General Physics I

chapter 7

Sharif University of Technology
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7-1 What is Physics? Energy:

Technically, energy is a scalar quantity associated with the state of one or more objects.

Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is *conserved*). No exception to this *principle of energy conservation* has ever been found.

Energy:

Forms of energy:

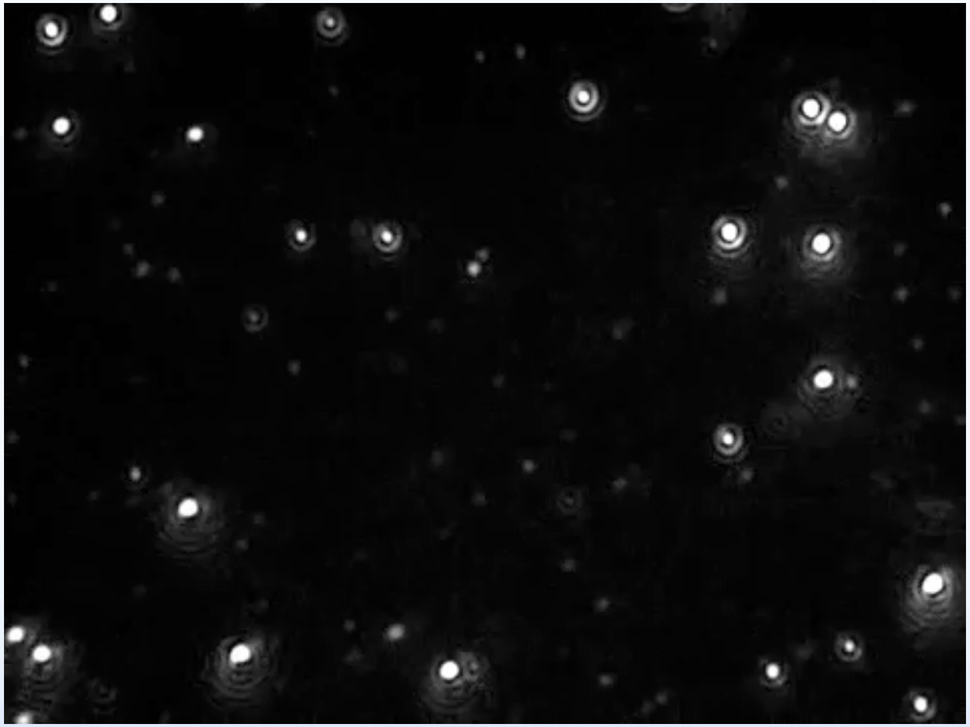
- ✓ Mechanical
- ✓ Chemical
- ✓ Electromagnetic
- ✓ Internal

- Energy can be transformed from one form to another

Essential to the study of physics, chemistry, biology, geology, astronomy

- Can be used in place of Newton's laws to solve certain problems more simply

Internal Energy



https://www.youtube.com/watch?time_continue=62&v=grqktnmAO3c

Kinetic Energy

For an object of mass m whose speed **v** is well below the speed of light,

$$K = \frac{1}{2}m(\mathbf{v}.\mathbf{v}) = \frac{1}{2}m|\mathbf{v}|^2$$

$$[K] = kg.m^2/s^2 = J$$

Sample Problem 7-1

In 1896 in Waco, Texas, William Crush of the "Katy" railroad parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed in front of 30,000 spectators.

- Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration along the track was a constant 0.26 m/s^2, what was the total kinetic energy of the two locomotives just before the collision?

SOLUTION:

$$v^2 = v_0{}^2 + 2a(x - x_0)$$

$$v^2 = 0 + 2(0.26\ m/s^2)(3.2*10^3\ m)$$

$$v = 40.8\ m/s$$

$$m = \frac{1.2*10^6\ N}{9.8\ m/s^2} = 1.22*10^5\,kg$$

$$K = 2(\tfrac{1}{2}mv^2) = (1.22*10^5\ kg)(40.8\ m/s)^2$$

$$= 2.0*10^8\,J$$

Section 7-2

7-2 WORK AND KINETIC ENERGY

Learning Objectives

After reading this module, you should be able to . . .

7.03 Apply the relationship between a force (magnitude and direction) and the work done on a particle by the force when the particle undergoes a displacement.

7.04 Calculate work by taking a dot product of the force vector and the displacement vector, in either magnitude-angle or unit-vector notation.

7.05 If multiple forces act on a particle, calculate the net work done by them.

7.06 Apply the work–kinetic energy theorem to relate the work done by a force (or the net work done by multiple forces) and the resulting change in kinetic energy.

Key Ideas

- Work W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.
- The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

$$W = Fd\cos\phi = \vec{F}\cdot\vec{d} \quad \text{(work, constant force)},$$

in which ϕ is the constant angle between the directions of $\vec{F}$ and $\vec{d}$.

- Only the component of $\vec{F}$ that is along the displacement $\vec{d}$ can do work on the object.
- When two or more forces act on an object, their net work is the sum of the individual works done by the forces, which is also equal to the work that would be done on the object by the net force $\vec{F}_{net}$ of those forces.
- For a particle, a change ΔK in the kinetic energy equals the net work W done on the particle:

$$\Delta K = K_f - K_i = W \quad \text{(work–kinetic energy theorem)},$$

in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done. The equation rearranged gives us

$$K_f = K_i + W.$$

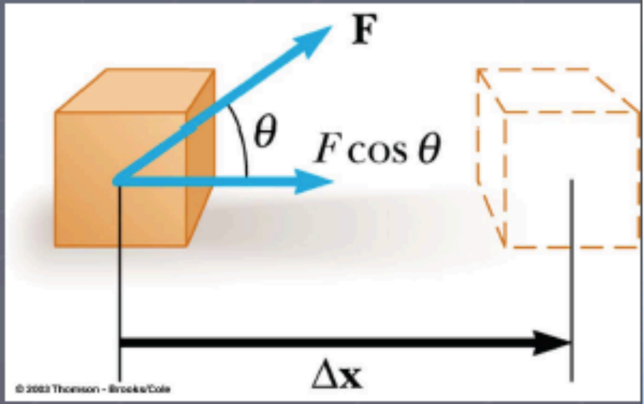
Work

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

Work
(Done by a Constant! Force)

- Provides a link between force and energy
- The work, *W*, done by a constant force on an object is defined as the product of the *component of the force along the direction of displacement* and *the magnitude of the displacement*

$$W \equiv (F\cos\theta)\Delta x$$



  - $(F\cos\theta)$ is the component of the force in the direction of the displacement
  - Δx is the displacement

Work

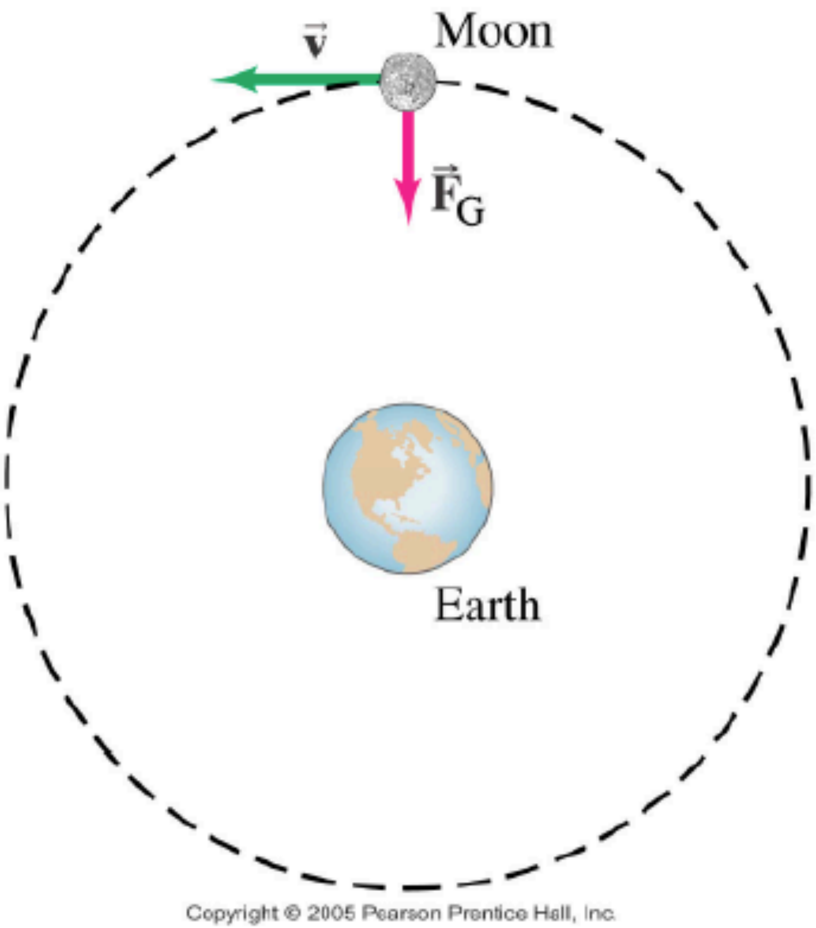
- This gives no information about
  - the time it took for the displacement to occur
  - the velocity or acceleration of the object
- Note: work **is** zero when
  - there is no displacement (holding a bucket)
  - force and displacement are perpendicular to each other, as $\cos 90° = 0$ (if we are carrying the bucket horizontally, gravity does no work)

