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An embedding theorem for Sobolev type functions with gradients in a Lorentz space. (English summary)

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In this paper the author generalizes the well-known Morrey embedding theorem

$$\sup_{x,y\in B(a,R)} |f(x)-f(y)| \le CR^{1-n/p}\|\nabla f\|_{L^p}, \quad n<p.$$

First he uses the definition of Sobolev spaces of the first order on measurable metric spaces and proves an analogue version of the previous inequality with the $L^{n,1}$-norm on the right-hand side.

Using this result the author proves some properties concerning the continuity and differentiability of functions with gradient from $L^{n,1}$.

Reviewed by *Aleš Nekvinda*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.