General Physics I

chapter 11

Sharif University of Technology
Mehr 1401 (2022-2023)

M. Reza Rahimi Tabar

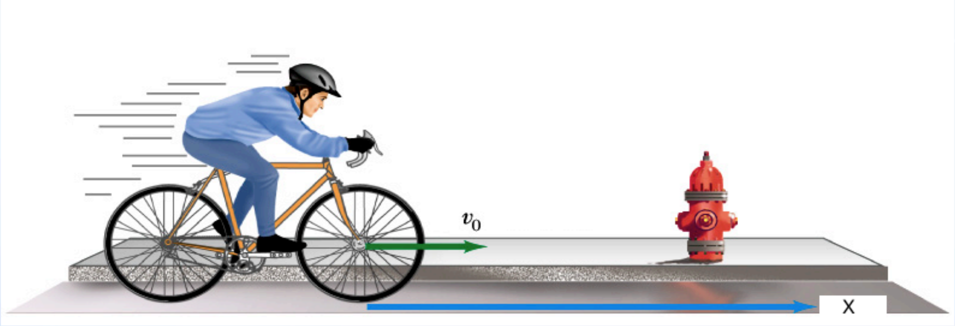
Chapter 11
Rolling, Torque, and Angular Momentum

- What Is Physics?
- Rolling as Translation and Rotation Combined ✔
- The Kinetic Energy of Rolling
- The Forces of Rolling
- Torque
- Angular Momentum ✔
- Newton's Second Law in Angular Form
- The Angular Momentum of a Rigid Body
- Conservation of Angular Momentum
- Precession of a Gyroscope ✔

What is Physics?

- The rolling motion of wheels

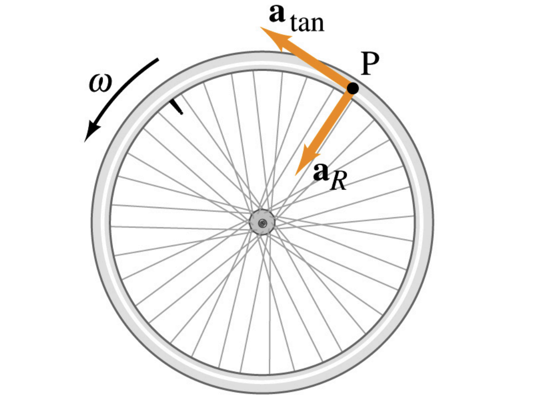
- Introducing of the “angular momentum’

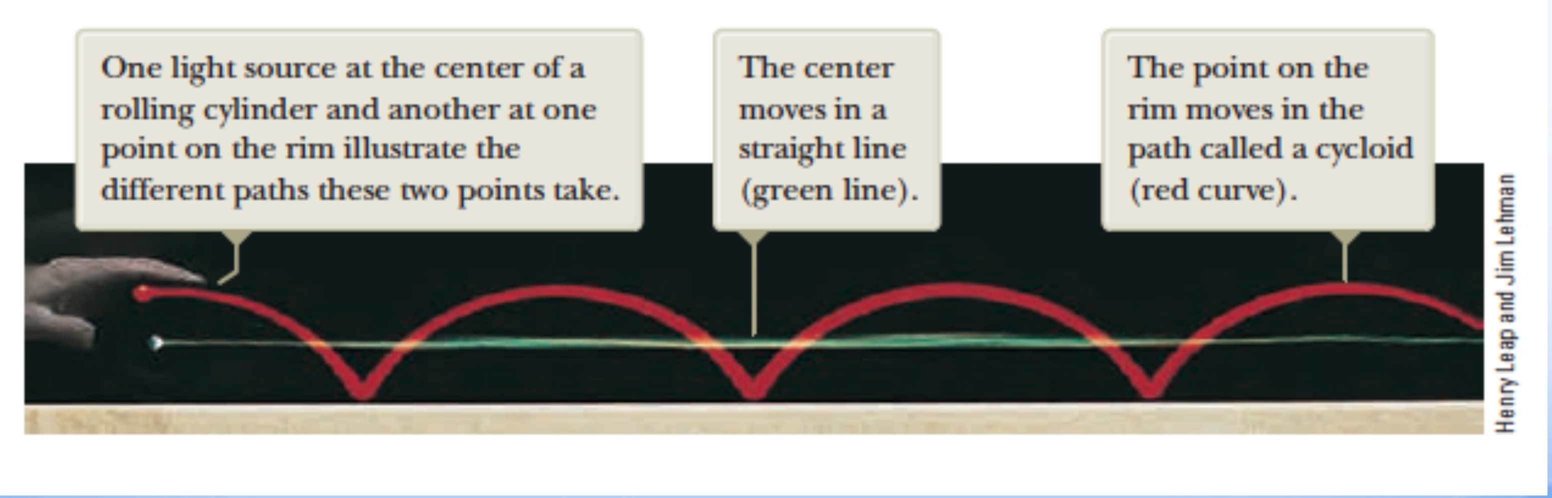


Segway PT

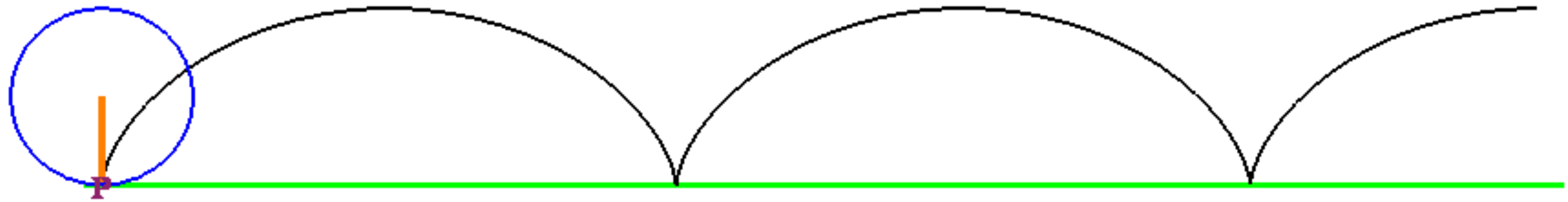
-

Rolling as Translation and Rotation Combined



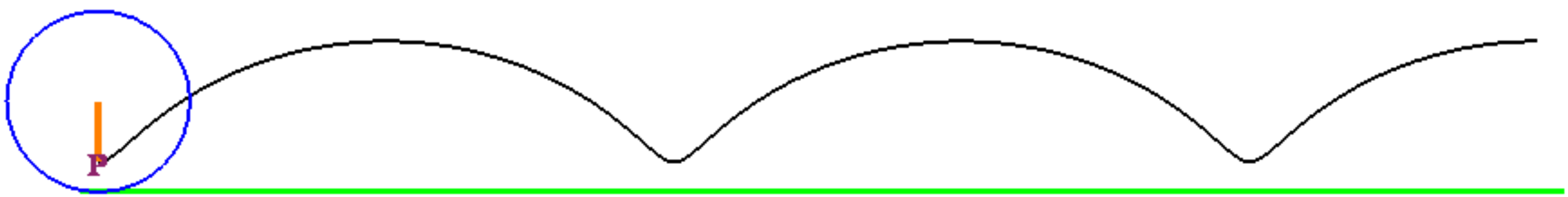


Cycloid



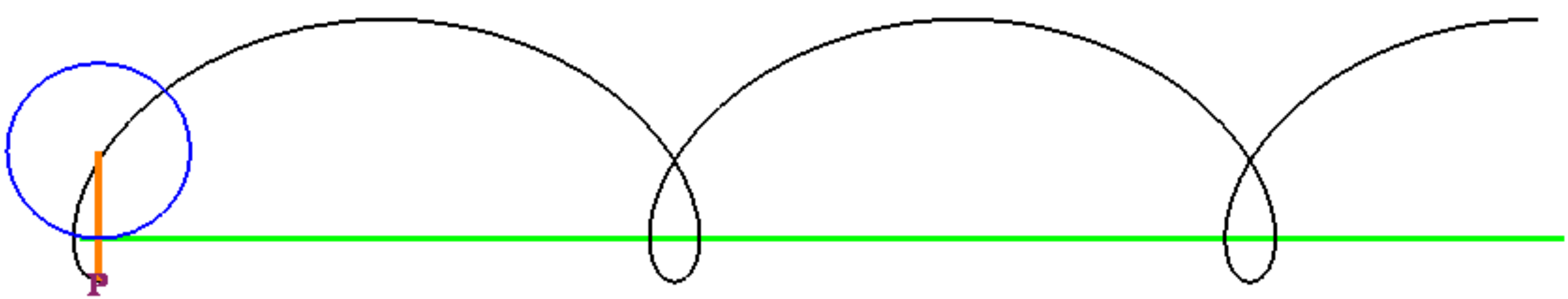
The general parametric equations for a ***cycloid*** are : **$x = at - a\sin(t)$, $y = a - a\cos(t)$**

Trochoid --- b < a

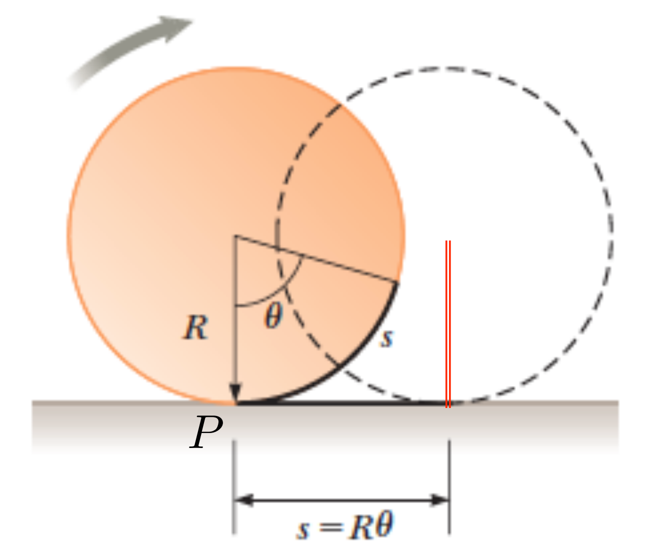


The general parametric equations for a ***trochoid*** are : **$x = at - b\sin(t)$, $y = a - b\cos(t)$**

Trochoid --- b > a



The Kinetic Energy of Rolling



$$v_{CM} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

$$a_{CM} = \frac{dv_{CM}}{dt} = R\frac{d\omega}{dt} = R\alpha$$

$$K = \tfrac{1}{2} I_P \omega^2$$

$$I_P = I_{CM} + MR^2$$

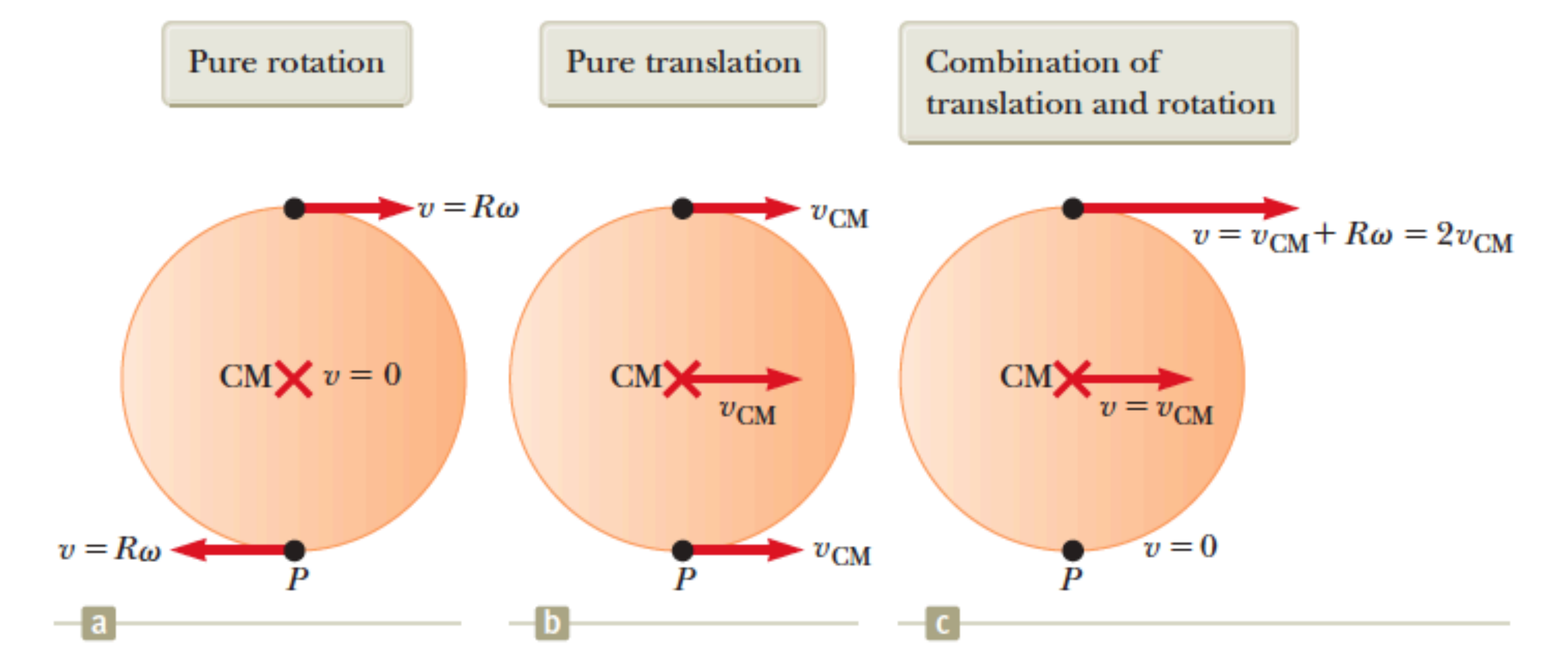
$$K = \tfrac{1}{2} I_{CM}\omega^2 + \tfrac{1}{2} MR^2\omega^2$$

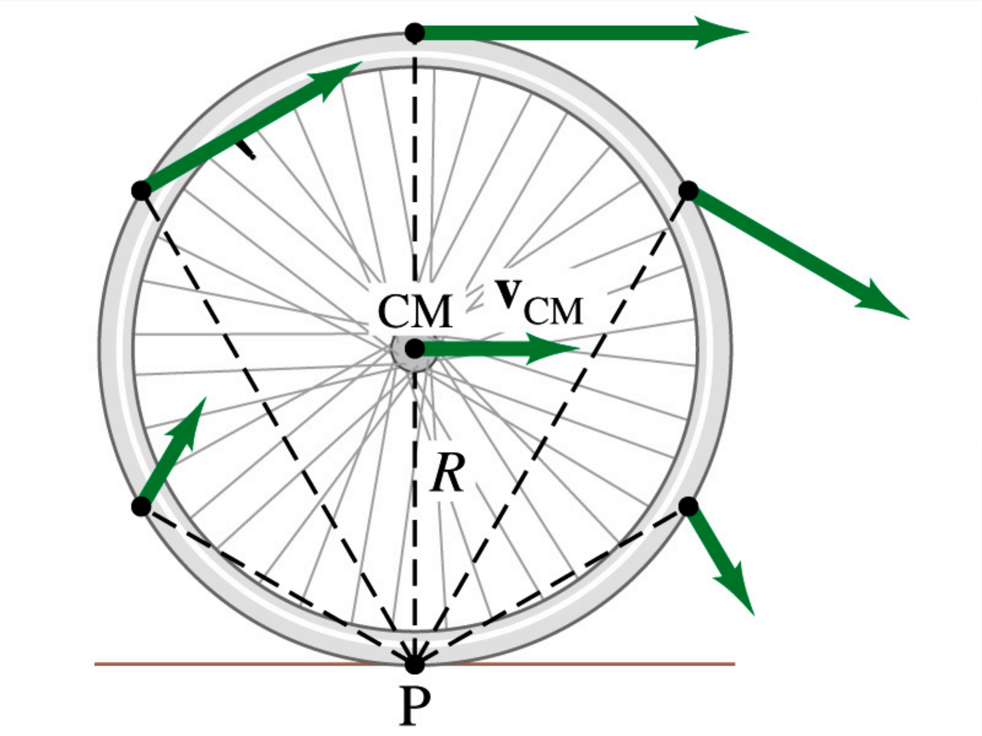
$$v_{CM} = R\omega,$$

$$K = \tfrac{1}{2} I_{CM}\omega^2 + \tfrac{1}{2} M{v_{CM}}^2$$

✔

- The motion of a rolling object can be modeled as a combination of pure translation and pure rotation