$$W \equiv (F\cos\theta)\Delta x$$

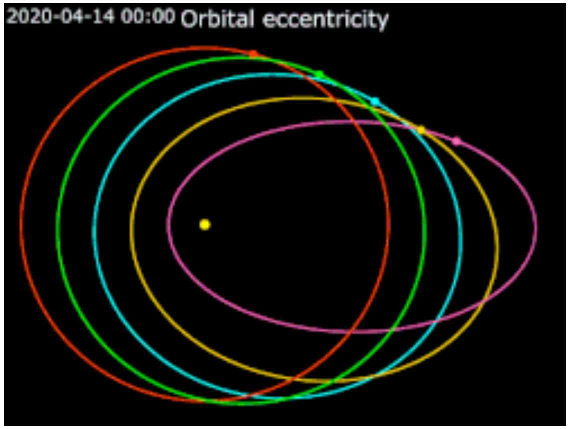
(different from everyday "definition" of work)

Example

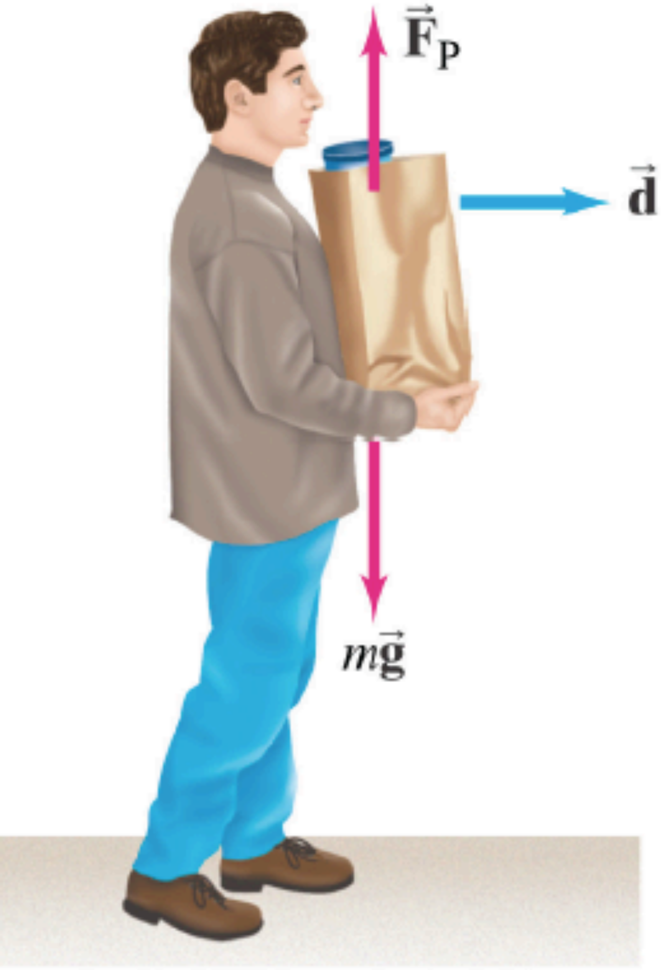
Work done by forces that oppose the direction of motion, such as friction, will be negative.



Centripetal forces do no work, as they are always perpendicular to the direction of motion.



Example



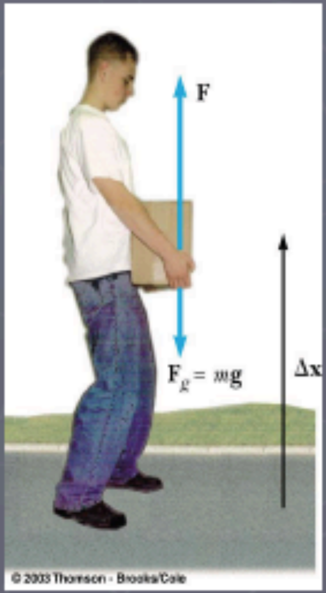
As long as this person does not lift or lower the bag of groceries, he is doing **no work** on it. The force he exerts has no component in the direction of motion.

Work

- ✓ If there are multiple forces acting on an object, the total work done is the algebraic sum of the amount of work done by each force.

Work

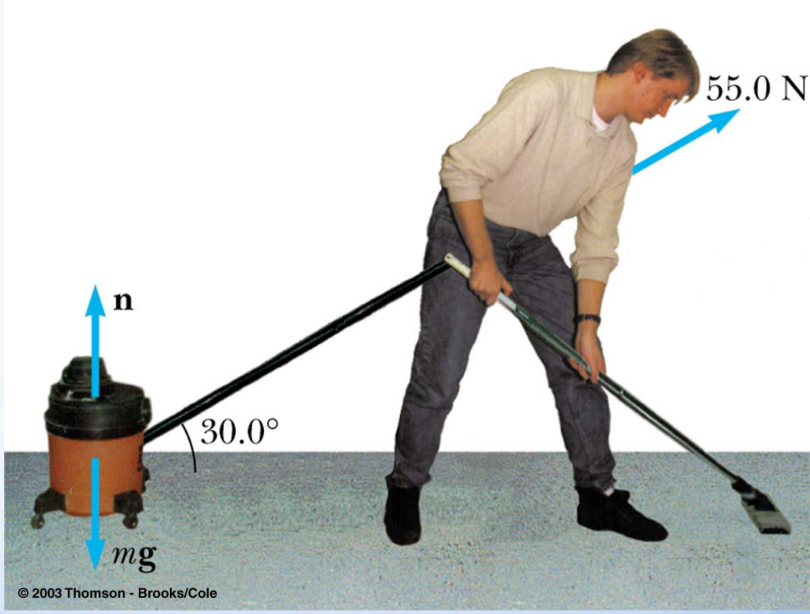
- Work can be positive or negative
  - Positive if the force and the displacement are in the same direction
  - Negative if the force and the displacement are in the opposite direction
- Example 1: lifting a cement block…
  - Work done by the person:
    - is positive when lifting the box
    - is negative when lowering the box
- Example 2: … then moving it horizontally
  - Work done by gravity:
    - is negative when lifting the box
    - is positive when lowering the box
    - is zero when moving it horizontally



$$\underline{Total\ work}: W = W_1 + W_2 + W_3 = \underbrace{-mgh}_{\text{lifting}} + \underbrace{mgh}_{\text{lowering}} + \underbrace{0}_{\text{moving}} = \underbrace{0}_{\text{total}}$$

Example: cleaning the dorm room

John decided to clean his dorm room with his vacuum cleaner. While doing so, he pulls the canister of the vacuum cleaner with a force of magnitude F=55.0 N at an angle 30.0°. He moves the vacuum cleaner a distance of 3.00 meters. Calculate the work done by all
the forces acting on the canister.



142.9 J

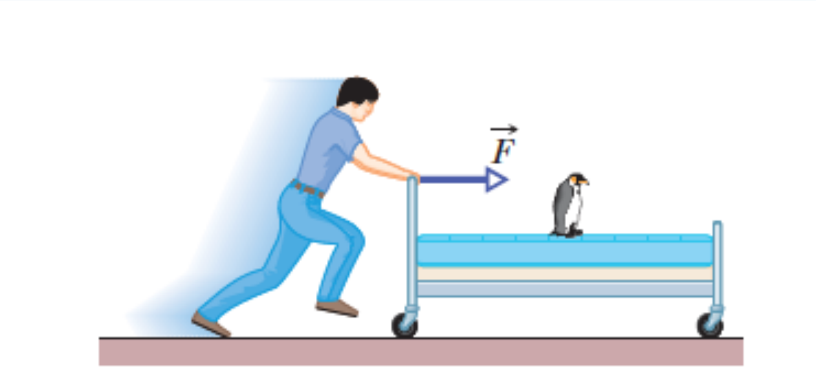
Work and Kinetic Energy

(Constant Force 1D)

$$F_x = ma_x$$

$$v^2 = v_0^2 + 2a_x d$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$$



Work and Kinetic Energy

(Constant Force 3D)

$$F_x = ma_x$$
$$v_x^2 - v_{0x}^2 = 2a_x x$$
$$\frac{1}{2}mv_x^2 - \frac{1}{2}mv_{0x}^2 = F_x x$$

$$F_y = ma_y$$
$$v_y^2 - v_{0y}^2 = 2a_y y$$
$$\frac{1}{2}mv_y^2 - \frac{1}{2}mv_{0y}^2 = F_y y$$

$$F_z = ma_z$$
$$v_z^2 - v_{0z}^2 = 2a_z z$$
$$\frac{1}{2}mv_z^2 - \frac{1}{2}mv_{0z}^2 = F_z z$$

$$\frac{1}{2}m|\mathbf{v}|^{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m}|\mathbf{v_0}|^{\mathbf{2}} = \mathbf{F} \cdot \mathbf{r}$$

