We shall investigate their actions and reactions; their concerns and their abilities; and also we shall know about the internal communication between these personifications during a mathematical conversation or in the process of problem solving which is completely internal. Then we will introduce a few educational perspectives. We shall first develop a mathematical formulation for T. Harris’s notions of ‘Parent, Child, Adult’[1].

‘Parent, Child, Adult’

Before the student is able to creatively engage in a subject matter, there is a mountain of raw data received by his brain which are personified in an internal structure which we call ‘Parent’. Most of the recorded data consist of the information, points of view, actions and reactions, and believes imposed by his/her teachers. Every one has this internal structure and student’s weakness in understanding the material and student’s needs to constant help of the teachers, forces him/her to keep this raw data as it is in his/her ‘Parent’ . All the mental rules, advises and encouragements are recorded in student’s brain too. ‘Parent’ never questions this recorded data. They could be true or false, but to the ‘Parent’ they are plain truth. They come from the teacher, the center of importance in the world. No one is able to clean this record. It is ready to be played over and over during the student’s lifetime. This repetition is very powerful, and affects the internal mathematical personality of the student. Most of this information is in the form of ‘how to do something’.

At the same time that information about external factors is recorded in ‘Parent’, internal feelings and reactions of the student is recorded and personified in an internal structure called ‘Child’. For example if the teacher is not skillful in the subject matter, this will be recorded as feeling uncertainty in being able to solve problems related to that subject. When ‘Parent’ is replayed, internal feelings recorded in ‘Child’ are also reminded. Most of these signs are negative, like ‘I am not understanding the relations’, ‘I don’t know the objects’, ‘I can’t figure out the implications’, ‘I can’t think as fast as the teacher’ and many similar questions.

Of course there are positive aspects in this recording too. Child is a powerful resource for the sense of curiosity, enthusiasm, magnificence in

discovering and learning, and many other signs of this type. The recordings of ‘Parent’ and ‘Child’ are stopped as soon as the first independent experience of personal creativity occurs.

The ‘Adult’ is born when the student is able to do the subject on his own for the first time. In this stage, he is able to guide himself, and is able to move forward on his own towards new discoveries. This is when the machinery of creativity starts to work. Although at first, ‘Adult’ is very breakable and can be easily overrun by ‘Parent’ and ‘Child’, but eventually the mainstream of data processing is a duty of ‘Adult’. Pieces of experiences are transformed to data and are stored by ‘Adult’ according to previous experiences. This is the exact difference between ‘Parent’ and ‘Adult’. Finding and applying data is also affected by previous experiences, which is very different from ‘Child’ whose actions are of the form of reactions. In fact, ‘Adult’ is the one who can distinguish between himself, ‘Child’ and ‘Parent’. He is like a computer, which is processing data and doing computations using information coming from three sources: ‘Parent’, ‘Child’ and himself. Besides, ‘Adult’ is intelligently checking if the information given by ‘Parent’ is still accurate or useful and also tries to decide if the reactions of ‘Child’ are still appropriate. We shall point out that, if ‘Adult’ finds the information coming from ‘Parent’ appropriate, this does not clean the negative information recorded in ‘Child’. If ‘Adult’ is weak or not awakened, it would not be able to keep the reactions of ‘Child’ in control.

So ‘Adult’ is continuously looking through old information and evaluating them; and then according to its evaluations, stores them again. If this goes smooth and there is not much contradiction between ‘Parent’ and truth, then ‘Adult’ is in good shape and is ready for important jobs like activating the process of creativity. This mechanism is born from the sense

of curiosity in the ‘Child’ which is present in ‘Adult’ too. ‘Child’ provides motivation for creation and ‘Adult’ provides the power and ability for creation. But this never happens, except if ‘Adult’ is provided with free time and energy. If ‘Adult’ is too busy processing old data and dealing with contradictions, there is no time and energy left for ‘Adult’ to spend on creation of new structures. The most productive situation is when ‘Adult’ is able to keep company of ‘Parent’ and ‘Child’ and use their help in the event of creation. ‘Parent’ provides ‘Adult’ with a rich resource of experiences, which would take a lifetime for ‘Adult’ to obtain them itself; and ‘Child’ is the most able help for ‘Adult’ in providing motivation and emotional encouragement. Creating this union is the outmost goal for education and since creation is a job for ‘Adult’, the educators should communicate students as ‘Adults’.

Educator’s Personality in Communication

The classification introduced above, not only introduces three major and determining aspects of student’s personality in the process of learning but also provides a more general scheme for human communication. In particular these three personifications, ‘Parent’, ‘Child’ and ‘Adult’, are also present in the educator’s strategy of communication with students. Information recorded from the teacher’s own experiences of being a student and being taught in a classroom are personified in the ‘Parent’ of the teacher. The ways in which his own teachers taught him, and their points of views and actions and believes, are all recorded in the educator’s brain and influence his communication skills through ‘Parent’.

At the same time that this external data is recorded, internal feelings and experiences of the educator as a student are stored and personified as ‘Child’. Positive experiences are a good motivation for him to be a good

teacher. He tries to awaken his own childhood experiences through educating his pupils. He is very anxious to see his pupils' curiosity, enthusiasm and magnificence in the events of discovery.

Negative experiences of 'Child' have a more complicated influence on educator's behavior towards students. Natural internal negative signs recorded by 'Child' are a result of educator's weakness of communication as a student, or being new to the subject matter. These signs force the educator to have in mind that students could be wondering around about what exactly objects in question are, and what properties they have, and how objects are related to each other, and how different steps in a proof are connected. He tries to approach problematic aspects in several directions so that, different minds be able to pick up the material according to their capacities. This positive attitude is when 'Adult' confirms that 'Parent' is consistent with truth. But in case negative signs are recorded in 'Child' as a result of recordings in 'Parent' which are recognized to be negative or illogical or forceful or primitive or not appropriate for any reason, the story is different. The pressures on the educator as a student, which are recorded in 'Child' could overcome 'Adult' and force unacceptable behavior while educator is communicating with students. In this case, any kind of interaction of 'Child' with the process of communication is problematic. This could be under control, only if educator's 'Adult' is on the stage in the process of communication.