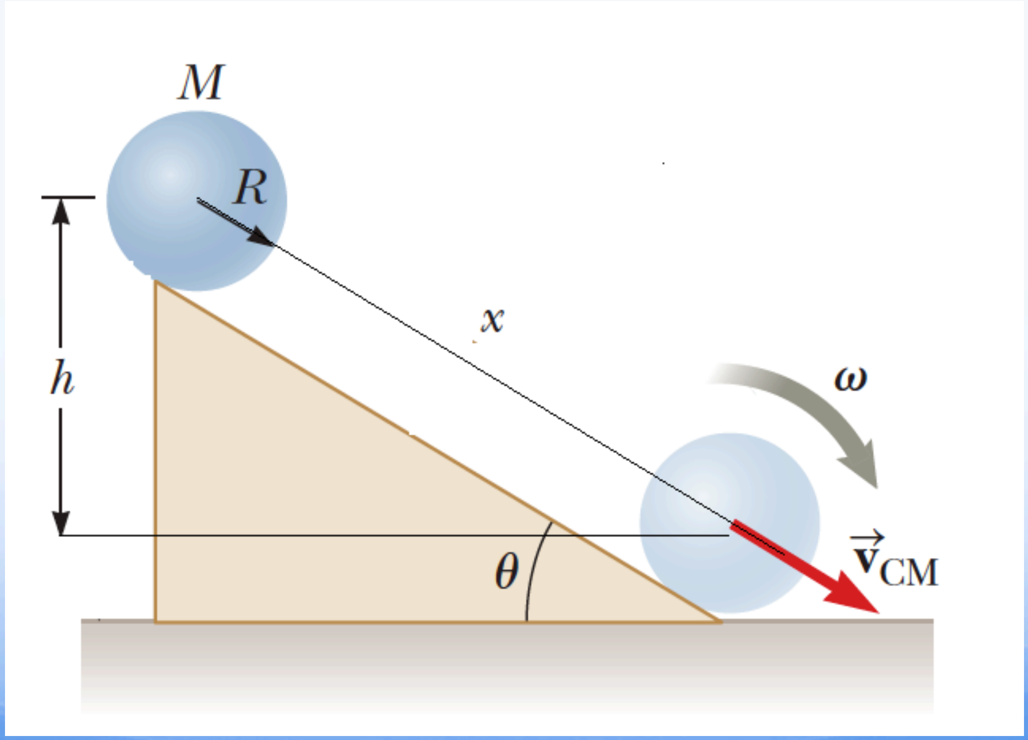


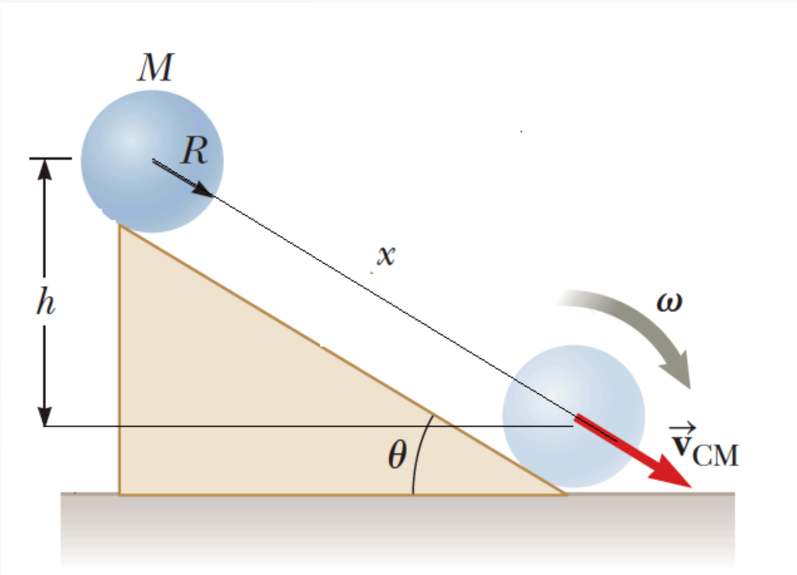
$$v = r\omega$$

$$v_{top} = (\omega)(2R) = 2(\omega R) = 2v_{com}$$

Example:

- The rolling motion of an object on a rough incline (energy method)





$$K = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} M v_{CM}^2$$

$$K = \frac{1}{2} I_{CM} \left(\frac{v_{CM}}{R} \right)^2 + \frac{1}{2} M v_{CM}^2$$

$$K = \frac{1}{2} \left(\frac{I_{CM}}{R^2} + M \right) v_{CM}^2$$

$$v_{CM} = r\omega$$

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2} \left(\frac{I_{CM}}{R^2} + M \right) v_{CM}^2 + 0 = 0 + Mgh$$

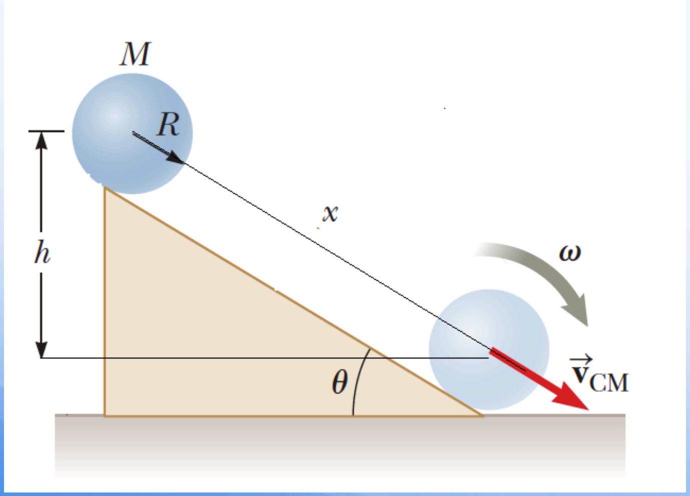
$$v_{CM} = \left[\frac{2gh}{1 + (I_{CM} / MR^2)} \right]^{1/2}$$

Example:

For the solid sphere shown in Active Figure 10.26, calculate the translational speed of the center of mass at the bottom of the incline and the magnitude of the translational acceleration of the center of mass.

$$v_{CM} = \left[\frac{2gh}{1 + (\frac{2}{5}MR^2/MR^2)}\right]^{1/2} = (\tfrac{10}{7}gh)^{1/2}$$

$$h = x\sin\theta.$$

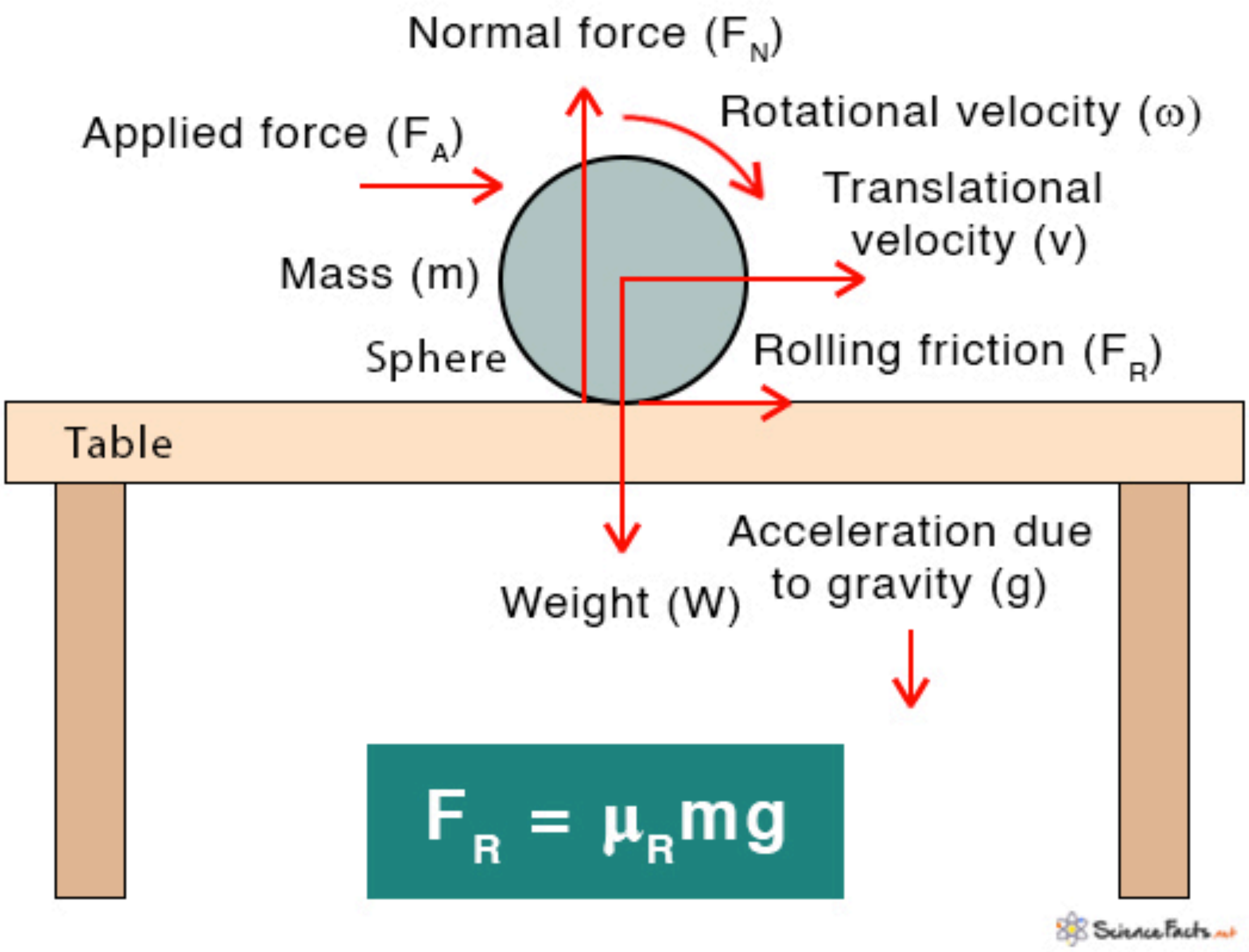


$$v_{CM}^2 = \tfrac{10}{7}gx\sin\theta$$

$$v_{CM}^2 = 2a_{CM}x$$

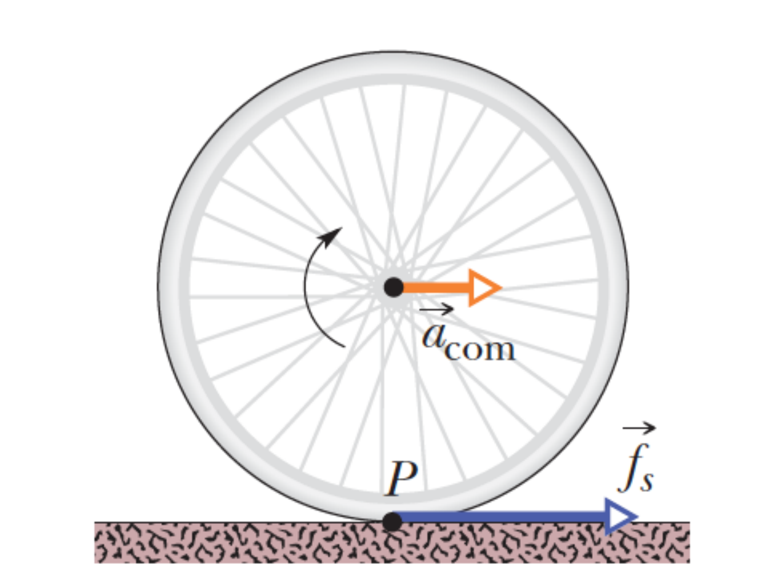
$$a_{CM} = \tfrac{5}{7}g\sin\theta$$

Rolling Friction



The Forces of Rolling

- Friction and Rolling

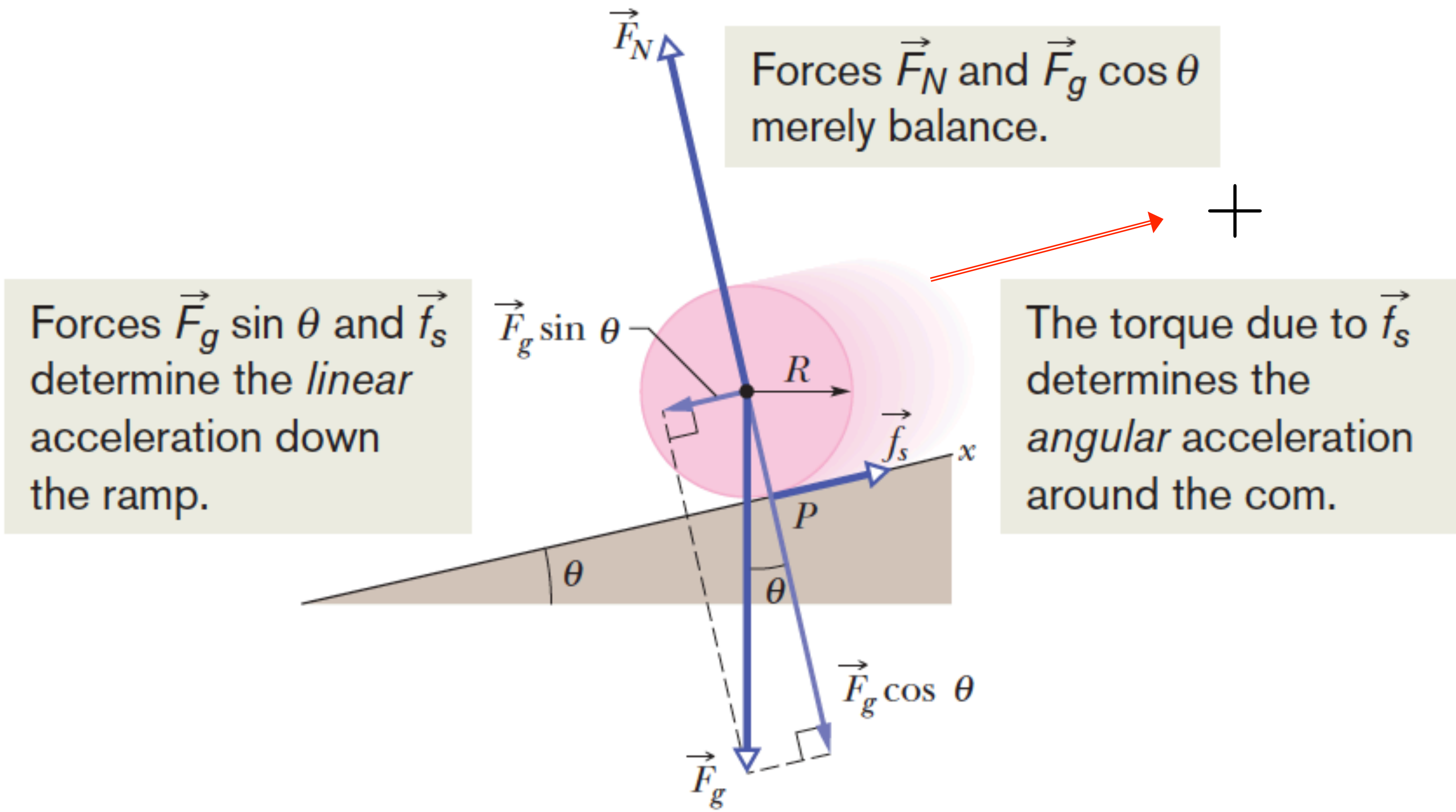


?