Work–Kinetic Energy Theorem

$$\Delta K = K_f - K_i = W$$

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}.$$

$$K_f = K_i + W$$

$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}.$$

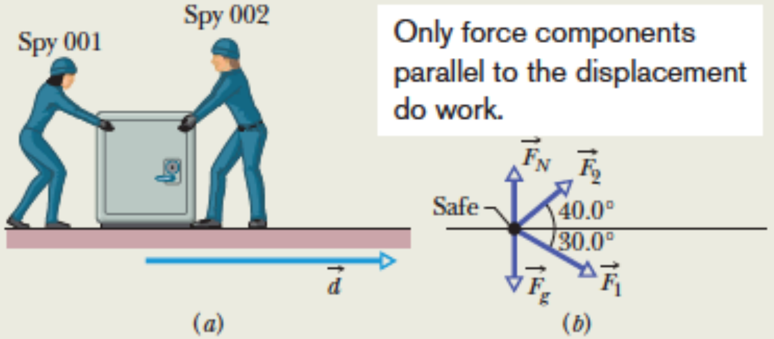
Checkpoint 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

Sample Problem 7.02 Work done by two constant forces, industrial spies

Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m. The push $\vec{F}_1$ of spy 001 is 12.0 N at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?



Only force components parallel to the displacement do work.

Figure 7-4 (*a*) Two spies move a floor safe through a displacement $\vec{d}$. (*b*) A free-body diagram for the safe.

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

KEY IDEAS

(1) The net work W done on the safe by the two forces is the sum of the works they do individually. (2) Because we can treat the safe as a particle and the forces are constant in both magnitude and direction, we can use either Eq. 7-7 ($W = Fd\cos\phi$) or Eq. 7-8 ($W = \vec{F}\cdot\vec{d}$) to calculate those works. Let's choose Eq. 7-7.

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4*b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d\cos\phi_1 = (12.0\ \text{N})(8.50\ \text{m})(\cos 30.0^\circ) = 88.33\ \text{J},$$

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d\cos\phi_2 = (10.0\ \text{N})(8.50\ \text{m})(\cos 40.0^\circ) = 65.11\ \text{J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33\ \text{J} + 65.11\ \text{J} = 153.4\ \text{J} \approx 153\ \text{J}. \quad \text{(Answer)}$$

During the 8.50 m displacement, therefore, the spies transfer 153 J of energy to the kinetic energy of the safe.

KEY IDEA

Because these forces are constant in both magnitude and direction, we can find the work they do with Eq. 7-7.

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd\cos 90^\circ = mgd(0) = 0 \quad \text{(Answer)}$$

and

$$W_N = F_N d\cos 90^\circ = F_N d(0) = 0. \quad \text{(Answer)}$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

KEY IDEA

The speed of the safe changes because its kinetic energy is changed when energy is transferred to it by $\vec{F}_1$ and $\vec{F}_2$.

Calculations: We relate the speed to the work done by combining Eqs. 7-10 (the work–kinetic energy theorem) and 7-1 (the definition of kinetic energy):

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\ \text{J})}{225\ \text{kg}}} = 1.17\ \text{m/s}. \quad \text{(Answer)}$$

Sample Problem 7.03 Work done by a constant force in unit-vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

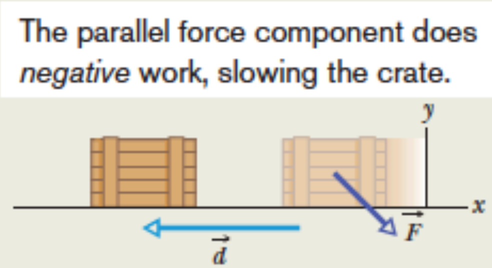


Figure 7-5 Force $\vec{F}$ slows a crate during displacement $\vec{d}$.

KEY IDEA

Because we can treat the crate as a particle and because the wind force is constant ("steady") in both magnitude and direction during the displacement, we can use either Eq. 7-7 ($W = Fd\cos\phi$) or Eq. 7-8 ($W = \vec{F}\cdot\vec{d}$) to calculate the work. Since we know $\vec{F}$ and $\vec{d}$ in unit-vector notation, we choose Eq. 7-8.

Calculations: We write

$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}]\cdot[(-3.0\text{ m})\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i}\cdot\hat{i}$, $\hat{j}\cdot\hat{j}$, and $\hat{k}\cdot\hat{k}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{i}\cdot\hat{i} + (-6.0\text{ N})(-3.0\text{ m})\hat{j}\cdot\hat{i}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

KEY IDEA

Because the force does negative work on the crate, it reduces the crate's kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}. \qquad \text{(Answer)}$$

Less kinetic energy means that the crate has been slowed.

WILEY PLUS Additional examples, video, and practice available at *WileyPLUS*

7-3 WORK DONE BY THE GRAVITATIONAL FORCE

Learning Objectives

After reading this module, you should be able to . . .

7.07 Calculate the work done by the gravitational force when an object is lifted or lowered.

7.08 Apply the work–kinetic energy theorem to situations where an object is lifted or lowered.

Key Ideas

- The work W_g done by the gravitational force $\vec{F}_g$ on a particle-like object of mass m as the object moves through a displacement $\vec{d}$ is given by

$$W_g = mgd \cos \phi,$$

in which ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

- The work W_a done by an applied force as a particle-like object is either lifted or lowered is related to the work W_g done by the gravitational force and the change ΔK in the object's kinetic energy by

$$\Delta K = K_f - K_i = W_a + W_g.$$

If $K_f = K_i$, then the equation reduces to

$$W_a = -W_g,$$

which tells us that the applied force transfers as much energy to the object as the gravitational force transfers from it.

Work Done by the Gravitational Force

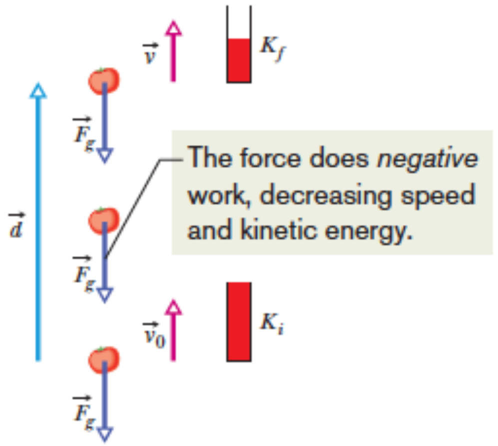
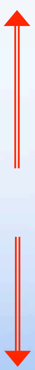


Figure 7-6 Because the gravitational force $\vec{F}_g$ acts on it, a particle-like tomato of mass m thrown upward slows from velocity $\vec{v}_0$ to velocity $\vec{v}$ during displacement $\vec{d}$. A kinetic energy gauge indicates the resulting change in the kinetic energy of the tomato, from $K_i (= \frac{1}{2}mv_0^2)$ to $K_f (= \frac{1}{2}mv^2)$.

$$W_g = mgd \cos \phi \quad \text{(work done by gravitational force).}$$



$$W_g = mgd \cos 180° = mgd(-1) = -mgd.$$

$$W_g = mgd \cos 0° = mgd(+1) = +mgd.$$

Work Done in Lifting and Lowering an Object

$$\Delta K = K_f - K_i = W_a + W_g,$$

$$W_a + W_g = 0$$

$$W_a = -W_g.$$

$$W_a = -mgd\cos\phi \quad \text{(work done in lifting and lowering; } K_f = K_i\text{)},$$

Sample Problem 7.05 Work done on an accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-9*a*).

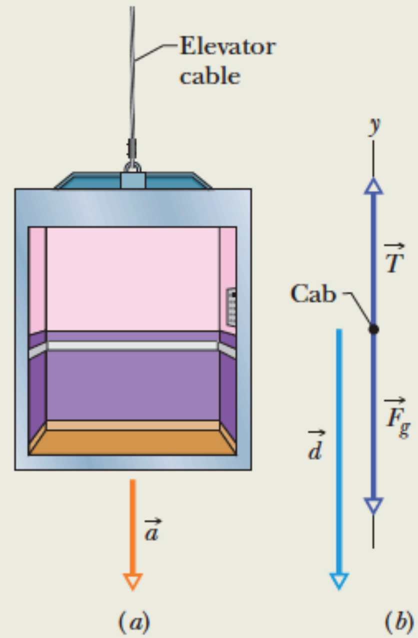
(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

(c) What is the net work W done on the cab during the fall?