Educator's 'Adult' should not only be conscious of 'Parent', 'Child' and 'Adult' of the student during a mathematical communication, but also it should keep educator's own 'Child' and 'Parent' in fruitful access. Since 'Adult' is the only one who can recognize 'Parent' and 'Child', it is unacceptable if either 'Parent' or 'Child' overcome educator's 'Adult' and

try to have control on the process of communication. Because, educator shall evaluate all the relevant internal data in which he/she has access on, so that his/her approach be chosen out of reason and be influenced by reactions of students to his/her standpoints, in order to ensure a fruitful atmosphere in the class. This role needs lots of creativity and only fits ‘Adult’. We shall discuss what goes wrong if ‘Parent’ or ‘Child’ make educational decisions.

‘Parent’ as an Educator

One may think that educator has the role of ‘Parent’ for the student anyway. But this would be a misunderstanding of the ‘Parent, Child, Adult’ terminology. If the educator’s behavior is influenced by his personal ‘Parent’, then he/she does not have self-control and recordings of the educator as a student are being replayed. This could cause many problems even if there is no conversation between educator and student. Not only the recorded data could be improper, but also it could only fit a specific situation, which is not exactly the same as the as the situation where the data is being replayed. This behavior will be stored in student’s ‘Parent’ and in future will cause contradictions while his ‘Adult’ will be processing this data. Student’s ‘Adult’ will regard the data in ‘Parent’ illogical and improper and dealing with this will take student’s energy. Problems are more serious if there is a mathematical conversation going on.

The goal of education is to help students to give birth to their ‘Adult’. If ‘Adult’ is born without the teacher knowing about its existence, ‘Adult’ will decide about the future of the mathematical communication. When ‘Parent’ is trying to teach students, it could be naturally considering them as a ‘Child’ or ‘Parent’, who is recording internal and external data without rethinking them. Before the birth of ‘Adult’, student is always engaged in a mathematical conversation as a ‘Child’ if he is convinced of the teacher’s

knowledge about the material. This is usually the case. Because ‘Child’ is not able to judge on his own about the abilities of its teacher. Student’s ‘Parent’ could be speaking only when he is talking to his fellow students. ‘Child’ will always regard the teacher as ‘Parent’. So communication will go on fluently. The communication is a ‘Parent’ to ‘Child’ and ‘Child’ to ‘Parent’ conversation.

In case ‘Adult’ is born in the student, ‘Child’ and ‘Parent’ stop recording data and ‘Adult’ will proceed the process of data storage, but this time according to his personal classifications. ‘Adult’ recognizes that, a ‘Parent’ to ‘Child’ and ‘Parent’ to ‘Parent’ communication can not proceed. It may be that ‘Adult’ decides to engage the ‘Child’ in the conversation with teacher’s ‘Parent’ to get more data and the process of education is not disconnected. But ‘Adult’ usually chooses to stop the conversation and start a new ‘Adult’ to ‘Adult’ communication. This is the best reaction, because it forces the educator’s ‘Adult’ to overcome ’Parent’ and engage in the conversation. Sometimes, student’s ‘Adult’ feels that a lot of its energy is being taken, as a result of problematic data being forced on ‘Adult’ by the ‘Parent’ of educator. ‘Adult’ may decide to respond with student’s ‘Parent’ as a negative reaction to educator’s ‘Parent’. He may convert the conversation to a ‘Parent’ to ‘Child’ and ‘Parent’ to ‘Child’ communication. But in this case, since ‘Adult’ is not present, educator will not be able to control his/her reactions. This could make disasters in classroom. Rarely happens that ‘Adult’ after being unsuccessful in awaking educator’s ‘Adult’, decides to convert the communication to a ‘Parent’ to ‘Parent’ conversation. This will be acceptable for educator’s ‘Parent’, but contradicts goals of education.

In short, ‘Parent’ is unable to be a good educator. There are many situations in which its presence could be problematic. We should think of manners, which keeps its uncontrolled presence away from the scene of education.

‘Child’ as an Educator

The first question coming into mind is how could it be that ‘Child’ occupies the role of educator instead of ‘Parent’ or ‘Adult’? One reason mentioned before is that ‘Child’ is eager to repeat its beautiful childish experiences. One way is to see them appear in other ‘Child’s, and this is through educating them. In fact student’s ‘Child’ encourages being educated by ‘Child’ because it no longer feels being weaker and uneducated. Students enjoy if they could share their uncertainty with their teacher. There are problems showing up in this case too. ‘Child’ being faced with ‘Child’ of educator is encouraged to be stronger and more knowledgeable. This will help him to do mathematics himself and give birth to ‘Adult’. But as soon as ‘Adult’ is born, it will react differently to ‘Child’. If it tries to make an ‘Adult’ to ‘Adult’ conversation and the ‘Child’ of educator leaves its position for educator’s ‘Adult’, then the process of education will proceed. But what usually happens is that educator’s ‘Child’ will be disturbed by the unwanted reaction of student. The educator will no longer receive data replaying his enjoyable memories. As a result mathematical conversation stops. Educator’s ‘Child’ is not able to make student’s ‘Adult’ stronger and more educated, so that he/she can do mathematics independently. It could also happen that, after the birth of ‘Adult’, the ‘Parent’ of student tries to show off and take advantage of the weak position that educator has chosen. This will stop the process of education too. The best way to take advantage of a ‘Child’ to ‘Child’ conversation is that educator’s ‘Adult’ tries to

communicate using ‘Child’ on purpose for the sake of educational goals, and be able to switch back as soon as this means of communication stops educating the student.

The above analysis holds consistent only when there is a single student in the classroom. Of course when ‘Child’ or ‘Parent’ faces a whole classroom, its performance would be much weaker than what is discussed above. It rarely happens that a classroom be so coherent that diversity of student’s thinking skills does not make a problem in the process of education. In any circumstances, presence of ‘Parent’ or ‘Child’ as an educator is acceptable only under supervision of ‘Adult’. It is the only personification who can decide which of ‘Parent’ or ‘Child’ could be of some use.

‘Adult’ as an Educator

When ‘Adult’ is controlling the process of education, the educator not only uses fruitful assistance of ‘Parent’ and ‘Child’, but also he/she is free of breathtaking contradictions recorded in his ‘Parent’ and ‘Child’. He is able to have a successful mathematical communication with many students, being conscious of their ‘Parent’s, ‘Child’s and ‘Adult’s. The question is, which of the communication frameworks fits the classroom; ‘Parent’ to ‘Child’ or ‘Adult’ to ‘Adult’? There is a disadvantage in both of the forms, because of the classroom being a non-homogenous field of education. The educator, in order to have a successful communication has to start with the ‘Parent’ to ‘Child’ scheme, and then right on time, switch to an ‘Adult’ to ‘Adult’ communication.