$$v_{com} = \omega R$$

$$a_{com} = \alpha R \qquad \text{(smooth rolling motion).}$$

Rolling Down a Ramp

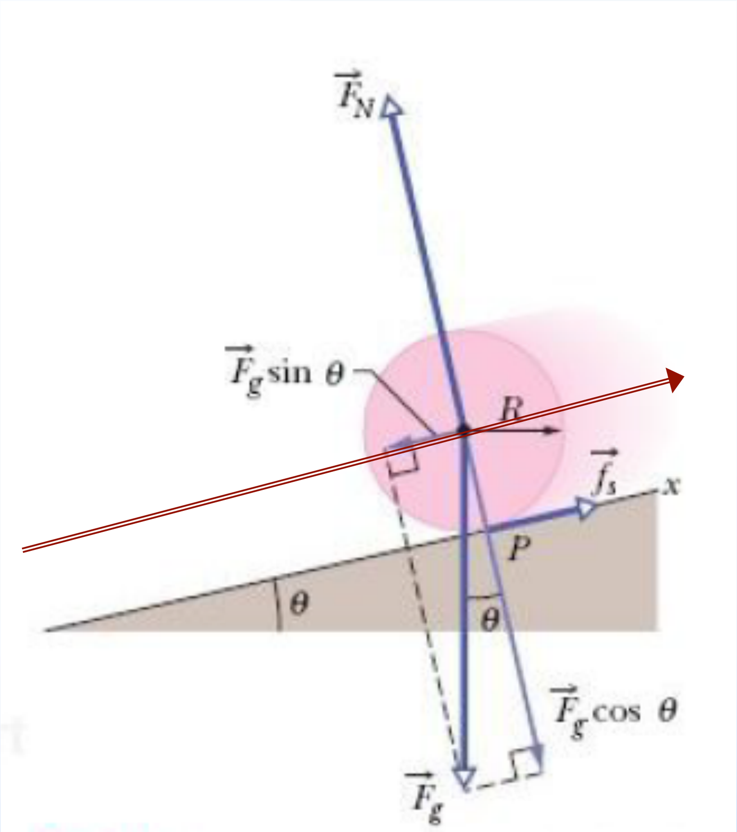


$$f_s - Mg\sin\theta = Ma_{\text{com},x}.$$

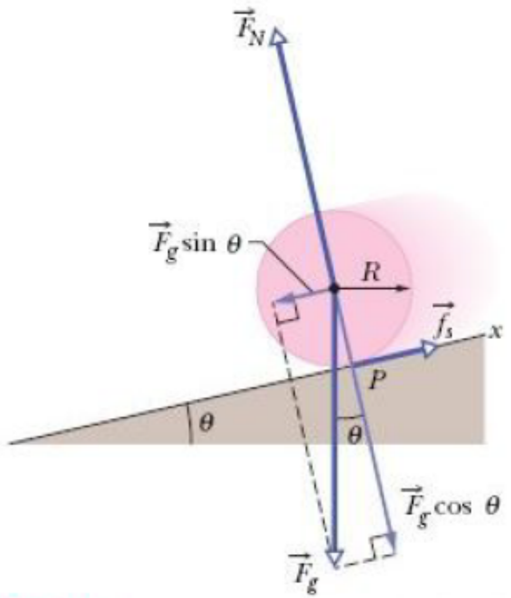
$$Rf_s = I_{\text{com}}\alpha.$$

$$f_s = -I_{\text{com}}\frac{a_{\text{com},x}}{R^2}.$$

$$a_{\text{com},x} = -\frac{g\sin\theta}{1 + I_{\text{com}}/MR^2}.$$

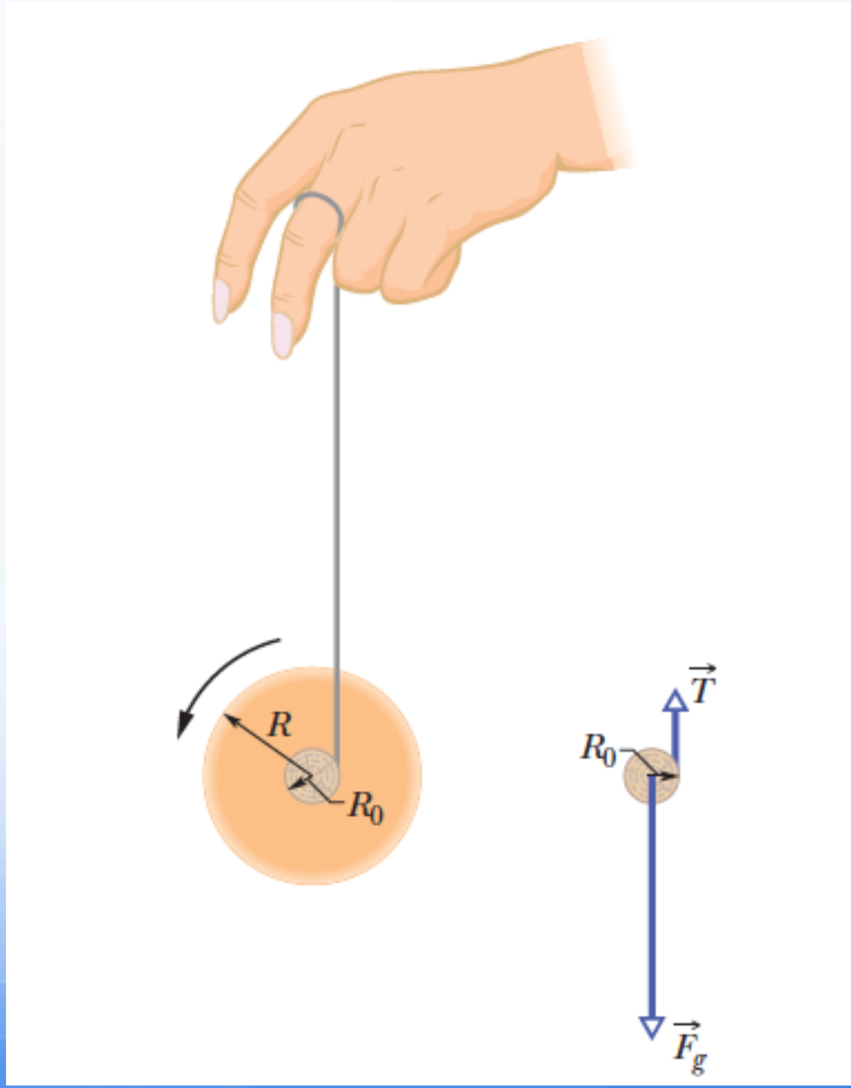


The Yo-Yo



$$a_{\text{com},x} = -\frac{g\sin\theta}{1 + I_{\text{com}}/MR^2}.$$

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2},$$



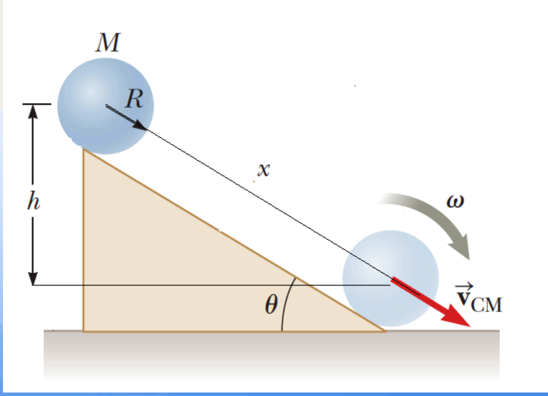
Example

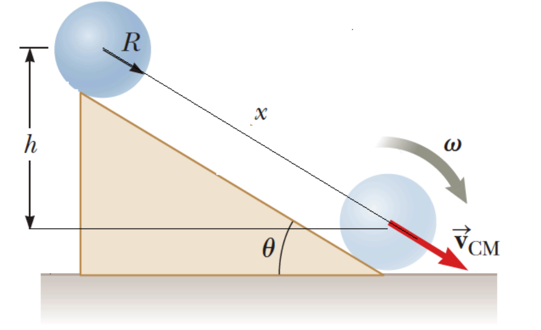
Sample Problem 11.01 Ball rolling down a ramp

A uniform ball, of mass $M = 6.00\,\text{kg}$ and radius R, rolls smoothly from rest down a ramp at angle $\theta = 30.0°$ (Fig. 11-8).

(a) The ball descends a vertical height $h = 1.20\,\text{m}$ to reach the bottom of the ramp. What is its speed at the bottom?

(b) What are the magnitude and direction of the frictional force on the ball as it rolls down the ramp?





$$K_f + U_f = K_i + U_i,$$

$$\left(\tfrac{1}{2}I_{com}\omega^2 + \tfrac{1}{2}Mv_{com}^2\right) + 0 = 0 + Mgh,$$

$$v_{com} = \sqrt{\left(\tfrac{10}{7}\right)gh} = \sqrt{\left(\tfrac{10}{7}\right)(9.8\ \text{m/s}^2)(1.20\ \text{m})}$$
$$= 4.10\ \text{m/s}. \qquad \text{(Answer)}$$

$$a_{com,x} = -\frac{g\sin\theta}{1 + I_{com}/MR^2} = -\frac{g\sin\theta}{1 + \frac{2}{5}MR^2/MR^2}$$
$$= -\frac{(9.8\ \text{m/s}^2)\sin 30.0^\circ}{1 + \frac{2}{5}} = -3.50\ \text{m/s}^2.$$

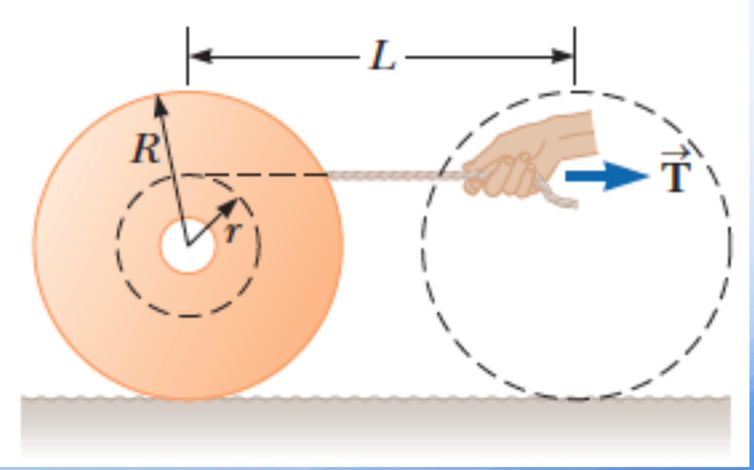
$$f_s = -I_{com}\frac{a_{com,x}}{R^2} = -\tfrac{2}{5}MR^2\frac{a_{com,x}}{R^2} = -\tfrac{2}{5}Ma_{com,x}$$

$$= -\tfrac{2}{5}(6.00\ \text{kg})(-3.50\ \text{m/s}^2) = 8.40\ \text{N.} \qquad \text{(Answer)}$$

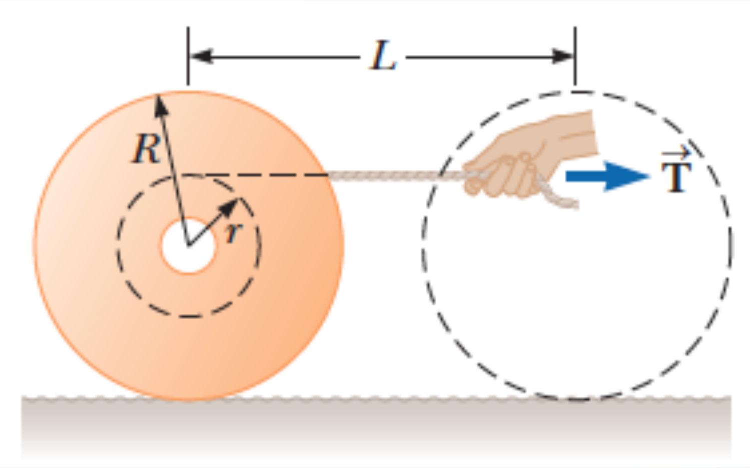
Example:

A cylindrically symmetric spool of mass m and radius R sits at rest on a horizontal table with friction (Fig. 10.27). With your hand on a massless string wrapped around the axle of radius r, you pull on the spool with a constant horizontal force of magnitude T to the right. As a result, the spool rolls without slipping a distance L along the table with no rolling friction.

(A) Find the final translational speed of the center of mass of the spool.



$$(1)\quad W = \Delta K = \Delta K_{trans} + \Delta K_{rot}$$



$$\ell = r\theta = \frac{r}{R}L$$

$$\ell + L = L(1 + r/R).$$

$$\omega = v_{CM}/R:$$

$$(2)\quad W = TL\left(1 + \frac{r}{R}\right)$$

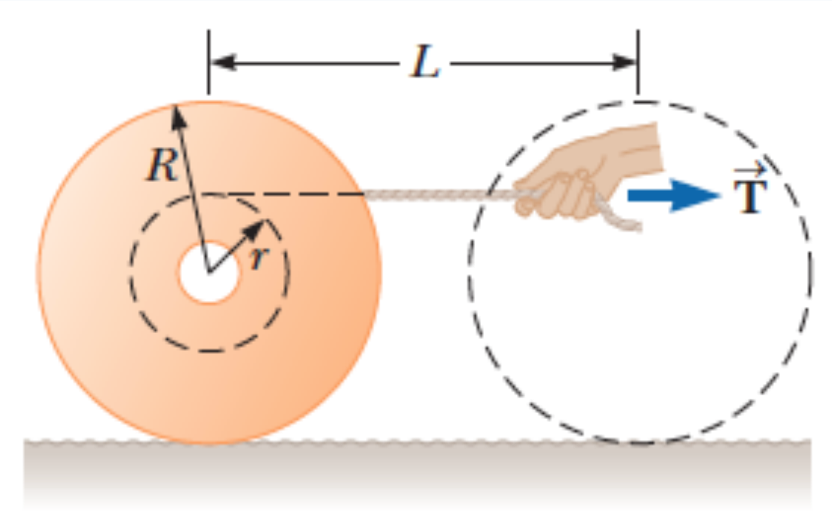
$$TL\left(1 + \frac{r}{R}\right) = \tfrac{1}{2}m{v_{CM}}^2 + \tfrac{1}{2}I\omega^2$$

$$TL\left(1 + \frac{r}{R}\right) = \tfrac{1}{2}m{v_{CM}}^2 + \tfrac{1}{2}I\frac{{v_{CM}}^2}{R^2}$$

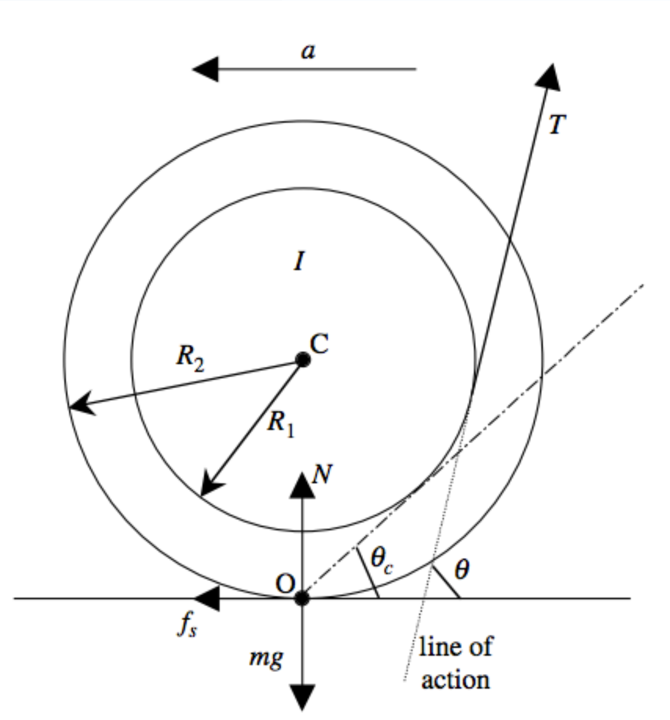
$$(3)\quad v_{CM} = \sqrt{\frac{2TL(1 + r/R)}{m(1 + I/mR^2)}}$$