(d) What is the cab's kinetic energy at the end of the 12 m fall?

Figure 7-9 An elevator cab, descending with speed v_i, suddenly begins to accelerate downward. (*a*) It moves through a displacement $\vec{d}$ with constant acceleration $\vec{a} = \vec{g}/5$. (*b*) A free-body diagram for the cab, displacement included.



Does negative work

Does positive work

Sample Problem 7.05 Work done on an accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-9*a*).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

KEY IDEA

We can treat the cab as a particle and thus use Eq. 7-12 ($W_g = mgd\cos\phi$) to find the work W_g.

Calculation: From Fig. 7-9*b*, we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. So,

$$W_g = mgd\cos 0° = (500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(1)$$
$$= 5.88 \times 10^4\text{ J} \approx 59\text{ kJ}. \quad \text{(Answer)}$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

KEY IDEA

We can calculate work W_T with Eq. 7-7 ($W = Fd\cos\phi$) by first writing $F_{net,y} = ma_y$ for the components in Fig. 7-9*b*.

Calculations: We get

$$T - F_g = ma. \quad (7\text{-}18)$$

Solving for T, substituting mg for F_g, and then substituting the result in Eq. 7-7, we obtain

$$W_T = Td\cos\phi = m(a+g)d\cos\phi. \quad (7\text{-}19)$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5} + g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$
$$= \frac{4}{5}(500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\cos 180°$$
$$= -4.70 \times 10^4\text{ J} \approx -47\text{ kJ}. \quad \text{(Answer)}$$

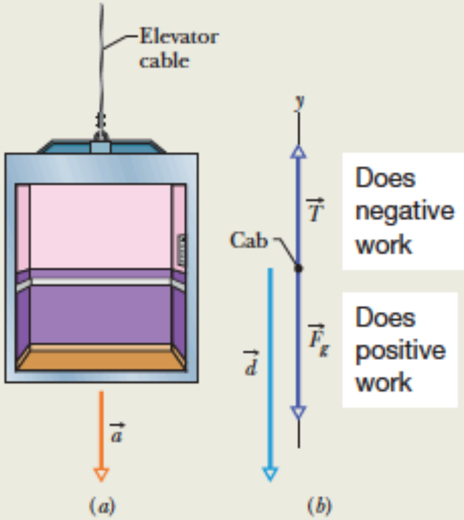


Figure 7-9 An elevator cab, descending with speed v_i, suddenly begins to accelerate downward. (*a*) It moves through a displacement $\vec{d}$ with constant acceleration $\vec{a} = \vec{g}/5$. (*b*) A free-body diagram for the cab, displacement included.

Caution: Note that W_T is not simply the negative of W_g because the cab accelerates during the fall. Thus, Eq. 7-16 (which assumes that the initial and final kinetic energies are equal) does not apply here.

(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88 \times 10^4\text{ J} - 4.70 \times 10^4\text{ J}$$
$$= 1.18 \times 10^4\text{ J} \approx 12\text{ kJ}. \quad \text{(Answer)}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

KEY IDEA

The kinetic energy changes *because* of the net work done on the cab, according to Eq. 7-11 ($K_f = K_i + W$).

Calculation: From Eq. 7-1, we write the initial kinetic energy as $K_i = \frac{1}{2}mv_i^2$. We then write Eq. 7-11 as

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500\text{ kg})(4.0\text{ m/s})^2 + 1.18 \times 10^4\text{ J}$$
$$= 1.58 \times 10^4\text{ J} \approx 16\text{ kJ}. \quad \text{(Answer)}$$

7-4 WORK DONE BY A SPRING FORCE

Key Ideas

- The force $\vec{F}_s$ from a spring is

$$\vec{F}_s = -k\vec{d} \quad \text{(Hooke's law)},$$

where $\vec{d}$ is the displacement of the spring's free end from its position when the spring is in its relaxed state (neither compressed nor extended), and k is the spring constant (a measure of the spring's stiffness). If an x axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, we can write

$$F_x = -kx \quad \text{(Hooke's law)}.$$

- A spring force is thus a variable force: It varies with the displacement of the spring's free end.
- If an object is attached to the spring's free end, the work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2.$$

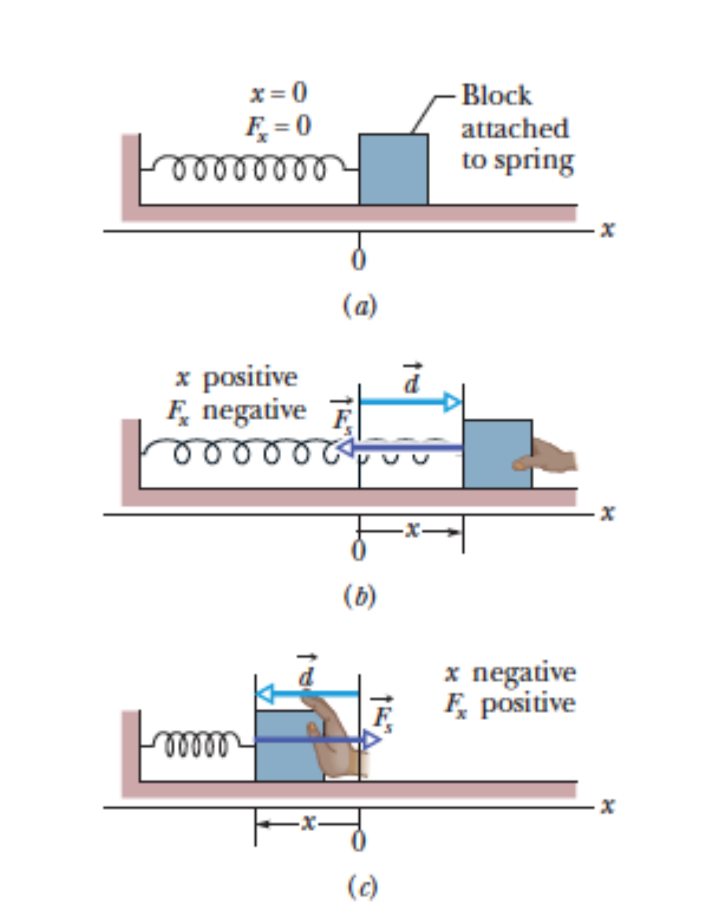
If $x_i = 0$ and $x_f = x$, then the equation becomes