Educational Systems emphasizing on problem solving

Since problem solving is the main goal of such an educational system, and there is no way to have a class coherent in subject of problem solving,

instant switch to an ‘Adult’ to ‘Adult’ scheme never works in such a system. If the educator follows the discussion through an ‘Adult’ to ‘Adult’ conversation, many of the students will not be able to follow. This forces the process of education to be discrete. In other words, after each step in solving a problem, teacher has to give time to the classroom to become homogenous. The need for homogenization encourages group working in class. In fact, problem solving is a practical tool to teach group thinking. Students communicate with each other through solving a common problem; get to understand each other better and tend to have a more coherent understanding of the material.. This process of homogenization does influence the teacher to prepare the material in a way that is compatible with the discrete scheme of education in the classroom. Textbooks are also influenced. They ought to have classroom exercise sections after introduction of each part of a new lesson to make sure that classroom is made coherent using them. Teacher education is also influenced. Teachers ought to be taught how to recognize all the different levels of sophistication in the process of learning a given material, so that they can divide the material in blocks, which can be coherently absorbed after group working. Recognizing these blocks is a difficult task. This question shall be studied in the framework of social psychology of mathematics education. Because, communication of students is an educational socialization and it shall be studied only with the methodology of social sciences.

Social Aspects of a Mathematical Communication

Here we open a new discussion in the framework of social sciences. We shall keep in mind that this social study is a completely applied investigation on the nature of mathematical homogenization in a classroom. So we carefully keep distant from philosophical aspects of social methods of

experimentation. Beforehand, we shall introduce a theoretical framework for the communication of two individuals, which forms the atom of a social communication, and then we can understand the communication between an individual and a social system. Since we are performing a social study, we shall no longer use the internal framework of ‘Parent, Adult, Child’ which was introduced to study an ‘individual’. This would be an attempt to build a social theory on psychology of individuals, which is not allowed in methodology of social sciences. We need a ‘social’ framework to work with.

Trial and error is the method used by a student to learn how to communicate. The student’s goal is to form a meaning for his/her observations of the society he/she has contact with. Student’s creativity in this level is only used to discover the relations and applications and meanings of raw social data stored in his/her brain. These meanings and explanations are stored and personified in the ‘Emotion’ of the student, which functions only in the form of reactions. ‘Emotion’ is not able to decide how to perform in a mathematical communication. ‘Emotion’ is not able to think and decide about its performance.

As soon as the first creative social response is formed, ‘Reason’ is born. ‘Reason’ chooses its functions by freewill. It is able to store meanings according to its own classifications. At first, ‘Reason’ is weak and can be easily overrun by ‘Emotion’. But eventually the mainstream of social communication is a duty of ‘Reason’. ‘Reason’ is the one who can distinguish between himself and ‘Emotion’. In better words, the social role of ‘Reason’ is similar to the internal creative role of ‘Adult’. The creativity of ‘Reason’ is used to define meanings for social actions not discovering them. Also ‘Reason’ is able to give birth to ‘Reason’ in other people. In fact, this is one of the goals of mathematical communication.

When two student are communicating by their ‘Emotion’s, they can not communicate meanings. The act of searching for meanings is a personal behavior for ‘Emotion’. When one of the students is communicating with his/her ‘Reason’ he/she is able to communicate his/her personal understandings by helping others to give birth to their ‘Reason’. So the goal of educator shall be making sure if the seed of knowledge has been accepted and given birth to ‘Reason’ in the mind of some members of each group, so that they can teach the material to their fellow students.

‘Emotion’ and ‘Reason’ as Educators

‘Emotion’ being an educator, can only communicate raw social data to students. This only works if there are many exceptionally creative and talented students in classroom who can give life to the raw material by their personal abilities. If the educator is skilled in the rules of dividing students into problem solving groups, it is possible that, in the light of presence of ‘Reason’ in some students, the classroom get a relatively acceptable homogenization after group discussions. This could also force the birth of ‘Reason’ in educator. But in a usual classroom, atmosphere is not as prepared. In that case, it is the educator who shall prepare the atmosphere. This is a creative social job and only ‘Reason’ is capable of doing it.

The most important job of ‘Reason’ as an educator is to make sure that the process of homogenization is smoothly going on. The educator can enforce his/her methods by properly dividing students into groups and choosing adequate problems for discussion in these groups. In time, the educator can develop an understanding of weaknesses and strengths of each group and take it into consideration in choosing problems which are posed to each group. In this manner, one can reduce a large classroom to a smaller classroom in which the teacher is communicating with a few groups.

The educator shall be prepared to teach according to a few different learning schemes which are personified by different groups. The first step is to recognize these schemes.

‘Reason’ can be an educator only if it is trained to be responsive to the mathematical behavior of the student groups and is able to keep record of the information it gains and is capable of analyzing the information and applying it in practical situations. Educator’s ‘Reason’ should have a methodology in order to check if some strategy is successful in giving birth to ‘Reason’ in students or not. In fact, one of the main sources that educates students is wise change of strategies by educator, which brings a broken unsuccessful system into life.

Strategy of ‘Reason’

‘Reason’ getting in touch with the classroom, knowing about the abilities of different groups has to follow a single algorithm while checking in each stage if an adequate response is given by each of the groups. The process of the birth of ‘Reason’ in students is out of control of the educator anyway. What it can do is to prepare the atmosphere and wait to see the result come out. Different groups have different abilities. Therefore, the educator has to have a measure to scale the difficulty of the material and the abilities of students, so that he/she can match student groups with appropriate problems by solving which they can act creatively. More precisely, every teacher knowing his/her students very well, develops expectations of the students’ abilities which enables the educator to predict if a new problem can be solved by them or not. This is a fact, and experience of majority of teachers confirms it. The question is, if this understanding of educator can be scientifically formulated. The higher the abilities of student groups are, the higher the jump of ‘Reason’ to new horizons would be. There