Movie: Pulled Spool

- https://www.youtube.com/watch?v=CRg39Bh6jfo



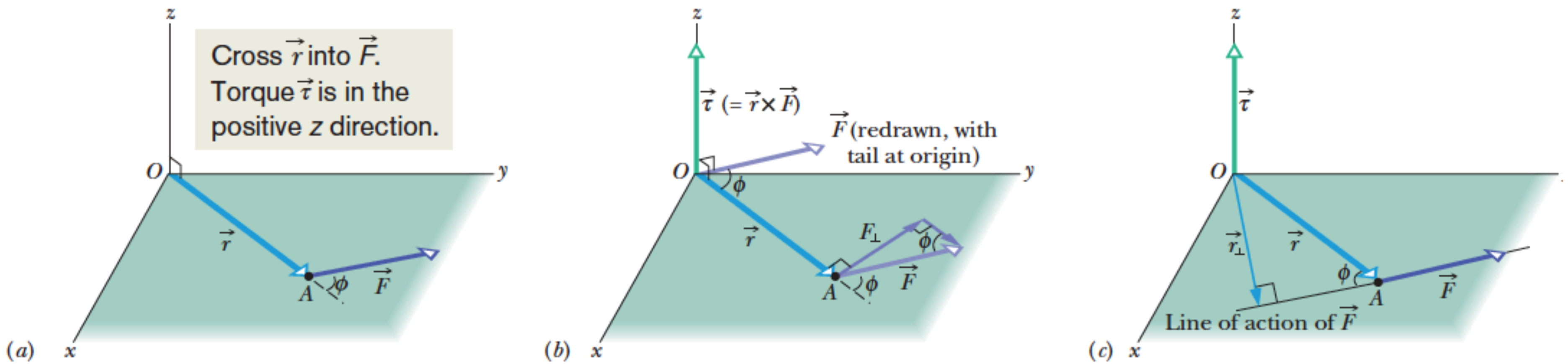
The critical angle:



$$\theta_c = \cos^{-1} R_1 / R_2.$$

- http://www.usna.edu/Users/physics/mungan/Publications/TPT.pdf

Torque



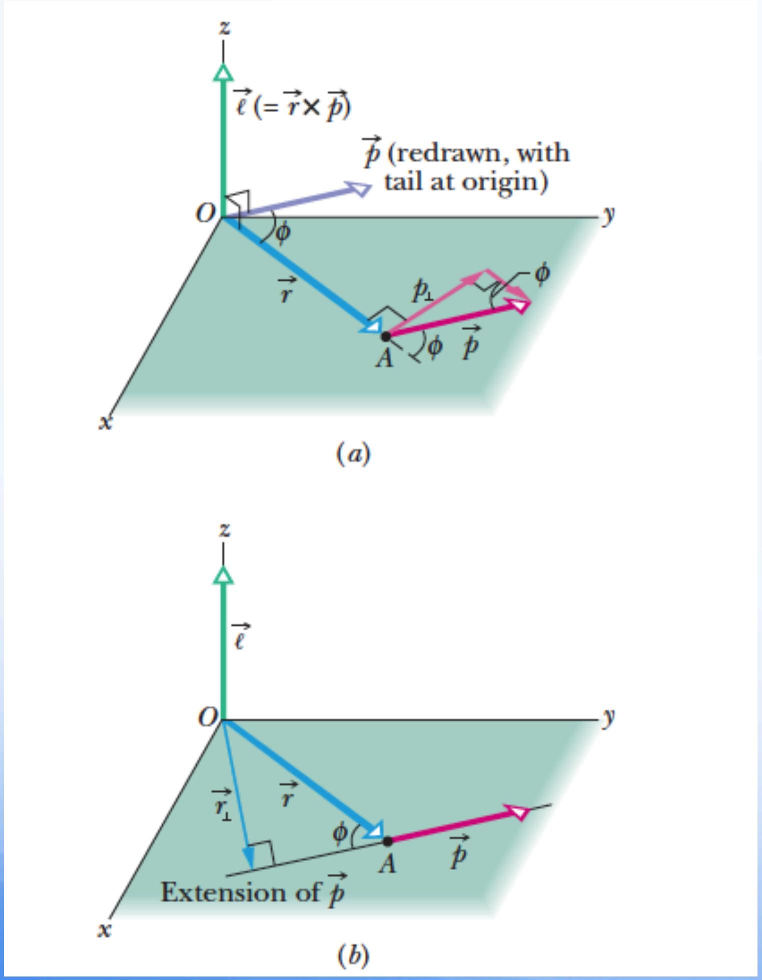
$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \text{(torque defined).}$$

$$\tau = rF \sin \phi,$$

$$\tau = rF_\perp,$$

$$\tau = r_\perp F,$$

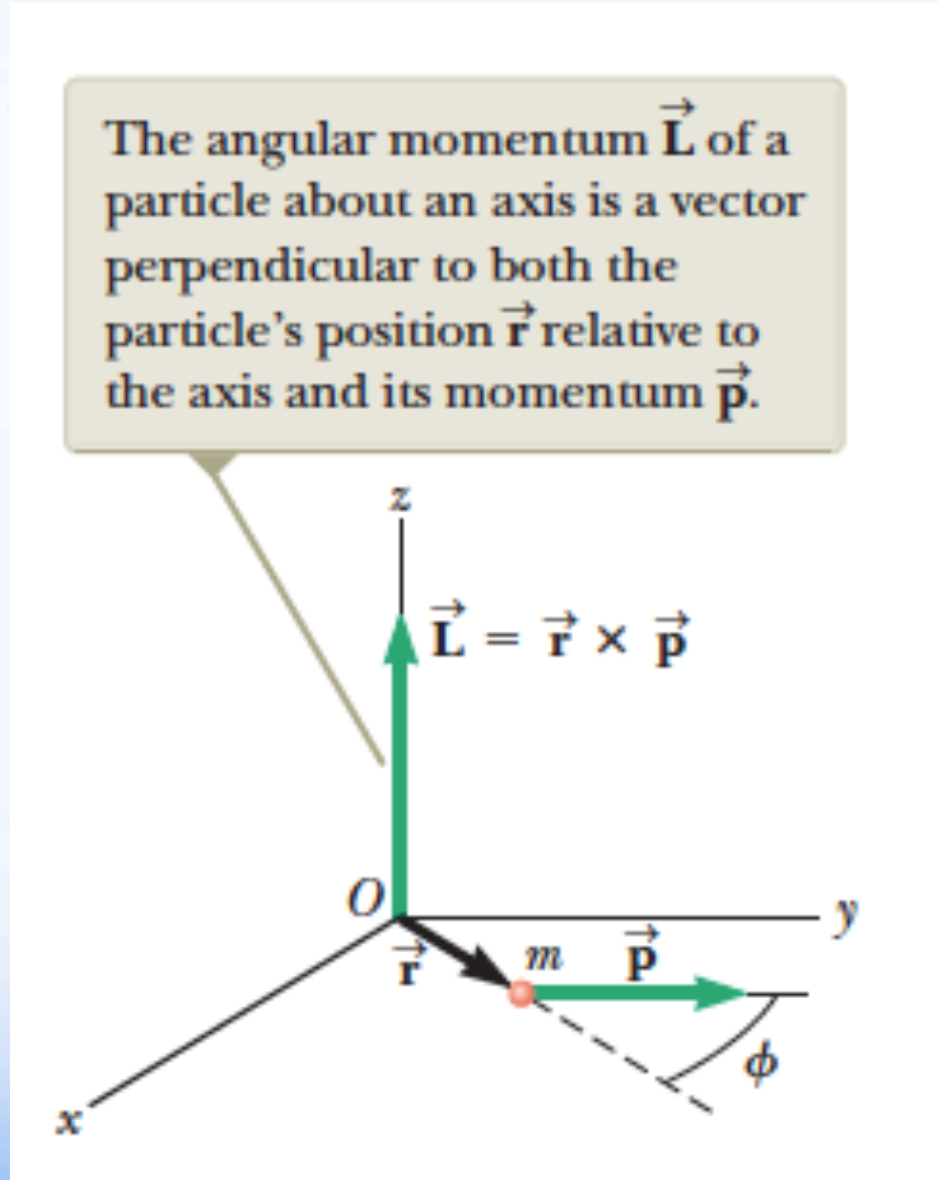
Angular Momentum



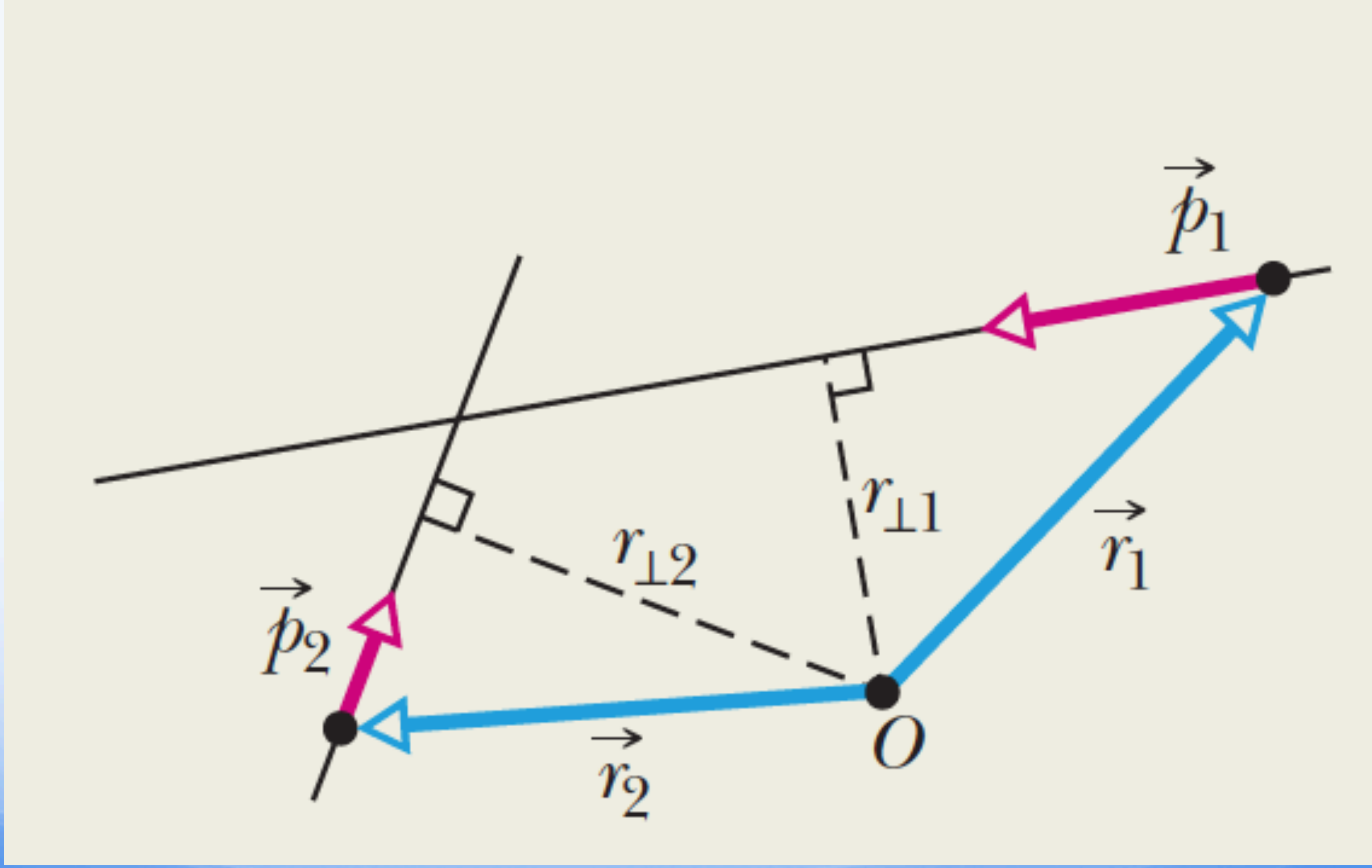
$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$\ell = rmv \sin \phi,$$

$$\ell = rp_\perp = rmv_\perp,$$


The angular momentum $\vec{\mathbf{L}}$ of a particle about an axis is a vector perpendicular to both the particle's position $\vec{\mathbf{r}}$ relative to the axis and its momentum $\vec{\mathbf{p}}$.
z
$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
O
y
$\vec{\mathbf{r}}$
m
$\vec{\mathbf{p}}$
ϕ
x

Example:



Newton's Second Law in Angular Form

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \text{(single particle)}$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt} \qquad \text{(single particle).}$$

Proof:

$$\vec{\ell} = m(\vec{r} \times \vec{v}),$$

$$\frac{d\vec{\ell}}{dt} = m\left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v}\right).$$

$$\frac{d\vec{\ell}}{dt} = m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v}).$$

$$\vec{v} \times \vec{v} = 0$$

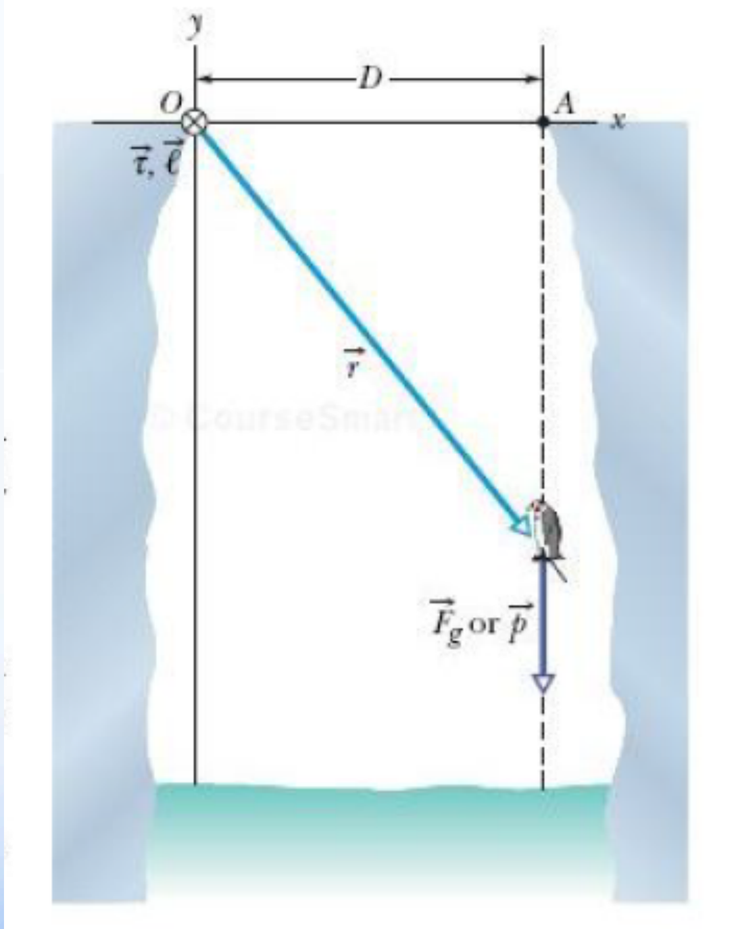
$$\frac{d\vec{\ell}}{dt} = m(\vec{r} \times \vec{a}) = \vec{r} \times m\vec{a}.$$

$$\frac{d\vec{\ell}}{dt} = \vec{r} \times \vec{F}_{\text{net}} = \sum(\vec{r} \times \vec{F}).$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt}.$$