$$W_s = -\tfrac{1}{2}kx^2.$$

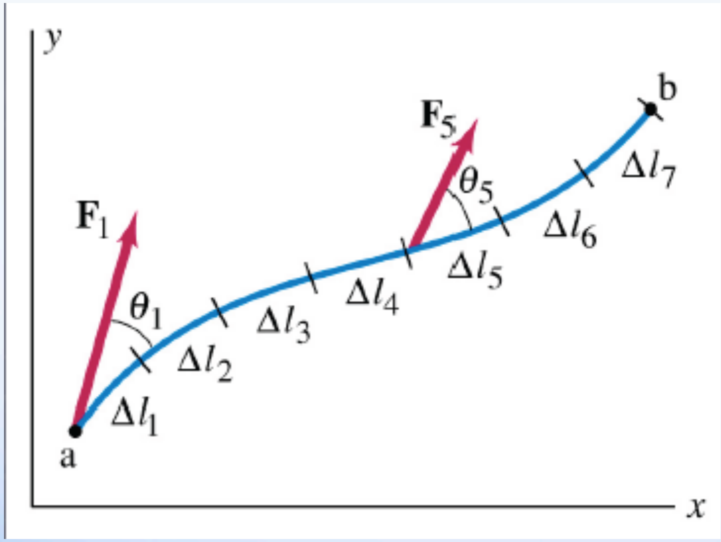
Work Done by a Spring Force

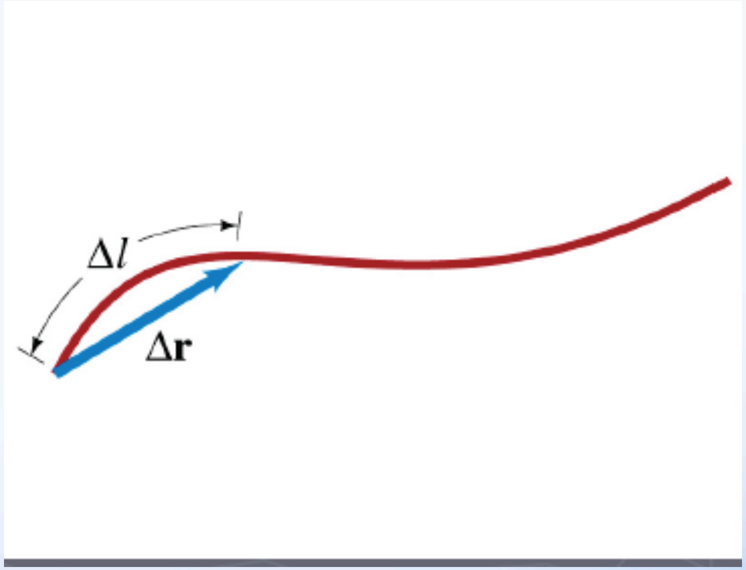
Hooke's law $\vec{F}_s = -k\vec{d}$

$$F_x = -kx$$



Work Done by a Varying Force





$$\Delta W = \mathbf{F} \cdot \Delta \mathbf{r}$$

Three-Dimensional Analysis

$$\vec{F} = F_x\hat{\mathrm{i}} + F_y\hat{\mathrm{j}} + F_z\hat{\mathrm{k}},$$

$$d\vec{r} = dx\hat{\mathrm{i}} + dy\hat{\mathrm{j}} + dz\hat{\mathrm{k}}.$$

$$dW = \vec{F}\cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz.$$

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

Example

Force $\vec{F} = (3x^2\ \text{N})\hat{\text{i}} + (4\ \text{N})\hat{\text{j}}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

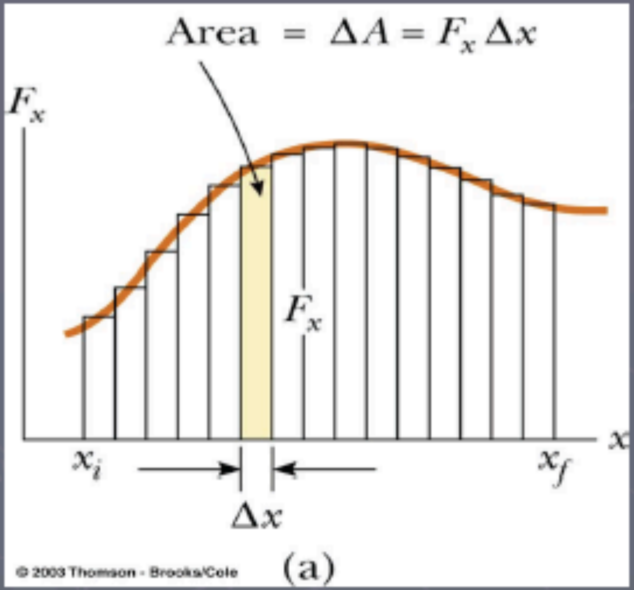
Calculation: We set up two integrals, one along each axis:

$$W = \int_2^3 3x^2\,dx + \int_3^0 4\,dy = 3\int_2^3 x^2\,dx + 4\int_3^0 dy$$

$$= 3[\tfrac{1}{3}x^3]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3]$$

$$= 7.0\text{ J.} \qquad \text{(Answer)}$$

In terms of components



- Split total displacement (x_f-x_i) into many small displacements Δx
- For each small displacement:

$$W_i = (F\cos\theta)\Delta x_i$$

- Thus, total work is:

$$W_{tot} = \sum_i W_i = \sum_i F_x \cdot \Delta x_i$$

$$W = \int_{x_i}^{x_f} F(x)\,dx \qquad \text{(work: variable force).}$$

which is total area under the F(x) curve!

In three dimensions

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy + \int_{z_i}^{z_f} F_z \, dz.$$

The Work Done by a Spring Force

$$W_s = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx$$

$$= (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2).$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \quad \text{(work by a spring force).}$$

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

The Work Done by a Spring Force

$x_i = 0$

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force).}$$

Work–Kinetic Energy Theorem

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx,$$

$$ma\,dx = m\frac{dv}{dt}\,dx.$$

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v,$$

$$ma\,dx = m\frac{dv}{dx}v\,dx = mv\,dv.$$

$$W = \int_{v_i}^{v_f} mv\,dv = m\int_{v_i}^{v_f} v\,dv$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

$$W = K_f - K_i = \Delta K,$$

Sample Problem 7.06 Work done by a spring to change kinetic energy

When a spring does work on an object, we *cannot* find the work by simply multiplying the spring force by the object's displacement. The reason is that there is no one value for the force—it changes. However, we can split the displacement up into an infinite number of tiny parts and then approximate the force in each as being constant. Integration sums the work done in all those parts. Here we use the generic result of the integration.

In Fig. 7-11, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

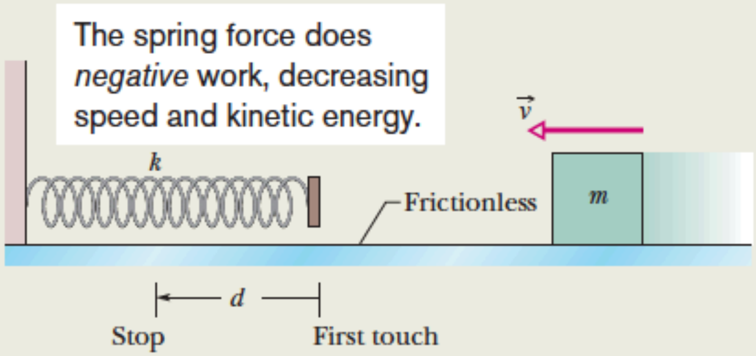


Figure 7-11 A canister moves toward a spring.

KEY IDEAS

1. The work W_s done on the canister by the spring force is related to the requested distance d by Eq. 7-26 ($W_s = -\frac{1}{2}kx^2$), with d replacing x.
2. The work W_s is also related to the kinetic energy of the canister by Eq. 7-10 ($K_f - K_i = W$).
3. The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression:

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50\text{ m/s})\sqrt{\frac{0.40\text{ kg}}{750\text{ N/m}}}$$

$$= 1.2 \times 10^{-2}\text{ m} = 1.2\text{ cm}. \qquad \text{(Answer)}$$

Sample Problem 7.07 Work calculated by graphical integration

In Fig. 7-13*b*, an 8.0 kg block slides along a frictionless floor as a force acts on it, starting at $x_1 = 0$ and ending at $x_3 = 6.5$ m. As the block moves, the magnitude and direction of the force varies according to the graph shown in Fig. 7-13*a*. For example, from $x = 0$ to $x = 1$ m, the force is positive (in the positive direction of the x axis) and increases in magnitude from 0 to 40 N. And from $x = 4$ m to $x = 5$ m, the force is negative and increases in magnitude from 0 to 20 N.

(Note that this latter value is displayed as −20 N.) The block's kinetic energy at x_1 is $K_1 = 280$ J. What is the block's speed at $x_1 = 0$, $x_2 = 4.0$ m, and $x_3 = 6.5$ m?

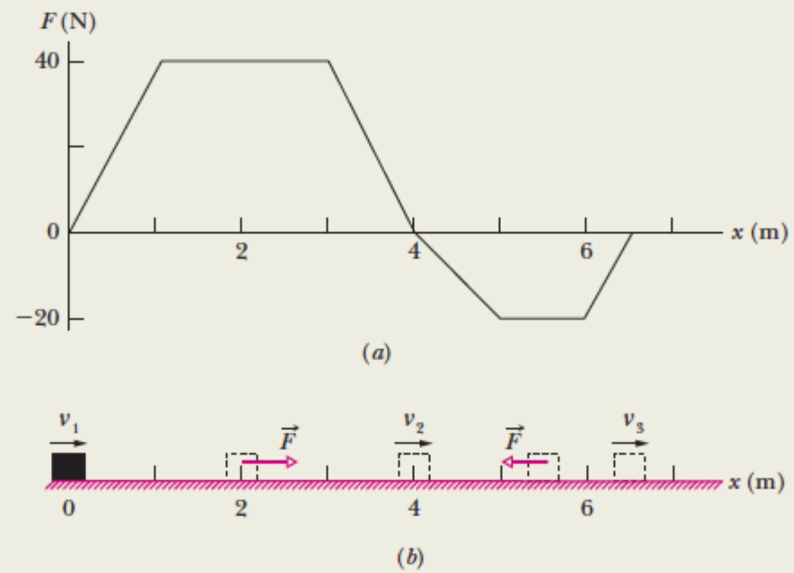


Figure 7-13 (*a*) A graph indicating the magnitude and direction of a variable force that acts on a block as it moves along an x axis on a floor, (*b*) The location of the block at several times.