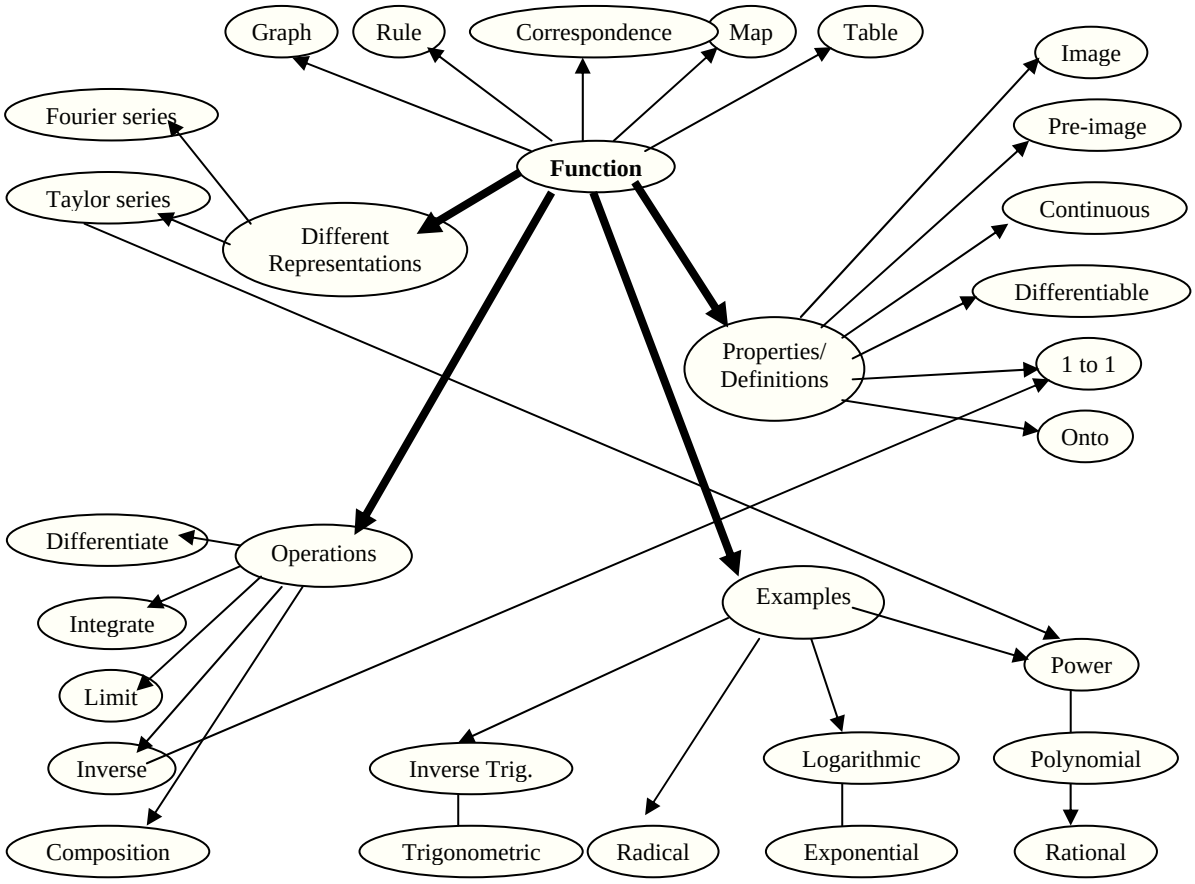
should be a standard measure on both sides to distinguish between different levels of capacities for learning and different levels of jumps. We try to construct a model for the student's problem solving process, which tries to explain what happens when 'Reason' is in action. The model should be simple enough, so that we can define different heights of jumps in mathematical creativity using the model. On the other hand, our model should be complicated enough, so that all different factors which take part in creativity effectively take part in the structure of our model. Such a model has the capacity to be improved, both by considering new factors relevant to the process of creation, and by the ways in which these factors take part in the model and affect the process of creativity.

A Mathematical Model for Creativity

Measuring different capacities for the act of creation and measuring conceptual jumps in a given material can only by spoken of, if we have a concrete understanding of what creative learning is. This is a difficult task. But we can suggest an approximation. We say that we have learned something, if we can apply it in computations. We say that we have learned something new, if we can apply it to perform computations easier and faster. Therefore, using this model, we shall recognize a mathematical knowledge, only if it has computational implications. This is a very limiting definition of learning, but it is appropriate since with this point of view, we have to deal with understanding computational skills of students, which is very practical. This approach is compatible with brain's function. Our brain makes new concepts so that it can compute with a minimum occupation of memory. For example, it conceptualizes three dimensions, because this way it can perform its everyday duties with less memory space occupation. This approach to the process of learning makes the above-mentioned measurements possible.

Creative Jumps in Computations

A curriculum which is developed according to computational goals is very different from a curriculum, which is organized according to general mathematical content. In order to apply our model, we have to be conscious of the exact computational reasons for the consideration of a mathematical concept in the curriculum. When the exact computational goals are identified, one of the tools for understanding the psychology of students' computations is the concept map of what they have learned. The concept map provides us a language in which we can speak of creative jumps in computations. Here is a summarized expert's concept map of "function" developed by Williams [2]:



One can recognize different levels of sophistication in computation by means of a concept map. Here we list a few elementary skills:

i. The ability to recognize each of the arrows related to a computational problem.
ii. Composition of two arrows without it being mentioned directly by the problem.
iii. Composition of three or more of the arrows.
iv. The ability to go through arrows backwards.
v. Composition of arrows some of which are being composed backwards.
vi. The ability to create new arrows in the concept map.
vii. Partial rearrangement of the concept map.
viii. Reformulation of the whole concept map to have fewer concepts and more structured relations.

If the educator can form a concept map of the background of students in a given subject, he/she would be able to propose appropriate problems to them and work on improving their sophistication in computations. Therefore it is reasonable to suggest that students should have a file of concept maps filled by their previous teachers, so that their teacher can refer to these files and use them in helping students. When students become able to perform independent studies, they should be taught how to draw their own concept mappings related to each subject.

Educational Systems Emphasizing on Mathematical Maturity

The process of education in such systems is a smoother process. An educational system which is directed towards educating students to have a mathematical character is very different from a system educating students to be a good problem solver. The student is expected to personally develop

mathematical insight out of computations. Contrary to a classroom in the previous system, abstract mathematical theories are the fruit of computations, different formulations, applications, and relations discovered between a group of problems. Trying to develop mathematical theories out of raw data has the advantage that it is consistent with the formation of the concept map in our mind. In an educational system based on problem solving, students are introduced to concepts and are asked to try for discovering arrows connecting these concepts. But in an educational system based on mathematical maturity, students are asked to create concepts themselves and to draw their own personal concept maps. The role of teacher will only be giving guidelines and asking the right questions. And at the end of each session the teacher summarizes what has been discussed and gives a precise formulation for the abstract mathematical perspectives introduced by students. We are interested in the psychology of mathematical communication in such a classroom.