✔

Example:



$$\ell = r_\perp mv \qquad v = gt$$

$$\ell = r_\perp mv = Dmgt.$$

$$\tau = DF_g = Dmg.$$

$$\ell = Dmgt.$$

$$\tau = \frac{d\ell}{dt} = \frac{d(Dmgt)}{dt} = Dmg,$$

The Angular Momentum of a System of Particles

$$\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3 + \cdots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i.$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{\ell}_i}{dt}.$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \vec{\tau}_{\text{net},i}.$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \quad \text{(system of particles)},$$

✔

The Angular Momentum of a Rigid Body Rotating About a Fixed Axis (1)

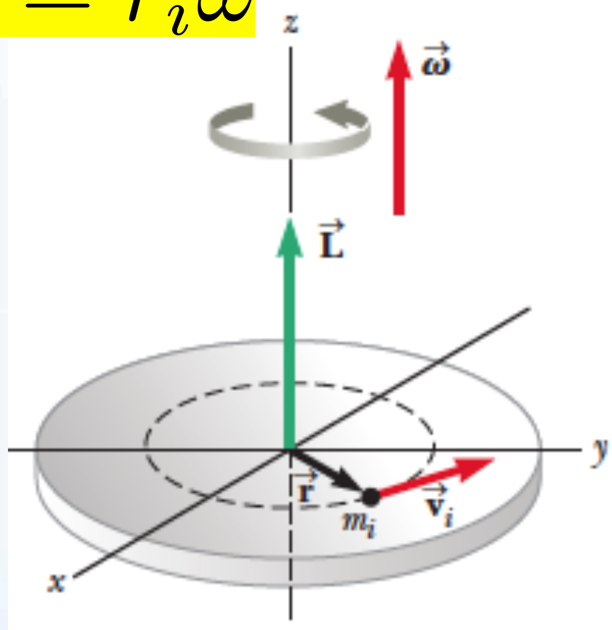
$$L_i = r_i p_i \qquad L_i = r_i(m_i v_i) \qquad v_i = r_i\omega$$

$$L_i = m_i r_i^2 \omega$$

$$L_z = \sum_i L_i = \sum_i m_i r_i^2 \omega = \left(\sum_i m_i r_i^2\right)\omega$$

$$L_z = I\omega$$

$$\frac{dL_z}{dt} = I\frac{d\omega}{dt} = I\alpha$$

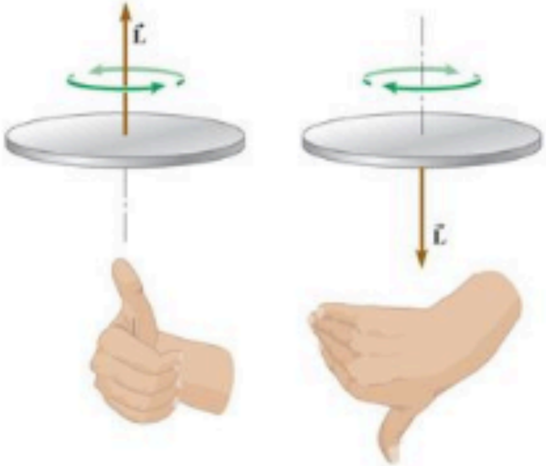


Rotational form of Newton's second law ▶

$$\sum \tau_{ext} = I\alpha$$

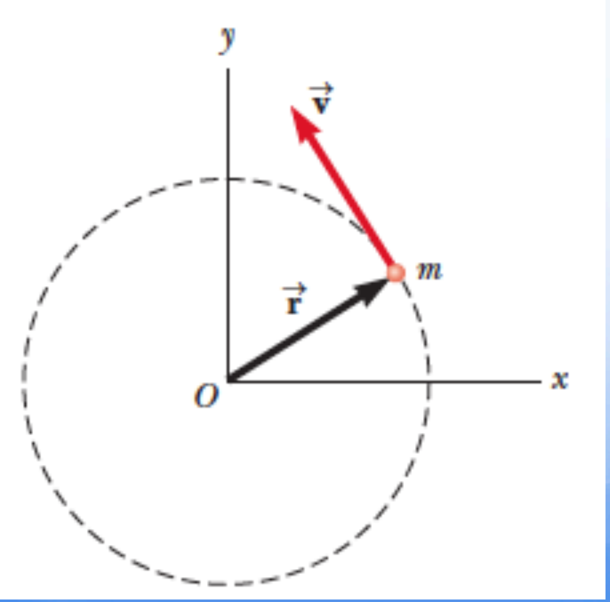
A right hand rule:

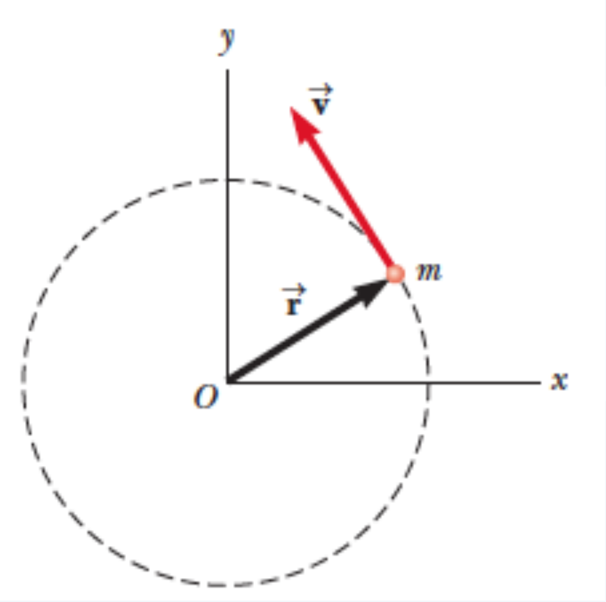
Using your right hand, 1) curl your fingers in the direction of the rotation
2) Your thumb points in the direction of the angular momentum.



Example:

A particle moves in the xy plane in a circular path of radius r as shown in Figure 11.5. Find the magnitude and direction of its angular momentum relative to an axis through O when its velocity is $\vec{\mathbf{v}}$.

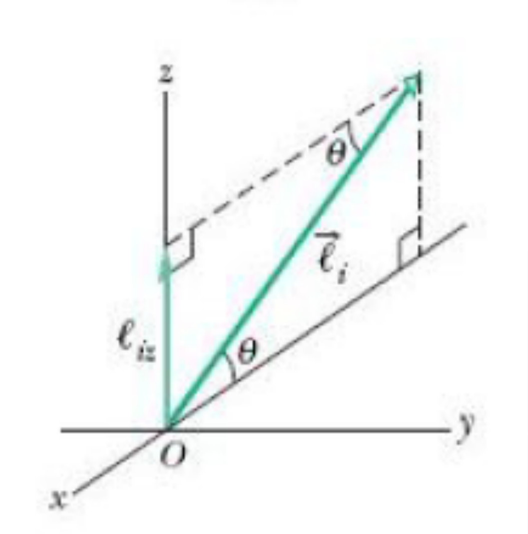




$$L = mvr\sin 90^\circ = mvr$$

This value of L is constant because all three factors on the right are constant. The direction of $\vec{L}$ also is constant, even though the direction of $\vec{p} = m\vec{v}$ keeps changing. To verify this statement, apply the right-hand rule to find the direction of $\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$ in Figure 11.5. Your thumb points out of the page, so that is the direction of $\vec{L}$. Hence, we can write the vector expression $\vec{L} = (mvr)\hat{k}$. If the particle were to move clockwise, $\vec{L}$ would point downward and into the page and $\vec{L} = -(mvr)\hat{k}$. A particle in uniform circular motion has a constant angular momentum about an axis through the center of its path.

The Angular Momentum of a Rigid Body Rotating About a Fixed Axis



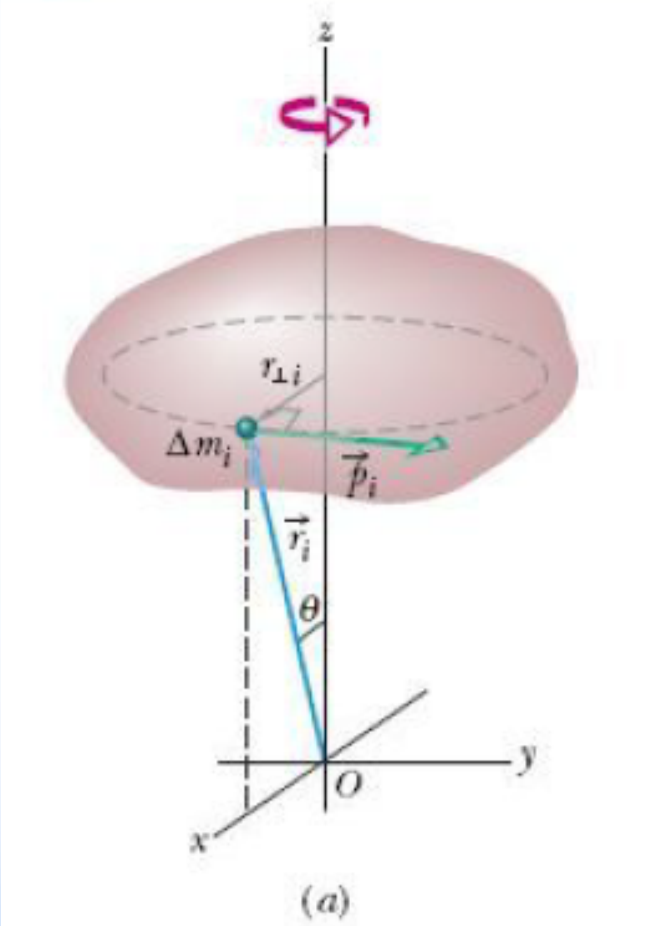
$$\ell_i = (r_i)(p_i)(\sin 90°) = (r_i)(\Delta m_i v_i),$$

$$\ell_{iz} = \ell_i \sin\theta = (r_i \sin\theta)(\Delta m_i v_i) = r_{\perp i}\Delta m_i v_i.$$

$$L_z = \sum_{i=1}^{n} \ell_{iz} = \sum_{i=1}^{n} \Delta m_i v_i r_{\perp i} = \sum_{i=1}^{n} \Delta m_i (\omega r_{\perp i}) r_{\perp i}$$

$$= \omega \left(\sum_{i=1}^{n} \Delta m_i r_{\perp i}^2 \right).$$

$$v = \omega r_\perp$$



$$L = I\omega$$

The Angular Momentum of a Rigid Body Rotating About a Fixed Axis (1)

$$L = I\omega \qquad \text{(rigid body, fixed axis)}.$$

More Corresponding Variables and Relations for Translational and Rotational Motion[a]

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\ (= \vec{r} \times \vec{F})$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\ (= \vec{r} \times \vec{p})$
Linear momentum[b]	$\vec{P}\ (= \Sigma \vec{p}_i)$	Angular momentum[b]	$\vec{L}\ (= \Sigma \vec{\ell}_i)$
Linear momentum[b]	$\vec{P} = M\vec{v}_{com}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\vec{F}_{net} = \dfrac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}$ = a constant	Conservation law[d]	$\vec{L}$ = a constant

Example:

A sphere of mass m_1 and a block of mass m_2 are connected by a light cord that passes over a pulley as shown in Figure 11.6. The radius of the pulley is R, and the mass of the thin rim is M. The spokes of the pulley have negligible mass. The block slides on a frictionless, horizontal surface. Find an expression for the linear acceleration of the two objects, using the concepts of angular momentum and torque.

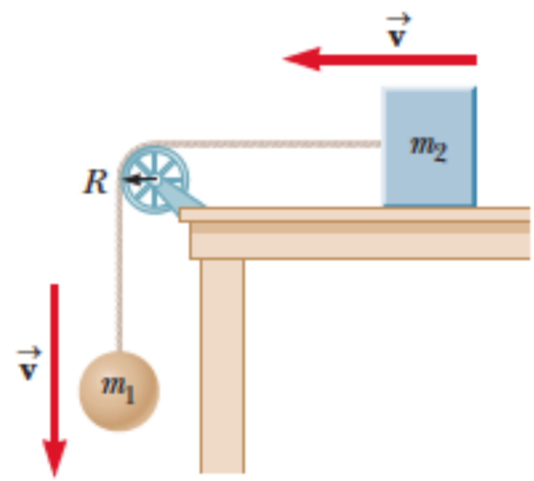


Figure 11.6 (Example 11.4) When the system is released, the sphere moves downward and the block moves to the left.

$$\sum \tau_{ext} = m_1 g R$$

$$L = I\omega = MR^2\omega = MR(R\omega) = MRv$$

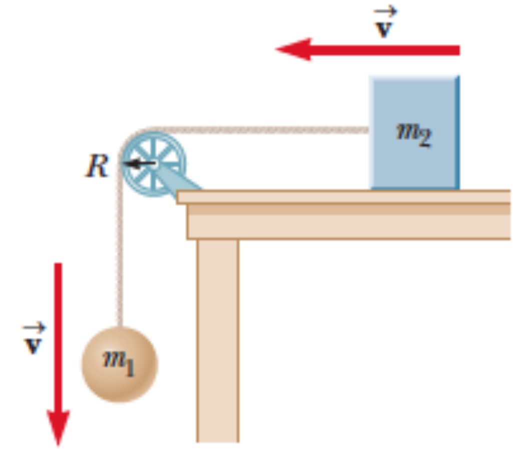


Figure 11.6 (Example 11.4) When the system is released, the sphere moves downward and the block moves to the left.

the total angular momentum of (1) $L = m_1 vR + m_2 vR + MvR = (m_1 + m_2 + M)vR$

$$\sum \tau_{ext} = \frac{dL}{dt}$$

$$m_1 g R = \frac{d}{dt}[(m_1 + m_2 + M)vR]$$

(2) $m_1 g R = (m_1 + m_2 + M)R\frac{dv}{dt}$

(3) $a = \dfrac{m_1 g}{m_1 + m_2 + M}$

Conservation of Angular Momentum

$$\vec{\tau}_{net} = d\vec{L}/dt)$$

$$\vec{\tau} = 0$$

$$d\vec{L}/dt = 0,$$

$$\vec{L} = \text{a constant} \qquad \text{(isolated system).}$$

$$\begin{pmatrix}\text{net angular momentum} \\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{net angular momentum} \\ \text{at some later time } t_f\end{pmatrix},$$

$$\vec{L}_i = \vec{L}_f \qquad \text{(isolated system).}$$

Conservation laws:

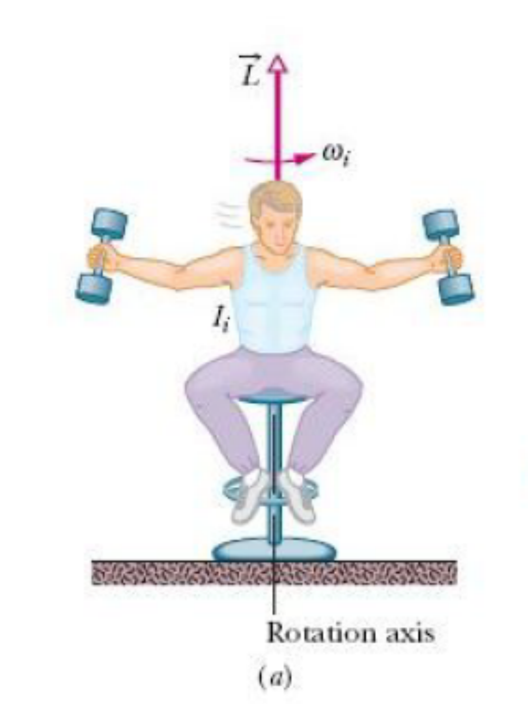
$E_i = E_f$ (if there are no energy transfers across the system boundary)

$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$ (if the net external force on the system is zero)