(1) At any point, we can relate the speed of the block to its kinetic energy with Eq. 7-1 ($K = \frac{1}{2}mv^2$). (2) We can relate the kinetic energy K_f at a later point to the initial kinetic K_i and the work W done on the block by using the work–kinetic energy theorem of Eq. 7-10 ($K_f - K_i = W$). (3) We can calculate the work W done by a variable force $F(x)$ by integrating the force versus position x. Equation 7-32 tells us that

$$W = \int_{x_i}^{x_f} F(x)\, dx.$$

We don't have a function $F(x)$ to carry out the integration, but we do have a graph of $F(x)$ where we can integrate by finding the area between the plotted line and the x axis. Where the plot is above the axis, the work (which is equal to the area) is positive. Where it is below the axis, the work is negative.

Calculations: The requested speed at $x = 0$ is easy because we already know the kinetic energy. So, we just plug the kinetic energy into the formula for kinetic energy:

$$K_1 = \tfrac{1}{2}mv_1^2,$$

$$280\text{ J} = \tfrac{1}{2}(8.0\text{ kg})v_1^2,$$

and then

$$v_1 = 8.37\text{ m/s} \approx 8.4\text{ m/s.} \qquad \text{(Answer)}$$

As the block moves from $x = 0$ to $x = 4.0$ m, the plot in Figure 7-13*a* is above the x axis, which means that positive work is being done on the block. We split the area under the plot into a triangle at the left, a rectangle in the center, and a triangle at the right. Their total area is

$$\tfrac{1}{2}(40\text{ N})(1\text{ m}) + (40\text{ N})(2\text{ m}) + \tfrac{1}{2}(40\text{ N})(1\text{ m}) = 120\text{ N}\cdot\text{m} = 120\text{ J.}$$

This means that between $x = 0$ and $x = 4.0$ m, the force does 120 J of work on the block, increasing the kinetic energy and speed of the block. So, when the block reaches $x = 4.0$ m, the work–kinetic energy theorem tells us that the kinetic energy is

$$K_2 = K_1 + W = 280\text{ J} + 120\text{ J} = 400\text{ J.}$$

Again using the definition of kinetic energy, we find

$$K_2 = \tfrac{1}{2}mv_2^2,$$

$$400\text{ J} = \tfrac{1}{2}(8.0\text{ kg})v_2^2,$$

and then

$$v_2 = 10\text{ m/s.} \qquad \text{(Answer)}$$

This is the block's greatest speed because from $x = 4.0$ m to $x = 6.5$ m the force is negative, meaning that it opposes the block's motion, doing negative work on the block and thus decreasing the kinetic energy and speed. In that range, the area between the plot and the x axis is

$$\tfrac{1}{2}(20\text{ N})(1\text{ m}) + (20\text{ N})(1\text{ m}) + \tfrac{1}{2}(20\text{ N})(0.5\text{ m}) = 35\text{ N}\cdot\text{m} = 35\text{ J.}$$

This means that the work done by the force in that range is -35 J. At $x = 4.0$, the block has $K = 400$ J. At $x = 6.5$ m, the work–kinetic energy theorem tells us that its kinetic energy is

$$K_3 = K_2 + W = 400\text{ J} - 35\text{ J} = 365\text{ J.}$$

Again using the definition of kinetic energy, we find

$$K_3 = \tfrac{1}{2}mv_3^2,$$

$$365\text{ J} = \tfrac{1}{2}(8.0\text{ kg})v_3^2,$$

and then

$$v_3 = 9.55\text{ m/s} \approx 9.6\text{ m/s.} \qquad \text{(Answer)}$$

The block is still moving in the positive direction of the x axis, a bit faster than initially.

Conservative Forces

A force is conservative if the work it does on an object moving between two points is independent of the path the objects take between the points

- ✓ The work depends only upon the initial and final positions of the object
- ✓ Any conservative force can have a potential energy function associated with it

Note: a force is conservative if the work it does on an object moving through any closed path is zero.

Nonconservative Forces

A force is nonconservative if the work it does on an object depends on the path taken by the object between its final and starting points.

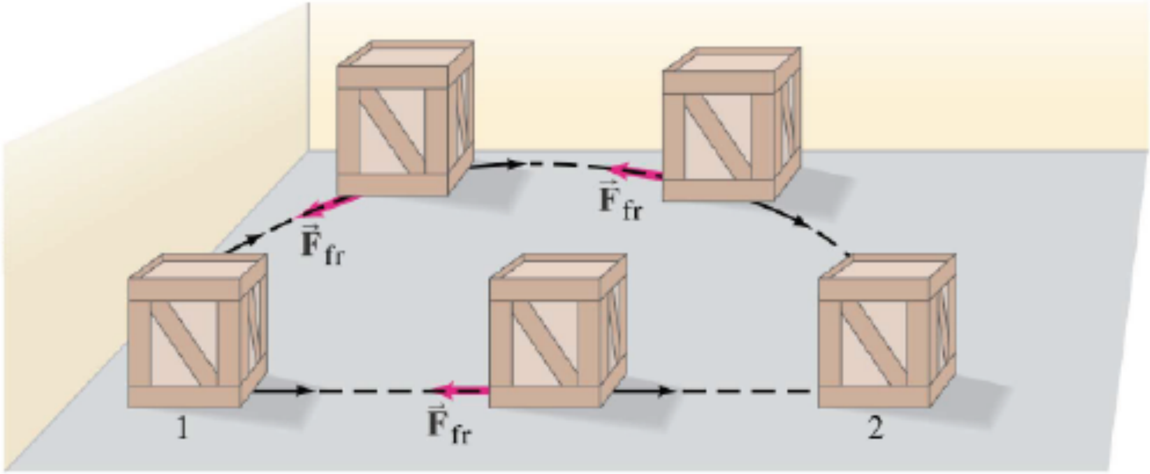
- Examples of nonconservative forces

✓ kinetic friction, air drag, ...

Example: Friction as a Nonconservative Force

- The friction force transforms kinetic energy of the object into a type of energy associated with temperature
  - the objects are warmer than they were before the movement
  - *Internal Energy* is the term used for the energy associated with an object's temperature

If friction is present, the work done depends not only on the starting and ending points, but also on the path taken. Friction is called a nonconservative force.



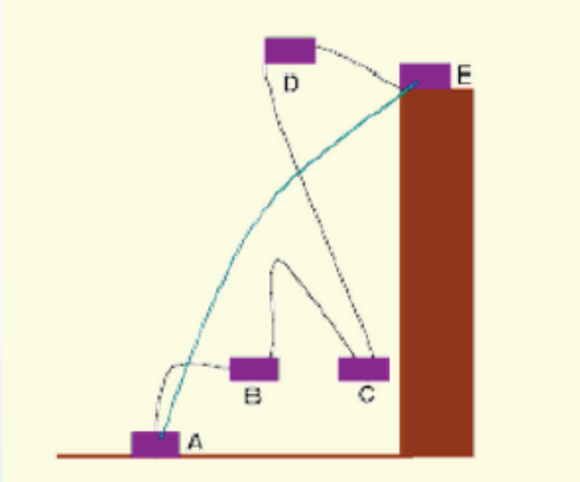


TABLE 6–1 Conservative and Nonconservative Forces

Conservative Forces	Nonconservative Forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propulsion
	Push or pull by a person

Potential energy can only be defined for conservative forces.

7-6 POWER

Key Ideas

- The power due to a force is the *rate* at which that force does work on an object.
- If the force does work W during a time interval Δt, the average power due to the force over that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}.$$

- Instantaneous power is the instantaneous rate of doing work:

$$P = \frac{dW}{dt}.$$

- For a force $\vec{F}$ at an angle ϕ to the direction of travel of the instantaneous velocity $\vec{v}$, the instantaneous power is

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v}.$$

Power

$$P_{avg} = \frac{W}{\Delta t} \quad \text{(average power).}$$

$$P = \frac{dW}{dt} \quad \text{(instantaneous power).}$$

1 watt = 1 W = 1 J/s = 0.738 ft · lb/s

1 horsepower = 1 hp = 550 ft · lb/s = 746 W.

$$P = \frac{dW}{dt} = \frac{F \cos\phi \, dx}{dt} = F \cos\phi \left(\frac{dx}{dt}\right),$$

$$P = Fv \cos\phi.$$

$$P = \vec{F} \cdot \vec{v} \quad \text{(instantaneous power).}$$

Sample Problem 7.09 Power, force, and velocity

Here we calculate an instantaneous work—that is, the rate at which work is being done at any given instant rather than averaged over a time interval. Figure 7-15 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

KEY IDEA

We want an instantaneous power, not an average power over a time period. Also, we know the box's velocity (rather than the work done on it).