‘Parent, Child, Adult’ and ‘Emotion, Reason’ Revisited

The teacher giving guidelines and asking questions in classroom could use the data recorded by his/her ‘Parent’ which consists of what he/she has been taught by his/her own teachers as a student. In an educational system emphasizing on mathematical maturity, questions being the main means of communication between the students and teacher are welcome. So the birth of ‘Adult’ in a student is not a matter that shall be urgently replied to by the educator. Of course the attention of the class could be misguided by questions asked by students. In this case, the teacher’s ‘Adult’ should come to action trying to analyze this new material which is not recorded in his/her ’Parent’. But still if there is delay, an ‘Adult’ to ‘Adult’ response is not as disturbing a question as the same situation in a problem solving system.

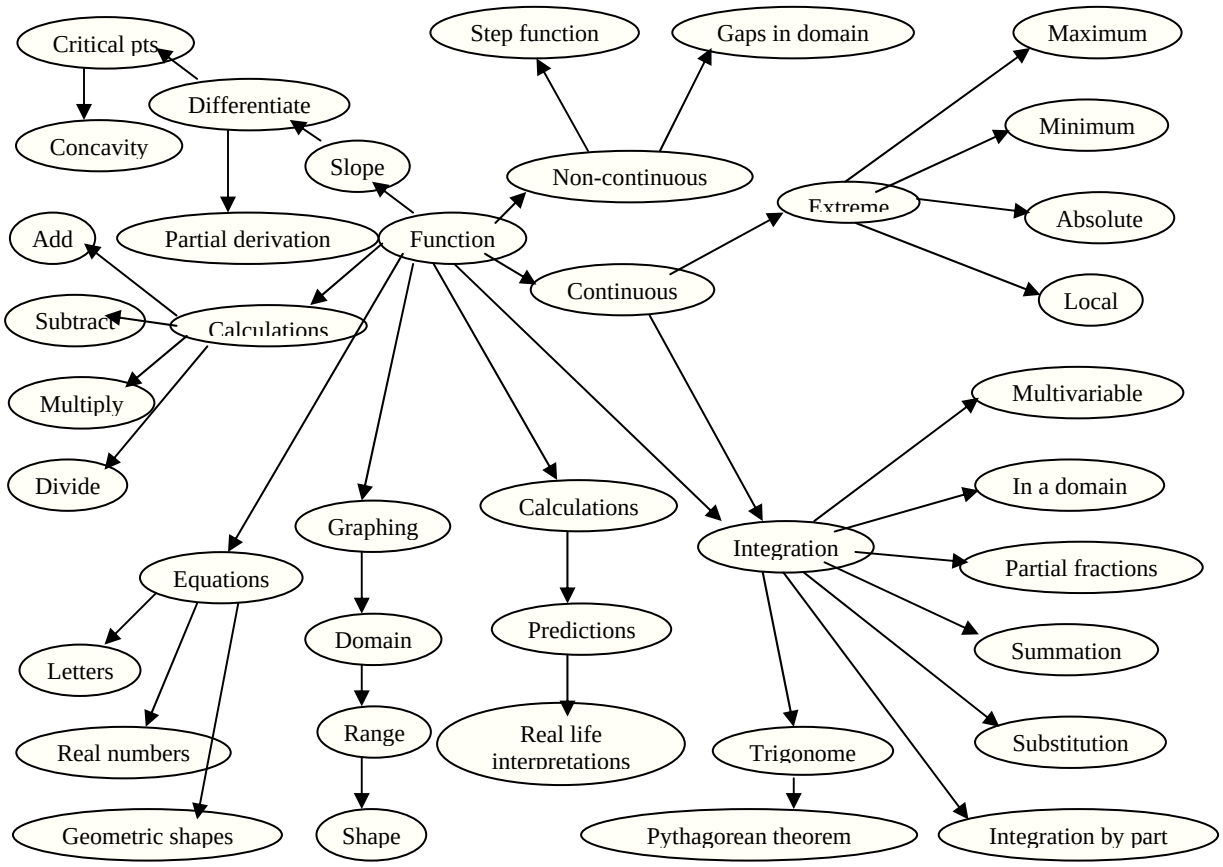
On the other hand, ‘Child’ appearing in the classroom as a teacher, represents personal wonders and achievements of the teacher, as a student. Since the goal is not teaching skills using some mental structure, but helping students develop an understanding with their personal systems, ‘Child’ as an educator is completely irrelevant to the classroom and its appearance in the education process is much more disturbing than in the problem solving system.

The point of view that birth of ‘Reason’ is the main goal of education is strongly implemented in such a system. But students need not be personally treated to give birth to their ‘Reason’. The social force of group thinking in the classroom makes a very creative atmosphere for the ‘Emotion’. This achievement of such an educational system is based on the fact that group learning is very common in human nature, but it is not the case about creativity and becoming skilled in previously defined aspects of problem solving. In order to understand the learning system in such a classroom, we shall give an explicit model approximating the process of mathematical group learning.

A Model for Group Thinking

The formation of concept maps which are very similar to human system of learning is not necessarily a good model for group learning as a social procedure. Since group thinking forces new concept maps on individuals, group thinking shall be related to concept maps anyway. The social process of group thinking can be understood and completely characterized by the social interaction between two persons. This kind of interaction shall be discovered and learned by ‘Emotion’ of students. As soon as everyone in the classroom is mature in this kind of interaction, the formation of a group concept map is a fluent process.

What we shall understand better is the nature of our mind in summarizing data, and changing personal concept map according to this data. The first process is based on language skills. So we shall concentrate on teaching students to compare concept maps and form their own. Let us start with an example. Here is another concept map for ‘function’ introduced by Williams [2]:



We want to know better about how our mind compares this concept mapping with the previous one. We recognize the following steps in comparison of two concept maps:

i. Identifying common concepts.

ii. Identifying common arrows in the two concept mappings.

iii. Representing one arrow as a combination of arrows in the other map.

iv. Adding new concepts which appear in these arrows to the concept mapping.