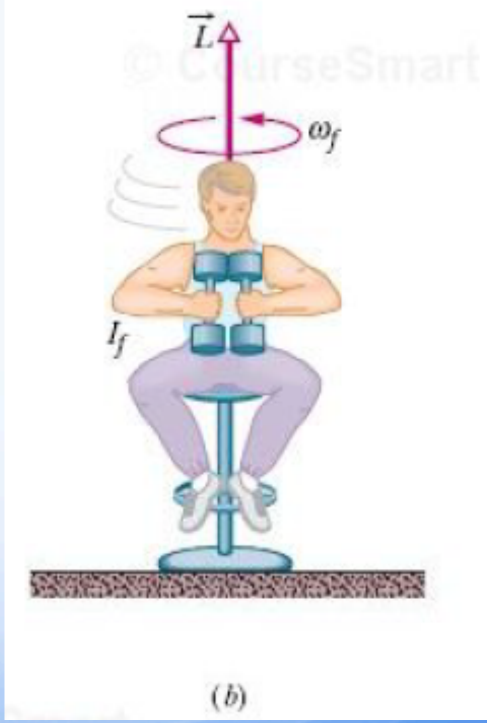
$\vec{\mathbf{L}}_i = \vec{\mathbf{L}}_f$ (if the net external torque on the system is zero)

Example:

$$\vec{L}_i = \vec{L}_f \qquad \text{(isolated system).}$$

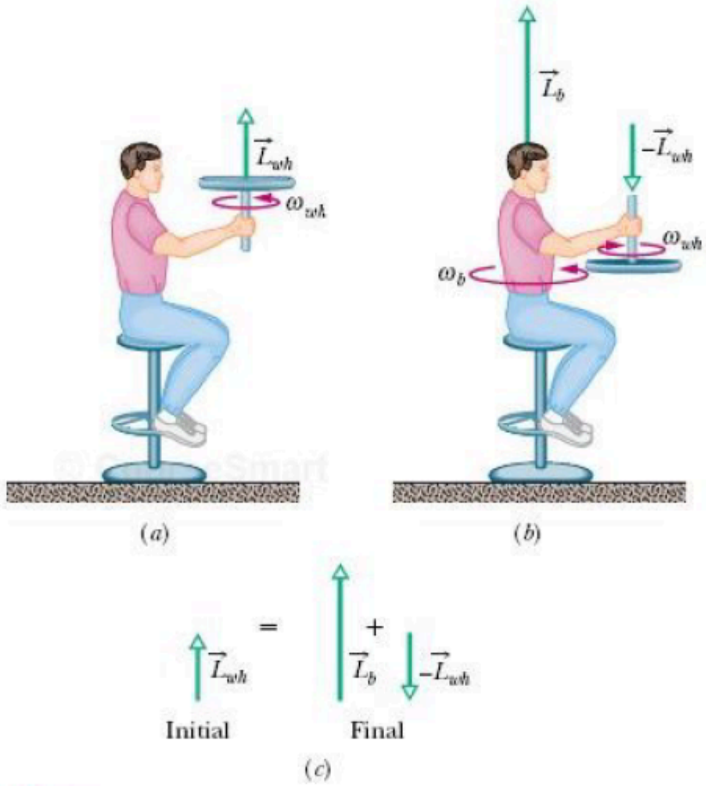


$$I_i\omega_i = I_f\omega_f.$$



Example:

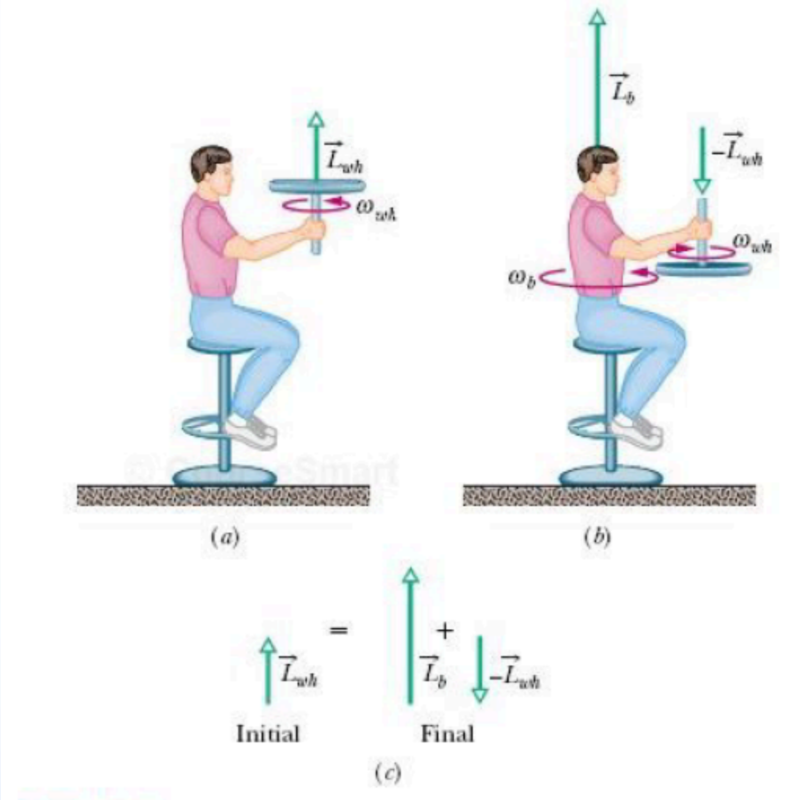
1. The angular speed ω_b we seek is related to the final angular momentum $\vec{L}_b$ of the composite body about the stool's rotation axis by Eq. 11-31 ($L = I\omega$).
2. The initial angular speed ω_{wh} of the wheel is related to the angular momentum $\vec{L}_{wh}$ of the wheel's rotation about its center by the same equation.
3. The vector addition of $\vec{L}_b$ and $\vec{L}_{wh}$ gives the total angular momentum $\vec{L}_{tot}$ of the system of student, stool, and wheel.
4. As the wheel is inverted, no net *external* torque acts on that system to change $\vec{L}_{tot}$ about any vertical axis. (Torques due to forces between the student and the wheel as the student inverts the wheel are *internal* to the system.) So, the system's total angular momentum is conserved about any vertical axis.



$$L_{b,f} + L_{wh,f} = L_{b,i} + L_{wh,i},$$

$$L_{b,f} = 2L_{wh,i}.$$

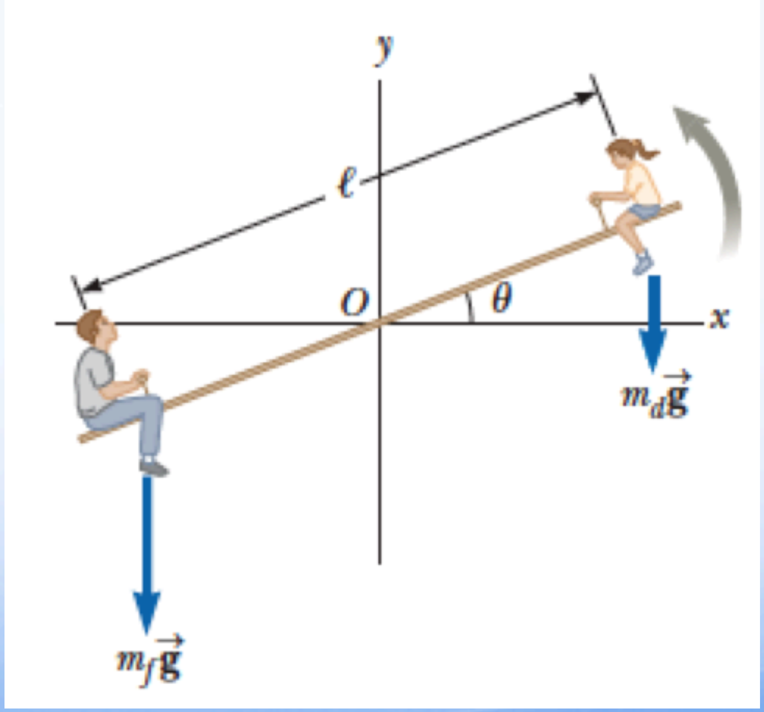
$$\omega_b = \frac{2I_{wh}}{I_b}\ \omega_{wh}$$

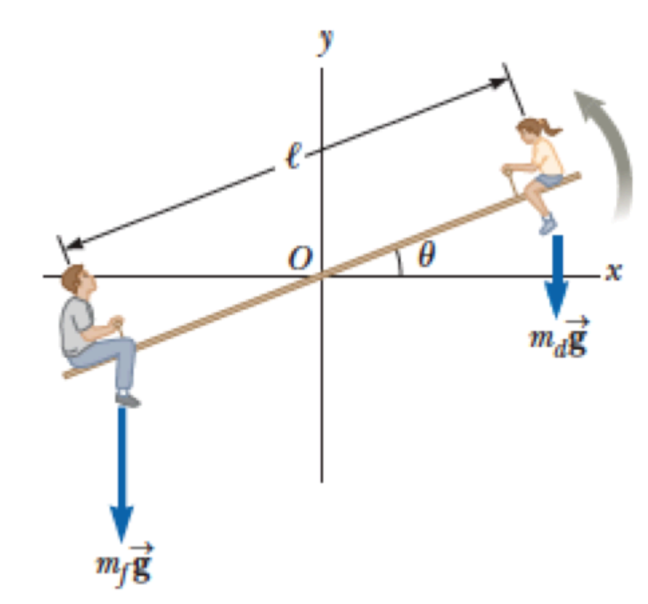


Example:

A father of mass m_f and his daughter of mass m_d sit on opposite ends of a seesaw at equal distances from the pivot at the center (Fig. 11.9, page 328). The seesaw is modeled as a rigid rod of mass M and length ℓ and is pivoted without friction. At a given moment, the combination rotates in a vertical plane with an angular speed ω.

(A) Find an expression for the magnitude of the system's angular momentum.





$$I = \frac{1}{12}M\ell^2 + m_f\left(\frac{\ell}{2}\right)^2 + m_d\left(\frac{\ell}{2}\right)^2 = \frac{\ell^2}{4}\left(\frac{M}{3} + m_f + m_d\right)$$

$$L = I\omega = \frac{\ell^2}{4}\left(\frac{M}{3} + m_f + m_d\right)\omega$$

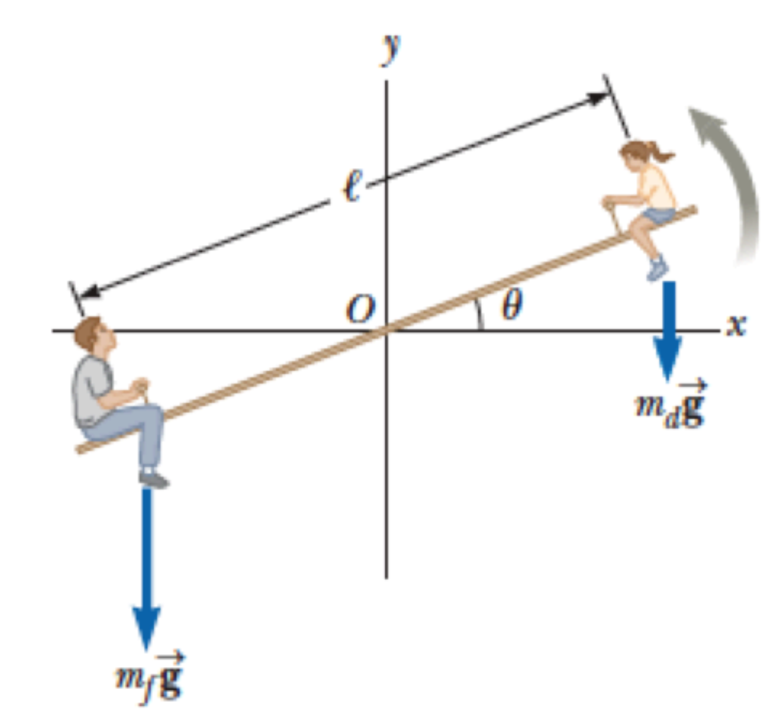
(B) Find an expression for the magnitude of the angular acceleration of the system when the seesaw makes an angle θ with the horizontal.

$$\tau_f = m_f g \frac{\ell}{2} \cos\theta \quad (\vec{\boldsymbol{\tau}}_f \text{ out of page})$$

$$\tau_d = -m_d g \frac{\ell}{2} \cos\theta \quad (\vec{\boldsymbol{\tau}}_d \text{ into page})$$

$$\sum \tau_{\text{ext}} = \tau_f + \tau_d = \tfrac{1}{2}(m_f - m_d) g \ell \cos\theta$$

$$\alpha = \frac{\sum \tau_{\text{ext}}}{I} = \boxed{\frac{2(m_f - m_d) g \cos\theta}{\ell\left[(M/3) + m_f + m_d\right]}}$$



$$I = \tfrac{1}{12}M\ell^2 + m_f d^2 + m_d\left(\frac{\ell}{2}\right)^2 = \frac{\ell^2}{4}\left(\frac{M}{3} + m_d\right) + m_f d^2$$

$$\sum \tau_{\text{ext}} = \tau_f + \tau_d = m_f g d \cos\theta - \tfrac{1}{2} m_d g \ell \cos\theta$$

$$\alpha = \frac{\sum \tau_{\text{ext}}}{I} = \frac{(m_f d - \frac{1}{2} m_d \ell) g \cos\theta}{(\ell^2/4)\left[(M/3) + m_d\right] + m_f d^2}$$

The seesaw is balanced when the angular acceleration is zero. In this situation, both father and daughter can push off the ground and rise to the highest possible point.

Find the required position of the father by setting $\alpha = 0$:

$$\alpha = \frac{(m_f d - \frac{1}{2} m_d \ell) g \cos\theta}{(\ell^2/4)[(M/3) + m_d] + m_f d^2} = 0$$

$$m_f d - \tfrac{1}{2} m_d \ell = 0 \quad \rightarrow \quad d = \left(\frac{m_d}{m_f}\right)\frac{\ell}{2}$$

In the rare case that the father and daughter have the same mass, the father is located at the end of the seesaw, $d = \ell/2$.