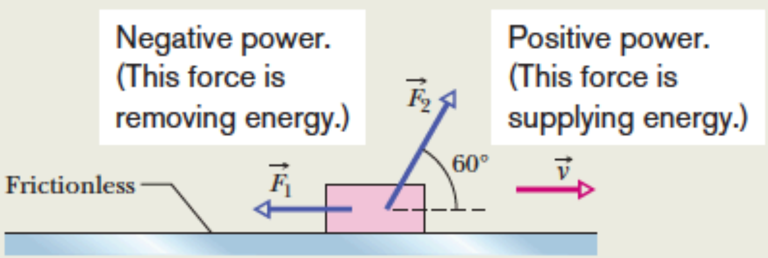


Figure 7-15 Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a box that slides rightward across a frictionless floor. The velocity of the box is $\vec{v}$.

Calculation: We use Eq. 7-47 for each force. For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v \cos \phi_1 = (2.0 \text{ N})(3.0 \text{ m/s}) \cos 180° = -6.0 \text{ W}. \qquad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

$$P_2 = F_2 v \cos \phi_2 = (4.0 \text{ N})(3.0 \text{ m/s}) \cos 60° = 6.0 \text{ W}. \qquad \text{(Answer)}$$

This positive result tells us that force $\vec{F}_2$ is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers (complete with their algebraic signs):

$$P_{net} = P_1 + P_2 = -6.0 \text{ W} + 6.0 \text{ W} = 0, \qquad \text{(Answer)}$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s. With neither the forces $\vec{F}_1$ and $\vec{F}_2$ nor the velocity $\vec{v}$ changing, we see from Eq. 7-48 that P_1 and P_2 are constant and thus so is P_{net}.

Example

Provided a funny car does not lose traction, the time it takes to race from rest through a distance D depends primarily on the engine's power P. Assuming the power is constant, derive the time in terms of D and P.

Example

62 A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of $k =$ 2.5 N/cm (Fig. 7-46). The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping. While the spring is being compressed, what work is done on the block by (a) the gravitational force on it and (b) the spring force? (c) What is the speed of the block just before it hits the spring? (Assume that friction is negligible.) (d) If the speed at impact is doubled, what is the maximum compression of the spring?

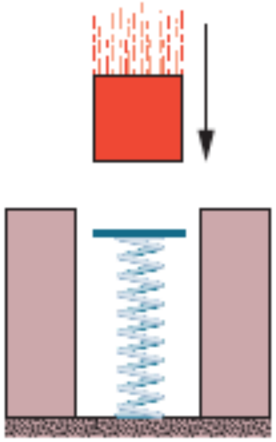


Figure 7-46 Problem 62.

Example

29. **S** A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a light cord that passes over the top of the cylinder as illustrated in Figure P7.29. (a) Assuming the particle moves at a constant speed, show that $F = mg \cos \theta$. *Note:* If the particle moves at constant speed, the component of its acceleration tangent to the cylinder must be zero at all times. (b) By directly integrating $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, find the work done in moving the particle at constant speed from the bottom to the top of the half-cylinder.

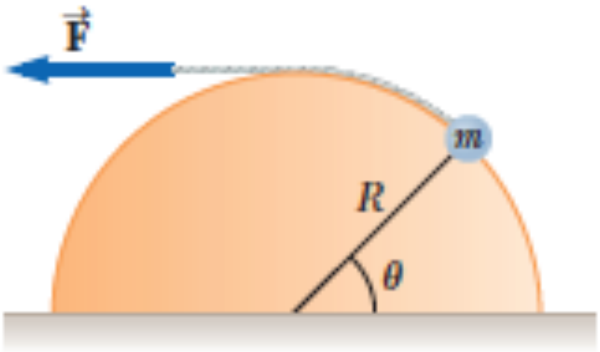


Figure P7.29

Example

•••42 GO Figure 7-41 shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left

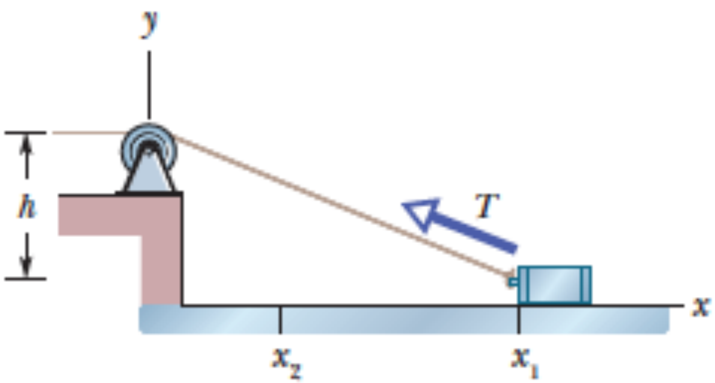


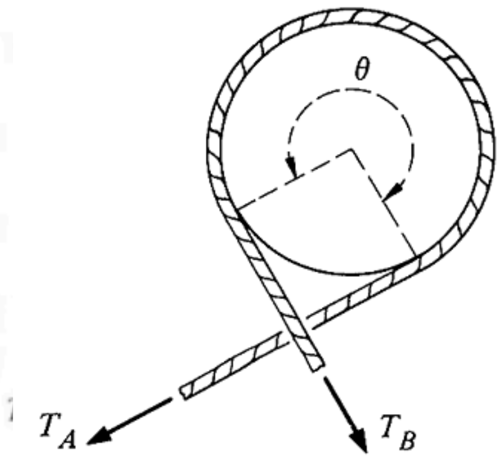
Figure 7-41 Problem 42.

end of the cord is pulled over a pulley, of negligible mass and friction and at cord height $h = 1.20$ m, so the cart slides from $x_1 = 3.00$ m to $x_2 = 1.00$ m. During the move, the tension in the cord is a constant 25.0 N. What is the change in the kinetic energy of the cart during the move?

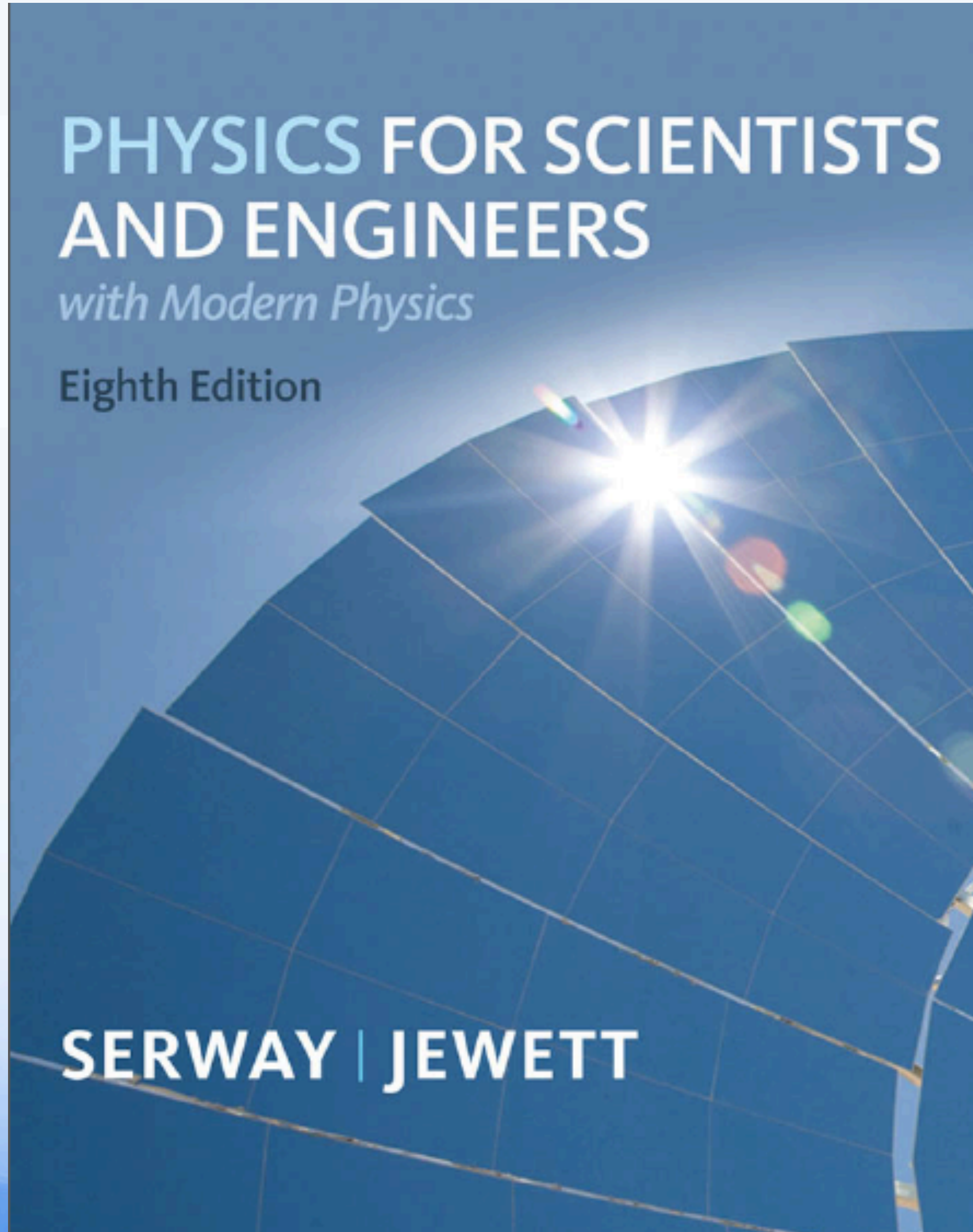
$41.8J$

Example:

A device called a capstan is used aboard ships in order to control a rope that is under great tension. The rope is wrapped around a fixed drum of radius R, usually for several turns (the drawing below shows about three fourths turn as seen from overhead).