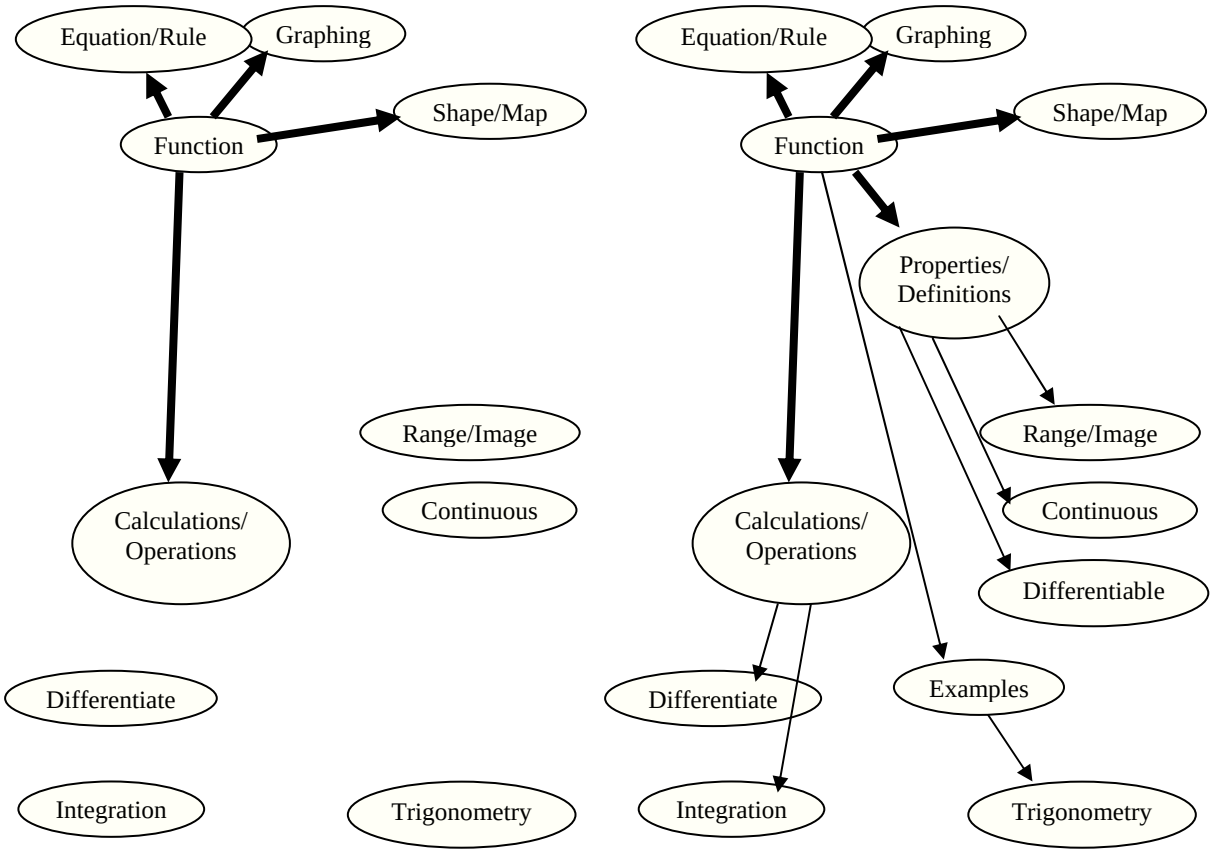
v. Adding new arrows relating the new concepts to the old concepts.

vi. Going back to the third step.

Let us go through the steps to see what happens to the student's concept map when it faces the expert's. The first step is recognition of common concepts between the two concept maps and drawing the common arrows, and the second step is to discover new concepts, which relate the old concepts by composition of arrows.

In the following diagrams, common concepts and common relations are indicated on the left side, and on the right, new concepts which relate the old ones are added.

One can immediately recognize that our algorithm can not develop student's concept mapping any further. This is because expert's concept mapping is not suited for educational purposes. So, what we shall do now is to introduce a suitable structure for a concept map which makes it easy to learn from it. Of course, we are limiting ourselves to the mathematical model we have introduced for learning.