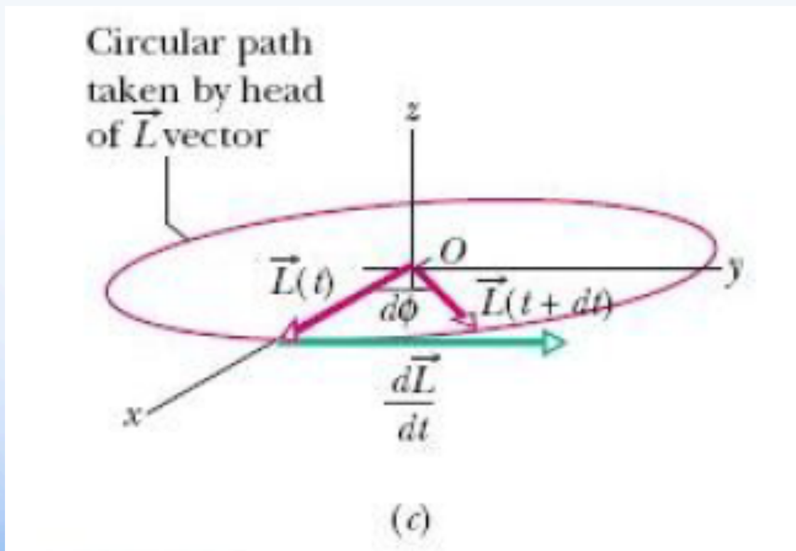
Bicycle Wheel Gyroscope

MIT Department of Physics
Technical Services Group

Precession of a Gyroscope (1)

$$\vec{\tau} = \frac{d\vec{L}}{dt}.$$

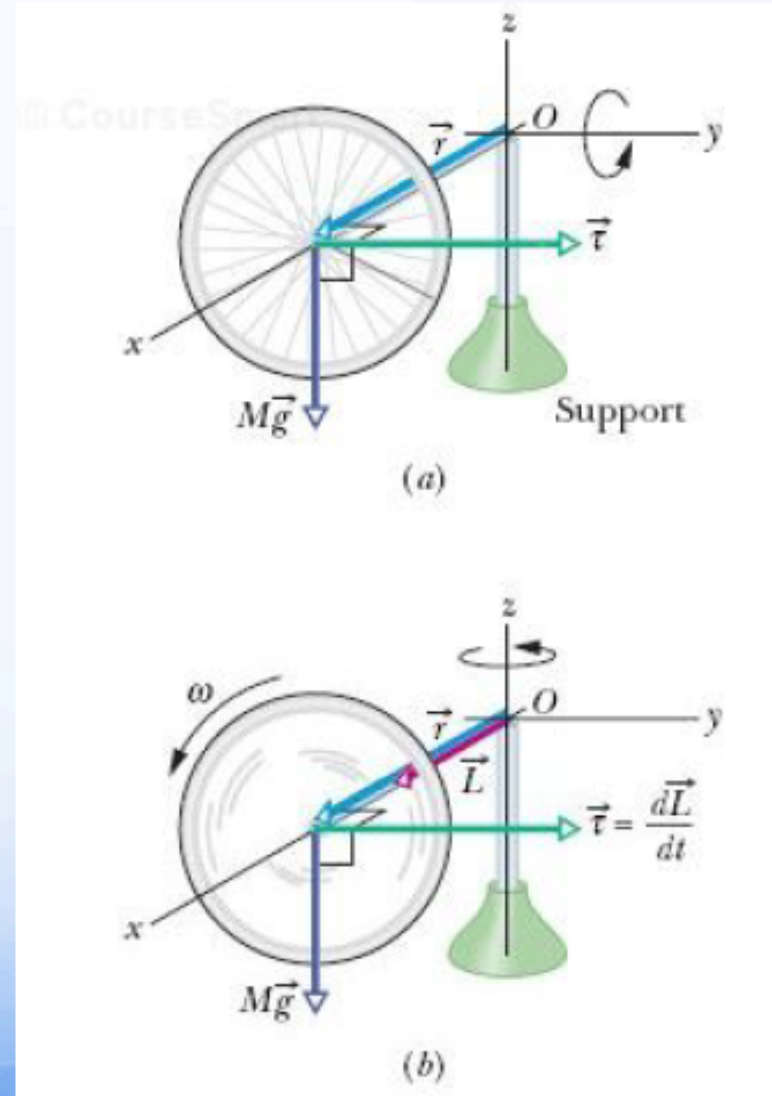
$$\tau = Mgr \sin 90^\circ = Mgr$$

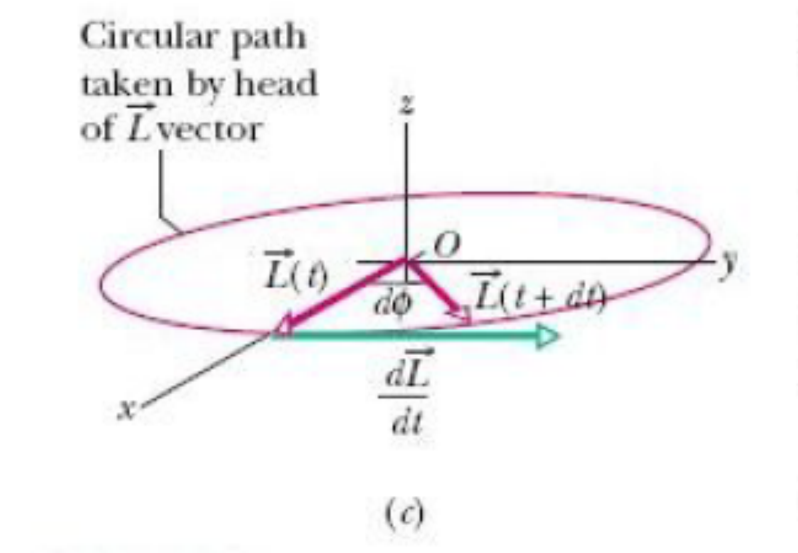


(c)

$$L = I\omega,$$

$$d\vec{L} = \vec{\tau}\,dt.$$





$$d\vec{L} = \vec{\tau}\,dt.$$

$$dL = \tau\,dt = Mgr\,dt.$$

$$d\phi = \frac{dL}{L} = \frac{Mgr\,dt}{I\omega}.$$

$$\Omega = d\phi/dt,$$

$$\Omega = \frac{Mgr}{I\omega} \qquad \text{(precession rate).}$$

Precession of a Gyroscope (2)

$$d\vec{\mathbf{L}} = \vec{\boldsymbol{\tau}}_{\text{net}}\, dt,$$

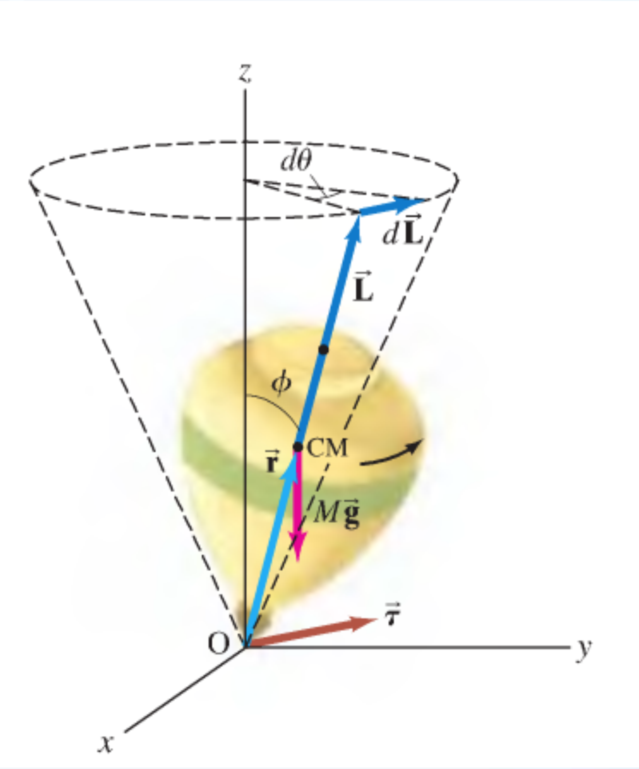
$$dL = L \sin\phi \, d\theta,$$

$$\Omega = d\theta/dt,$$

$$d\theta = dL/L \sin\phi$$

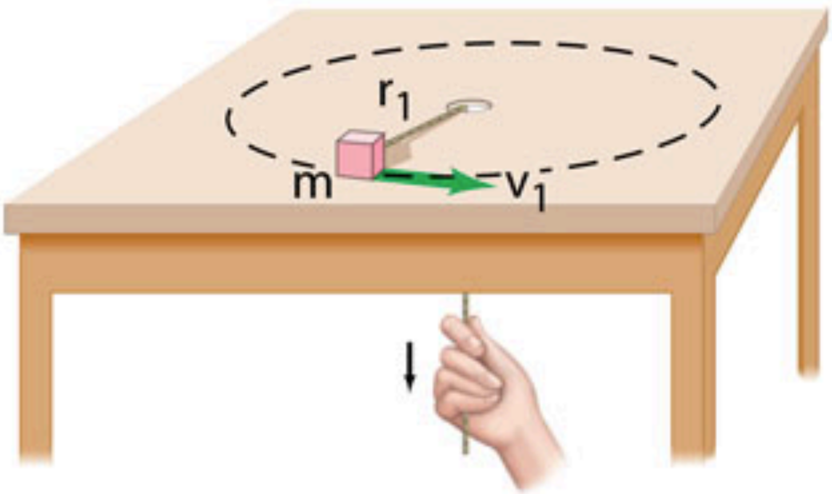
$$\Omega = \frac{1}{L\sin\phi}\frac{dL}{dt} = \frac{\tau}{L\sin\phi}.$$

$$\tau_{\text{net}} = \left|\vec{\mathbf{r}} \times M\vec{\mathbf{g}}\right| = rMg\sin\phi$$

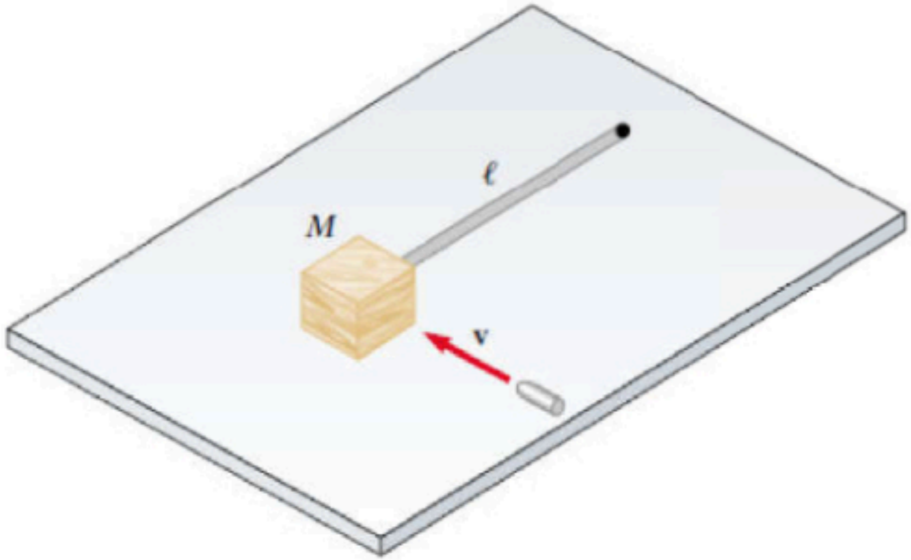


$$\Omega = \frac{Mgr}{L}.$$

A small mass m attached to the end of a string revolves in a circle on a frictionless tabletop. The other end of the string passes through a hole in the table (the figure (Figure 1)). Initially, the mass revolves with a speed v1 = 2.3 m/s in a circle of radius r1 = 0.80 m. The string is then pulled slowly through the hole so that the radius is reduced to r2 = 0.48 m.



What is the speed, v2, of the mass now?

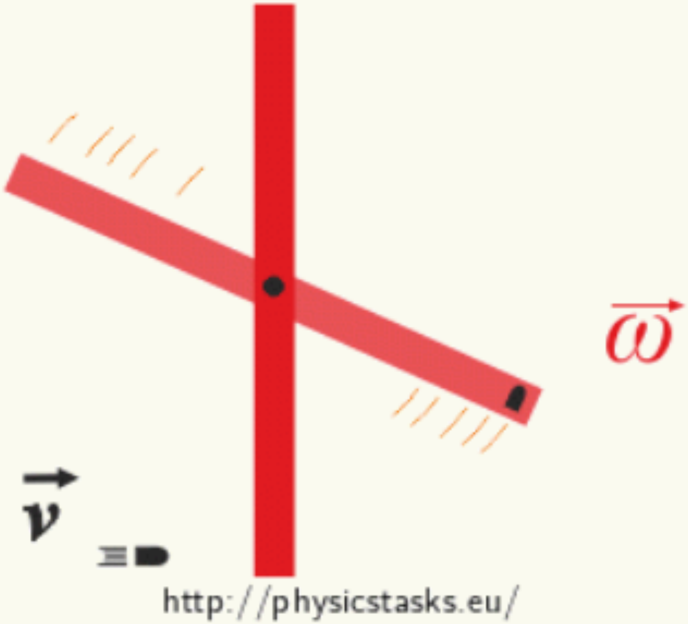


- A wooden block of mass M resting on a frictionless horizontal surface is attached to a rigid rod of length l and of negligible mass. The rod is pivoted at the other end. A bullet of mass m traveling parallel to the horizontal surface and perpendicular to the rod with speed v hits the block and becomes embedded in it.
- (a) What is the angular momentum of the bullet–block system?
- (b) What fraction of the original kinetic energy is lost in the collision?

Example:

A wooden rod of length of 40 cm and mass of 1 kg can rotate about the axis perpendicular to the rod and passing through the center of the rod. One end of the rod gets hit by a projectile of mass of 10 g at the speed of 200 $m \cdot s^{-1}$ in the direction perpendicular to both the axis and the rod. Determine the angular speed of the rod after the projectile gets stuck in it.

Note: We assume that the projectile first very quickly buries into the rod and then the system starts moving with the angular speed we are solving for.



Example:

A 2.0-kg disk traveling at 3.0 m/s strikes a 1.0-kg stick of length 4.0 m that is lying flat on nearly frictionless ice as shown in the overhead view of Figure 11.12a. The disk strikes at the endpoint of the stick, at a distance $r = 2.0$ m from the stick's center. Assume the collision is elastic and the disk does not deviate from its original line of motion. Find the translational speed of the disk, the translational speed of the stick, and the angular speed of the stick after the collision. The moment of inertia of the stick about its center of mass is $1.33\ \text{kg}\cdot\text{m}^2$.

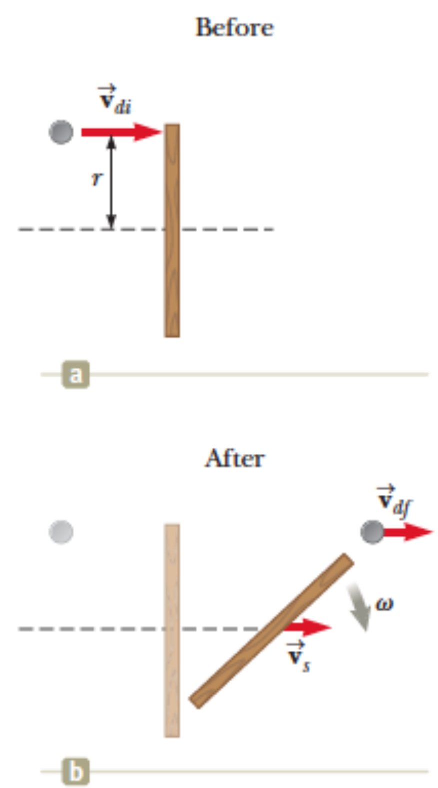


Figure 11.12 (Example 11.9) Overhead view of a disk striking a stick in an elastic collision. (a) Before the collision, the disk moves toward the stick. (b) The collision causes the stick to rotate and move to the right.

Analyze First notice that we have three unknowns, so we need three equations to solve simultaneously.