The load on the rope pulls it with a force T_A, and the sailor holds the other end of the rope with a much smaller force T_B. The coefficient of static friction between the rope and the drum is μ_s. The sailor is holding the rope so that it is just about to slip. Can you show that $T_B = T_A e^{-\mu_s \theta}$, where μ_s is the coefficient of static friction and θ is the total angle subtended by the rope on the drum?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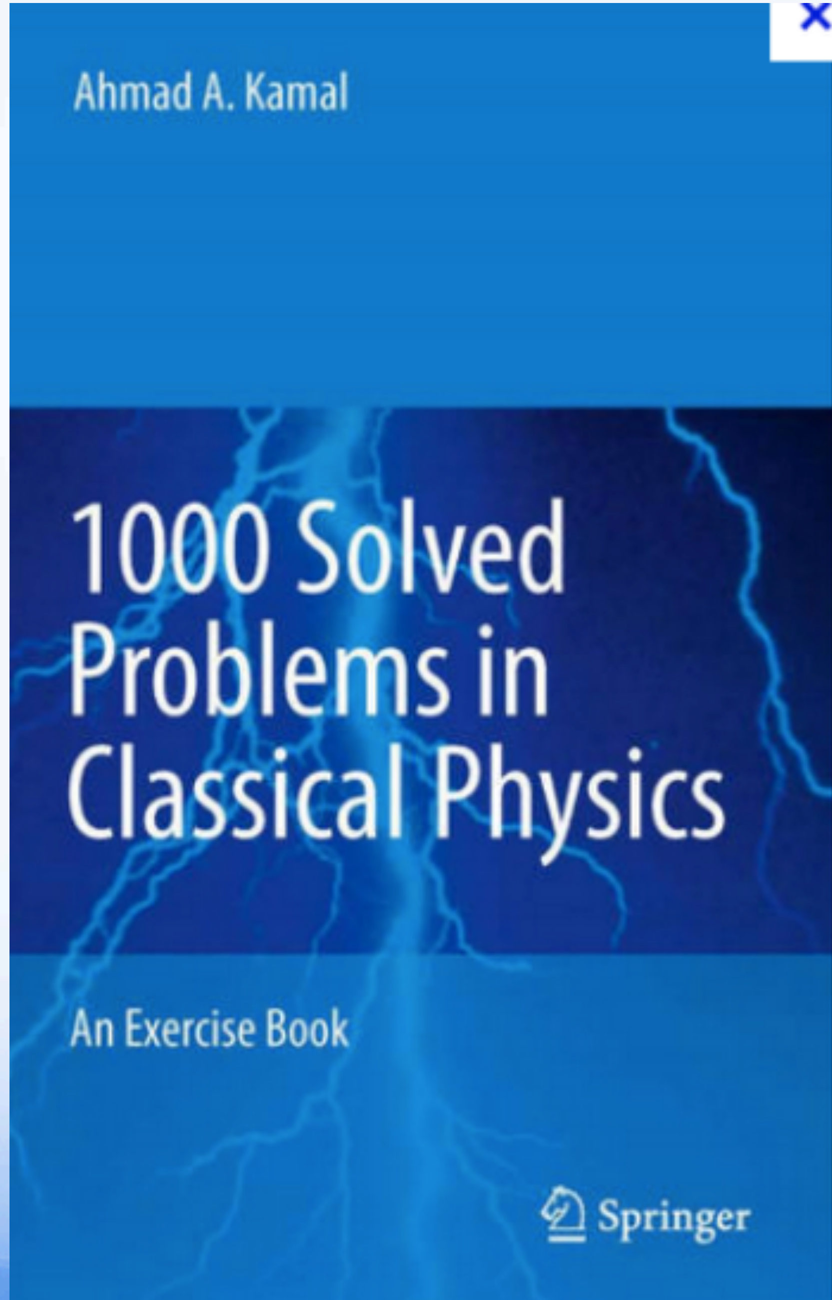
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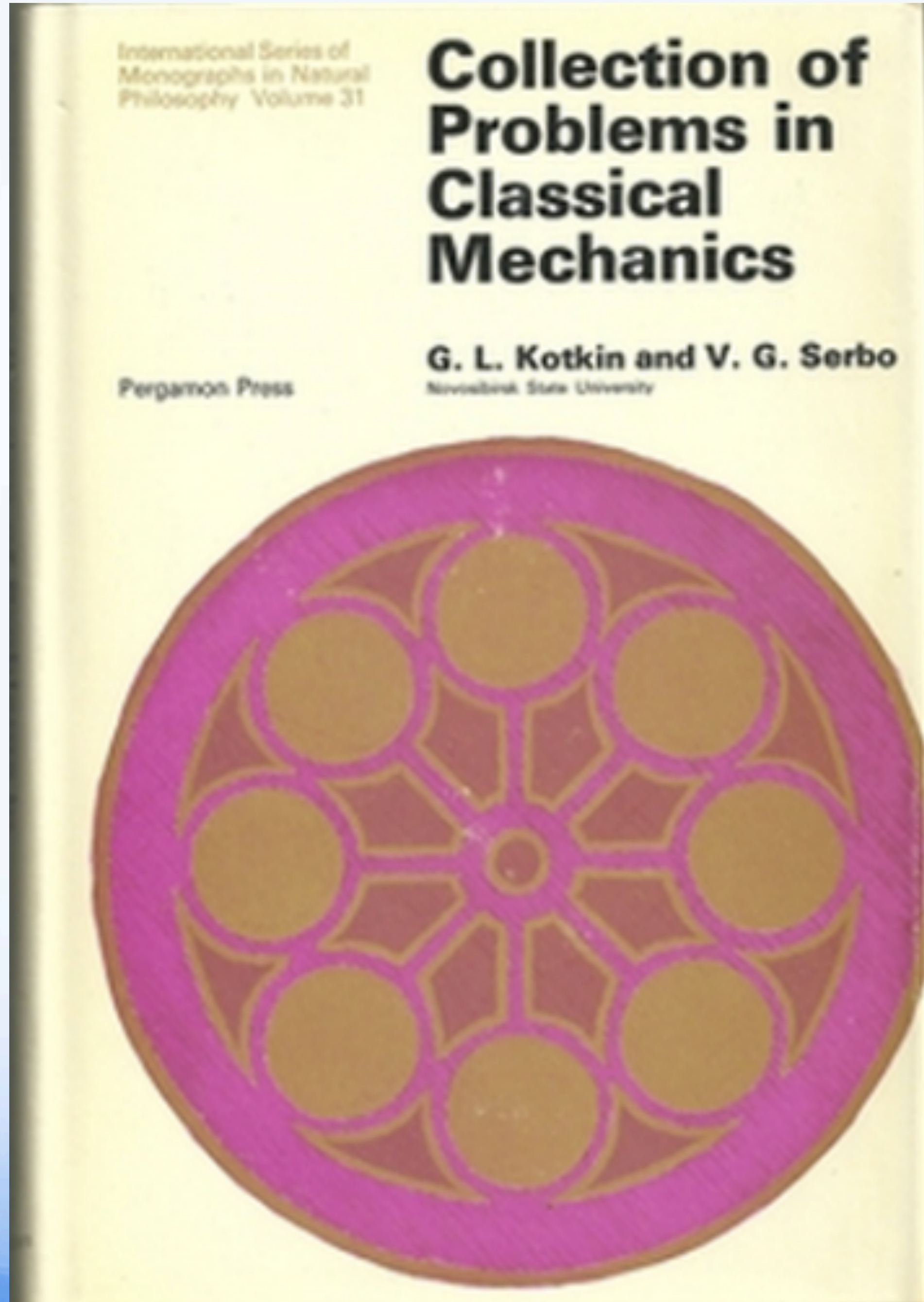
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