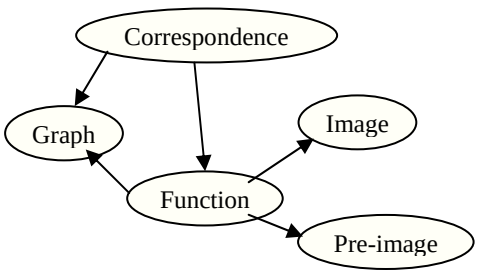


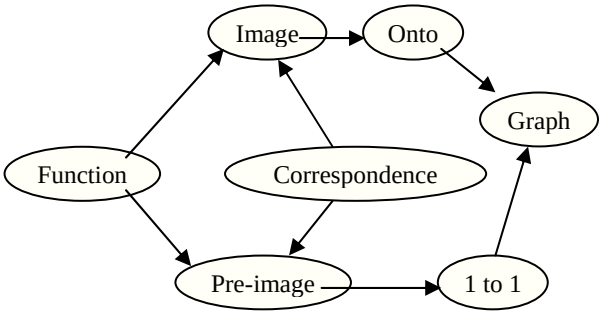
Concept Maps and Birth of Perspectives

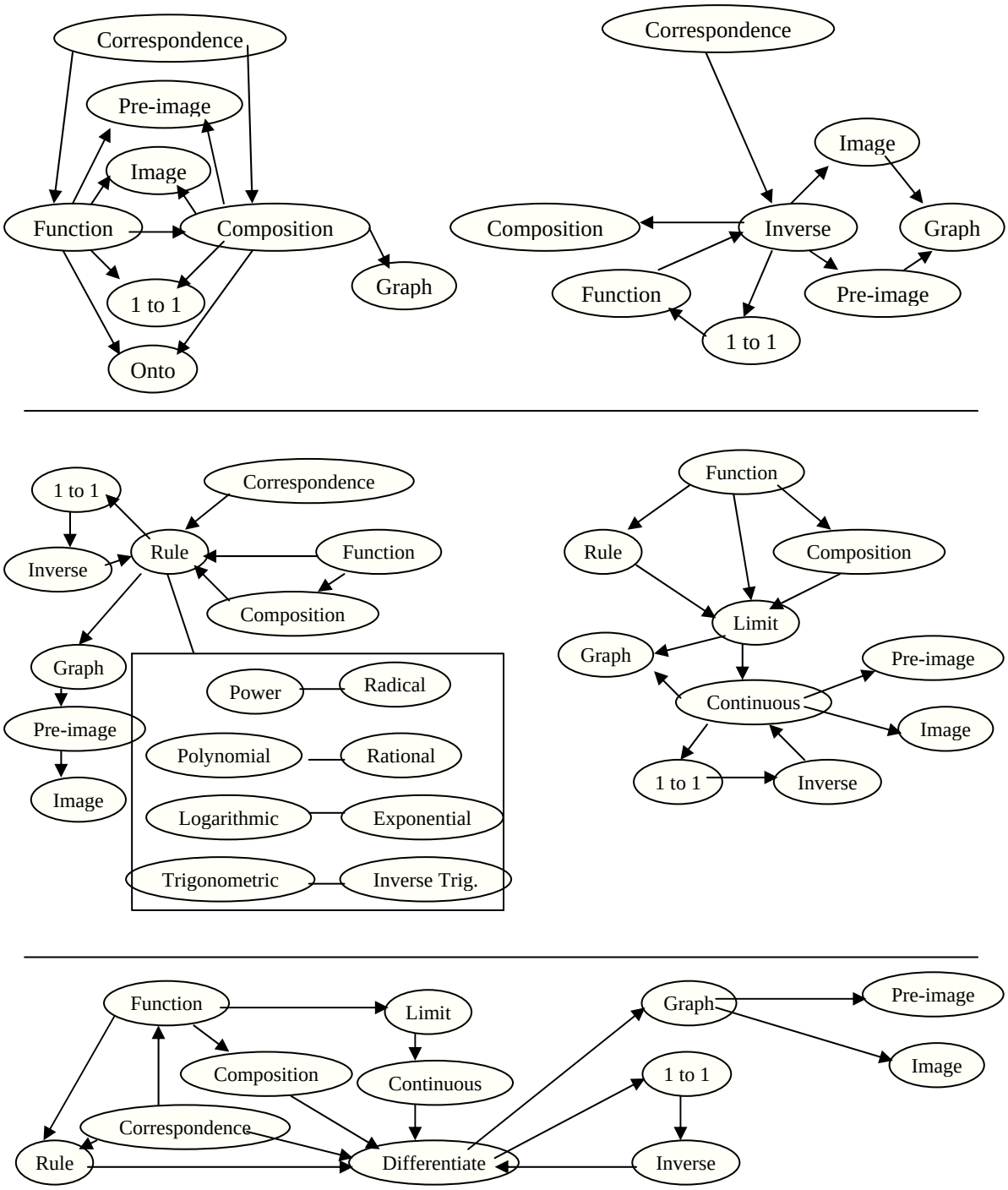
What we are interested in an educational system emphasizing on mathematical maturity is to help students to create their own concept maps. In order to give them enough knowledge so that they have freedom of choice in building their concept map, we have to follow the rules of nature in development of knowledge. It is not reasonable to expect that experts in education shall be experts in history of science too. But at least they shall be able to make a model of how the subject matter they are teaching came into existence. The element we find very important is the birth of new perspectives. We will see that this point of view will affect our concept mappings in a suitable manner.

From educational perspectives, it is important for students to see their knowledge from many different perspectives. This will help them to have a healthier feeling about scientific truth. This has definitely been the case in history of science. New concepts, which gave us new scientific power, always appeared after birth of new perspectives of elder concepts, which were motivated by new unsolved problems. To have this picture in our educational model, we have to replace a concept map by a series of concept maps, each being suggested by a new perspective. We call it “The Atlas of History of concepts”. The concepts in this Atlas shall be an increasing family with omissions. In each map, arrows are connected according to the perspective, which is introduced by that map.

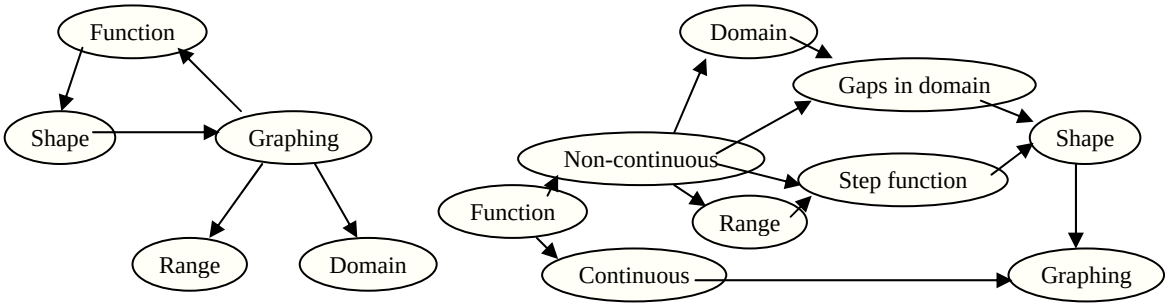
In such an atlas, every new concept will be connected to other concepts by several arrows in later maps. The atlas is organized in such a way that new concepts are always detected via the algorithm we introduced for comparison of two concept mappings in our mind. Indeed, in every new map, new concepts are supposed to be on the pathway between two concepts from the previous maps. An example of an atlas of concept would be the following:

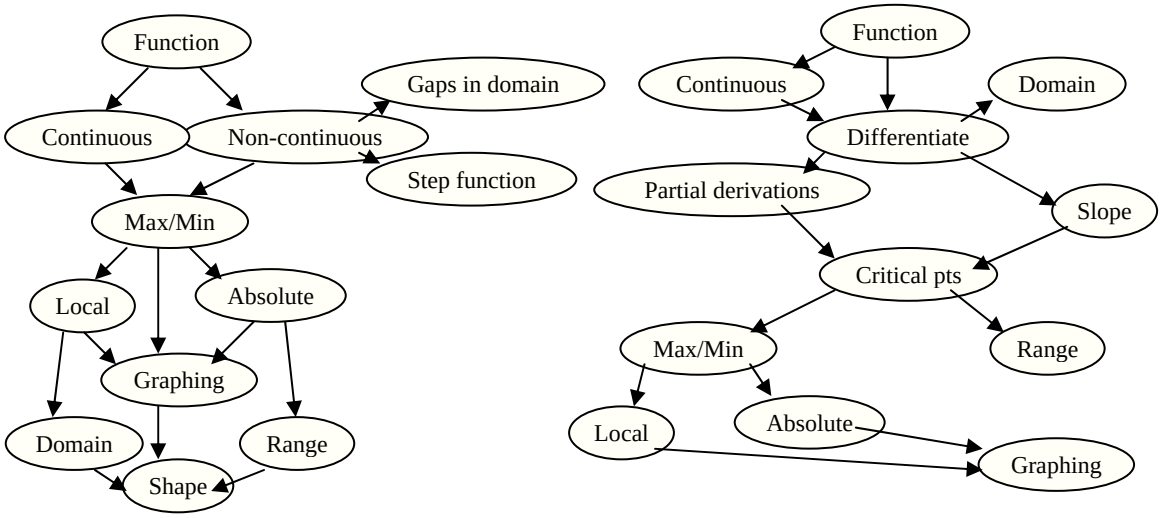


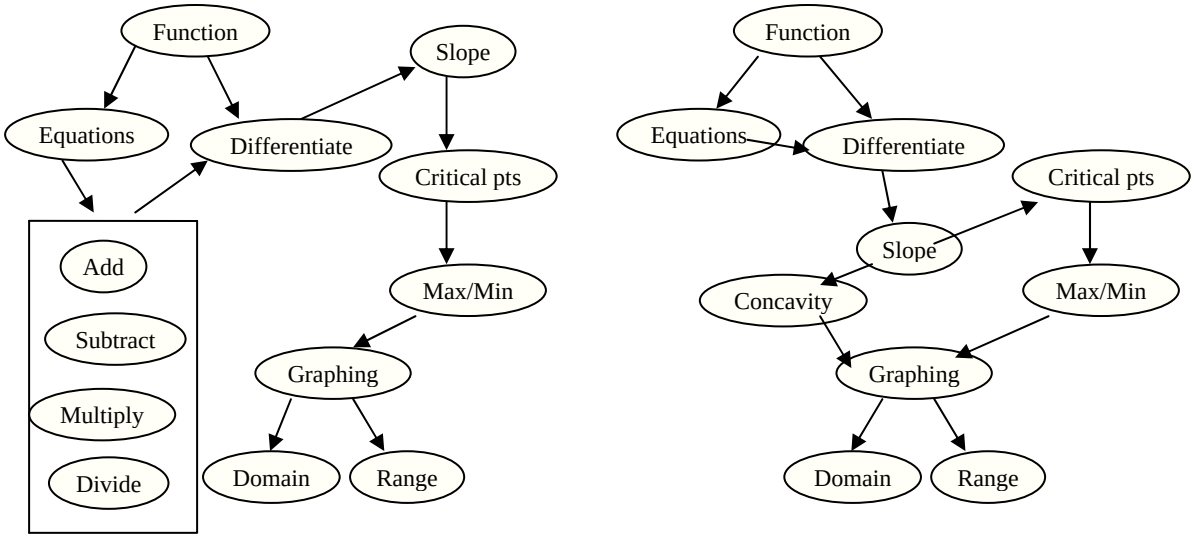


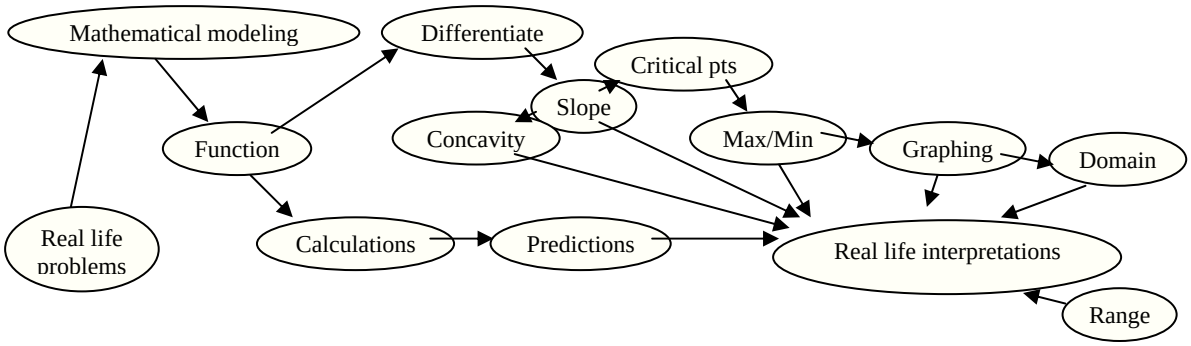


The above atlas is part of the atlas suggested for expert's concept mapping. We suggest this model for the data given to students by the class. We shall investigate how two atlases affect each other. In order to do such an investigation, we shall suggest another atlas. Here we give an atlas for student's concept map:


Function
Shape
Graphing
Range
Domain
Domain
Gaps in domain
Shape
Non-continuous
Step function
Function
Range
Continuous
Graphing


Function
Gaps in domain
Continuous
Non-continuous
Step function
Max/Min
Local
Absolute
Graphing
Domain
Range
Shape
Function
Continuous
Domain
Differentiate
Partial derivations
Slope
Critical pts
Max/Min
Range
Local
Absolute
Graphing


Function
Slope
Equations
Differentiate
Critical pts
Add
Subtract
Multiply
Divide
Max/Min
Graphing
Domain
Range
Function
Equations
Differentiate
Critical pts
Slope
Concavity
Max/Min
Graphing
Domain
Range



Now, we suggest an algorithm for student's atlas being changed under the influence of another atlas. This will be a model for the atom of social interaction in classroom. This process will cause formation of a global atlas as a result of group thinking. We recognize the following steps in comparison of two atlases:

i. Identifying concepts in the first map, which are not present in personal atlas.

ii. Considering each new concept as a concept relating elder concepts.

iii. If the old concepts fit in a single map in personal atlas, insert the new concept.

iv. If not, by taking union of two appropriate maps in the personal atlas, make a new map and then insert the new concept.

v. When all the new concepts in the first map are planted in the personal atlas, follow the same procedure for the next map in the given atlas.

This is a procedure, which supports absorption of most of the new concepts to the personal atlas of the student. Of course, basic concepts should be common. Otherwise many important parts of the atlas will not be copied by the above procedure. For example, the concept of "correspondence" is an elementary concept in the expert's atlas which does not appear in the student's atlas. So, from this point of view, it is important

to introduce basic concepts in full generality. Another good example is the concept of “pre-image”, which could be understood better in the presence of the concept of “correspondence”. But now it is supposed to replace the concept of “domain”, which is a more primitive concept. Here we face a new aspect of social communication: the process of evolution of concepts. There are students who formally accept new concepts in their atlas. But for some creative students, concepts are alive and are in constant change. We shall produce a mathematical model for this process.

References

1- T. Harris :”I’m OK –You’re OK”, Avon Books, NY 1973

2- Williams C.G.: Using concept maps to assess conceptual knowledge of function. J. for Research in Math. Education, 29(1998) pp.414-421.