Apply the isolated system model for momentum to the system and then rearrange the result:

$$m_d v_{di} = m_d v_{df} + m_s v_s$$

$$(1) \quad m_d(v_{di} - v_{df}) = m_s v_s$$

Apply the isolated system model for angular momentum to the system and rearrange the result. Use an axis passing through the center of the stick as the rotation axis so that the path of the disk is a distance $r = 2.0$ m from the rotation axis:

$$-rm_d v_{di} = -rm_d v_{df} + I\omega$$

$$(2) \quad -rm_d(v_{di} - v_{df}) = I\omega$$

Apply the isolated system model for energy to the system, rearrange the equation, and factor the left side:

$$\tfrac{1}{2}m_d v_{di}^2 = \tfrac{1}{2}m_d v_{df}^2 + \tfrac{1}{2}m_s v_s^2 + \tfrac{1}{2}I\omega^2$$

$$(3) \quad m_d(v_{di} - v_{df})(v_{di} + v_{df}) = m_s v_s^2 + I\omega^2$$

Multiply Equation (1) by r and add to Equation (2):

$$rm_d(v_{di} - v_{df}) = rm_s v_s$$

$$-rm_d(v_{di} - v_{df}) = I\omega$$

$$0 = rm_s v_s + I\omega$$

Solve for ω: (4) $\omega = -\frac{rm_s v_s}{I}$

Divide Equation (3) by Equation (1):

$$\frac{m_d(v_{di} - v_{df})(v_{di} + v_{df})}{m_d(v_{di} - v_{df})} = \frac{m_s v_s^2 + I\omega^2}{m_s v_s}$$

$$(5) \quad v_{di} + v_{df} = v_s + \frac{I\omega^2}{m_s v_s}$$

Substitute Equation (4) into Equation (5): (6) $v_{di} + v_{df} = v_s\left(1 + \frac{r^2 m_s}{I}\right)$

Substitute v_{df} from Equation (1) into Equation (6):

$$v_{di} + \left(v_{di} - \frac{m_s}{m_d} v_s\right) = v_s\left(1 + \frac{r^2 m_s}{I}\right)$$

Solve for v_s and substitute numerical values:

$$v_s = \frac{2v_{di}}{1 + (m_s/m_d) + (r^2 m_s/I)}$$

$$= \frac{2(3.0 \text{ m/s})}{1 + (1.0 \text{ kg}/2.0 \text{ kg}) + [(2.0 \text{ m})^2(1.0 \text{ kg})/1.33 \text{ kg}\cdot\text{m}^2]} = 1.3 \text{ m/s}$$

Substitute numerical values into Equation (4):

$$\omega = -\frac{(2.0 \text{ m})(1.0 \text{ kg})(1.3 \text{ m/s})}{1.33 \text{ kg}\cdot\text{m}^2} = -2.0 \text{ rad/s}$$

Solve Equation (1) for v_{df} and substitute numerical values:

$$v_{df} = v_{di} - \frac{m_s}{m_d} v_s = 3.0 \text{ m/s} - \frac{1.0 \text{ kg}}{2.0 \text{ kg}}(1.3 \text{ m/s}) = 2.3 \text{ m/s}$$

35. Find the net torque on the wheel in Figure P10.35 about the axle through O, taking $a = 10.0$ cm and $b = 25.0$ cm.

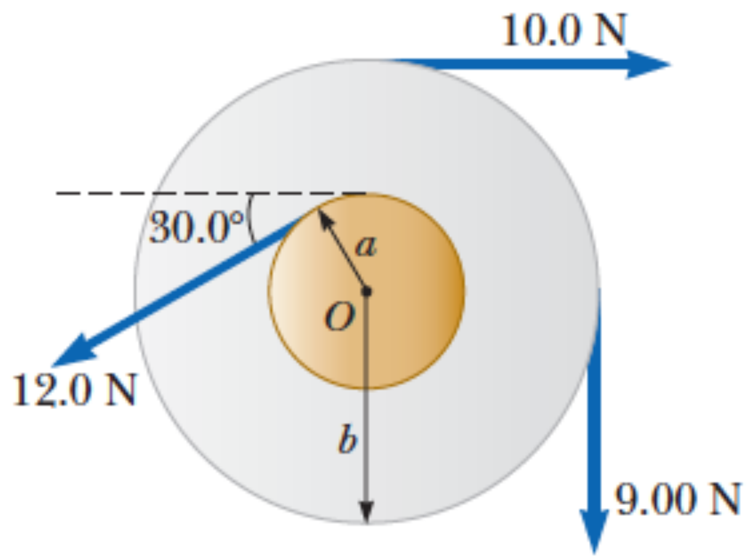


Figure P10.35

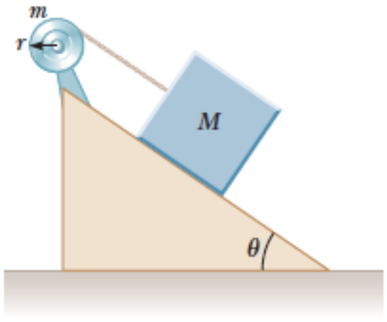


Figure P10.86

Example:

S A cord is wrapped around a pulley that is shaped like a disk of mass m and radius r. The cord's free end is connected to a block of mass M. The block starts from rest and then slides down an incline that makes an angle θ with the horizontal as shown in Figure P10.86. The coefficient of kinetic friction between block and incline is μ. (a) Use energy methods to show that the block's speed as a function of position d down the incline is

$$v = \sqrt{\frac{4Mgd(\sin\theta - \mu\cos\theta)}{m + 2M}}$$

(b) Find the magnitude of the acceleration of the block in terms of μ, m, M, g, and θ.

Example

S A uniform, hollow, cylindrical spool has inside radius $R/2$, outside radius R, and mass M (Fig. P10.81). It is mounted so that it rotates on a fixed, horizontal axle. A counterweight of mass m is connected to the end of a string wound around the spool. The counterweight falls from rest at $t = 0$ to a position y at time t. Show that the torque due to the friction forces between spool and axle is

$$\tau_f = R\left[m\left(g - \frac{2y}{t^2}\right) - M\frac{5y}{4t^2}\right]$$

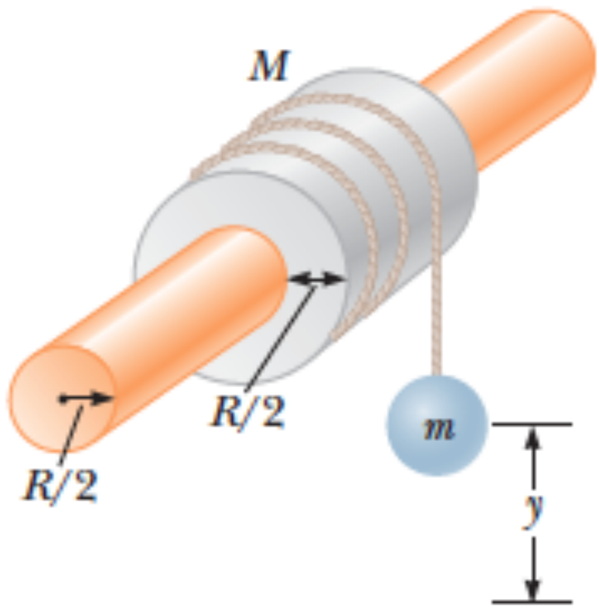


Figure P10.81

Example:

S **Review.** A conical pendulum consists of a bob of mass m in motion in a circular path in a horizontal plane as shown in Figure P11.16. During the motion, the supporting wire of length ℓ maintains a constant angle θ with the vertical. Show that the magnitude of the angular momentum of the bob about the vertical dashed line is

$$L = \left(\frac{m^2 g \ell^3 \sin^4 \theta}{\cos \theta}\right)^{1/2}$$

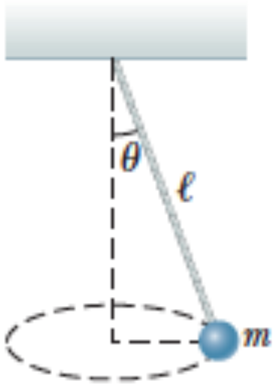


Figure P11.16

Example:

QC S A wad of sticky clay with mass m and velocity $\vec{\mathbf{v}}_i$ is fired at a solid cylinder of mass M and radius R (Fig. P11.39). The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axle and at a distance $d < R$ from the center. (a) Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. (b) Is the mechanical energy of the clay–cylinder system constant in this process? Explain your answer. (c) Is the momentum of the clay–cylinder system constant in this process? Explain your answer.

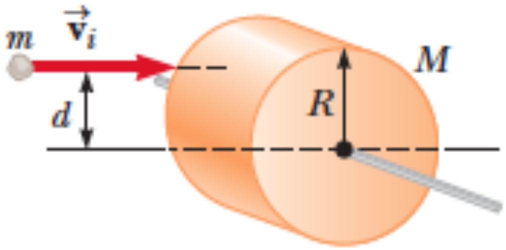


Figure P11.39

its frictionless hinges. (a) Before it hits the door, does the bullet have angular momentum relative to the door's axis of rotation? (b) If so, evaluate this angular momentum. If not, explain why there is no angular momentum. (c) Is the mechanical energy of the bullet–door system constant during this collision? Answer without doing a calculation. (d) At what angular speed does the door swing open immediately after the collision? (e) Calculate the total energy of the bullet–door system and determine whether it is less than or equal to the kinetic energy of the bullet before the collision.

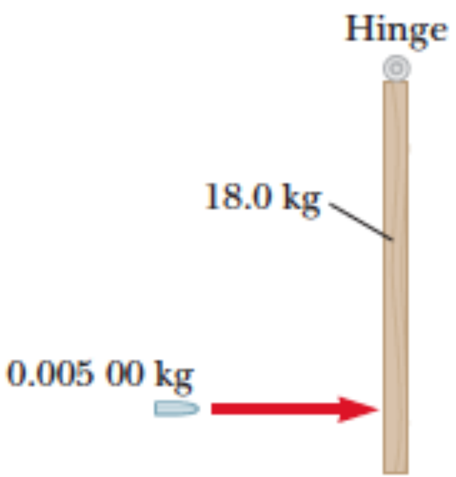


Figure P11.41 An overhead view of a bullet striking a door.

Example:

S A rigid, massless rod has three particles with equal masses attached to it as shown in Figure P11.49. The rod is free to rotate in a vertical plane about a frictionless axle perpendicular to the rod through the point P and is released from rest in the horizontal position at $t = 0$. Assuming m and d are known, find (a) the moment of inertia of the system of three particles about the pivot, (b) the torque acting on the system at $t = 0$, (c) the angular acceleration of the system at $t = 0$, (d) the linear acceleration of the particle labeled 3 at $t = 0$, (e) the maximum kinetic energy of the system, (f) the maximum angular speed reached by the rod, (g) the maximum angular momentum of the system, and (h) the maximum speed reached by the particle labeled 2.

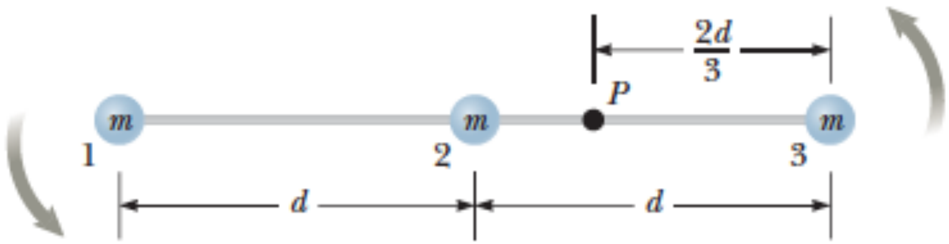


Figure P11.49

S **GP** A projectile of mass m moves to the right with a speed v_i (Fig. P11.51a). The projectile strikes and sticks to the end of a stationary rod of mass M, length d, pivoted about a frictionless axle perpendicular to the page through O (Fig. P11.51b). We wish to find the fractional change of kinetic energy in the system due to the collision. (a) What is the appropriate analysis model to describe the projectile and the rod? (b) What is the angular momentum of the system before the collision about an axis through O? (c) What is the moment of inertia of the system about an axis through O after the projectile sticks to the rod? (d) If the angular speed of the system after the collision is ω, what is the angular momentum of the system after the collision? (e) Find the angular speed ω after the collision in terms of the given quantities. (f) What is the kinetic energy of the system before the collision? (g) What is the kinetic energy of the system after the collision? (h) Determine the fractional change of kinetic energy due to the collision.

Example:

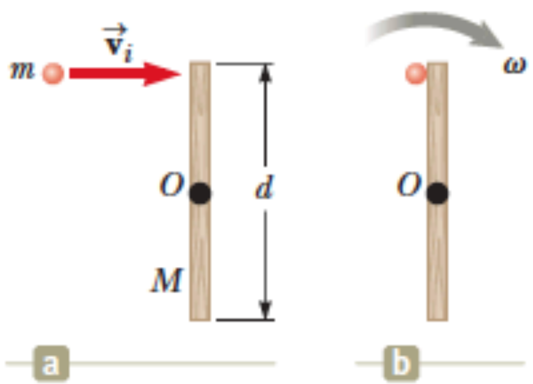


Figure P11.51

Example:

S A solid cube of wood of side $2a$ and mass M is resting on a horizontal surface. The cube is constrained to rotate about a fixed axis AB (Fig. P11.62). A bullet of mass m and

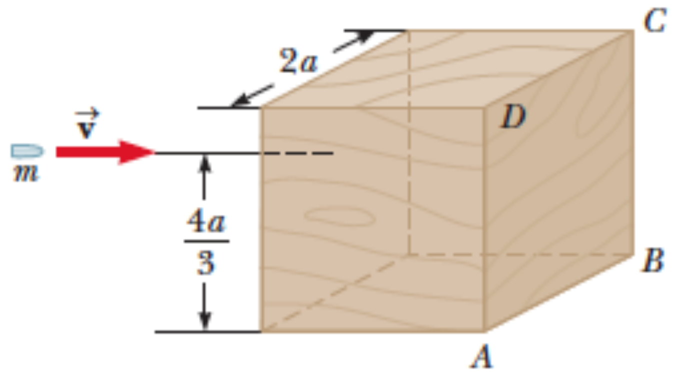


Figure P11.62

speed v is shot at the face opposite $ABCD$ at a height of $4a/3$. The bullet becomes embedded in the cube. Find the minimum value of v required to tip the cube so that it falls on face $ABCD$. Assume $m << M$.

Example:

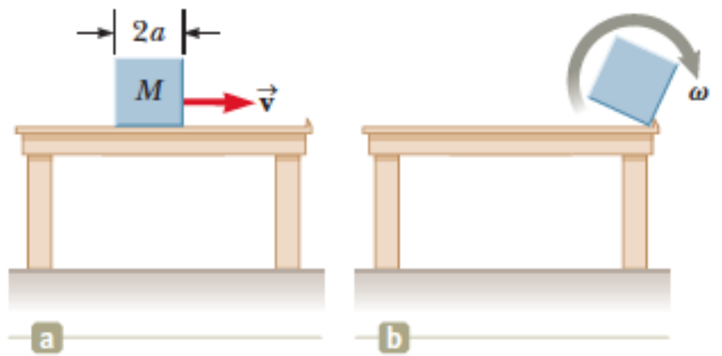


Figure P11.63

S A solid cube of side $2a$ and mass M is sliding on a frictionless surface with uniform velocity $\vec{v}$ as shown in Figure P11.63a. It hits a small obstacle at the end of the table that causes the cube to tilt as shown in Figure P11.63b. Find the minimum value of the magnitude of $\vec{v}$ such that the cube tips over and falls off the table. *Note:* The cube undergoes an inelastic collision at the edge.

Example:

•••13 GO *Nonuniform ball.* In Fig. 11-36, a ball of mass M and radius R

Figure 11-36 Problem 13.

rolls smoothly from rest down a ramp and onto a circular loop of radius 0.48 m. The initial height of the ball is $h = 0.36$ m. At the loop bottom, the magnitude of the normal force on the ball is $2.00Mg$. The ball consists of an outer spherical shell (of a certain uniform density) that is glued to a central sphere (of a different uniform density). The rotational inertia of the ball can be expressed in the general form $I = \beta MR^2$, but β is not 0.4 as it is for a ball of uniform density. Determine β.

Example:

•••15 A bowler throws a bowling ball of radius $R = 11$ cm along a lane. The ball (Fig. 11-38) slides on the lane with initial speed $v_{com,0} = 8.5$ m/s and initial angular speed $\omega_0 = 0$. The coefficient of kinetic friction between the ball and the lane is 0.21. The kinetic frictional force $\vec{f}_k$ acting on the ball causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When speed v_{com} has decreased enough and angular speed ω has increased enough, the ball stops sliding and then rolls smoothly. (a) What then is v_{com} in terms of ω? During the sliding, what are the ball's (b) linear acceleration and (c) angular acceleration? (d) How long does the ball slide? (e) How far does the ball slide? (f) What is the linear speed of the ball when smooth rolling begins?

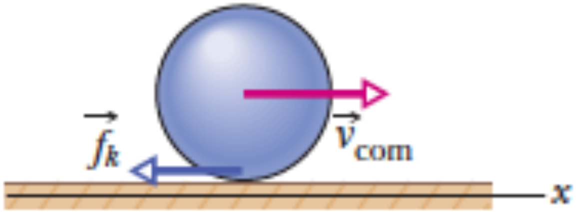


Figure 11-38 Problem 15.

Example:

•••67 GO Figure 11-59 is an overhead view of a thin uniform rod of length 0.600 m and mass M rotating horizontally at 80.0 rad/s counterclockwise about an axis through its center. A particle of mass $M/3.00$ and traveling horizontally at speed 40.0 m/s hits the rod and sticks. The particle's path is perpendicular to the rod at the instant of the hit, at a distance d from the rod's center. (a) At what value of d are rod and particle stationary after the hit? (b) In which direction do rod and particle rotate if d is greater than this value?

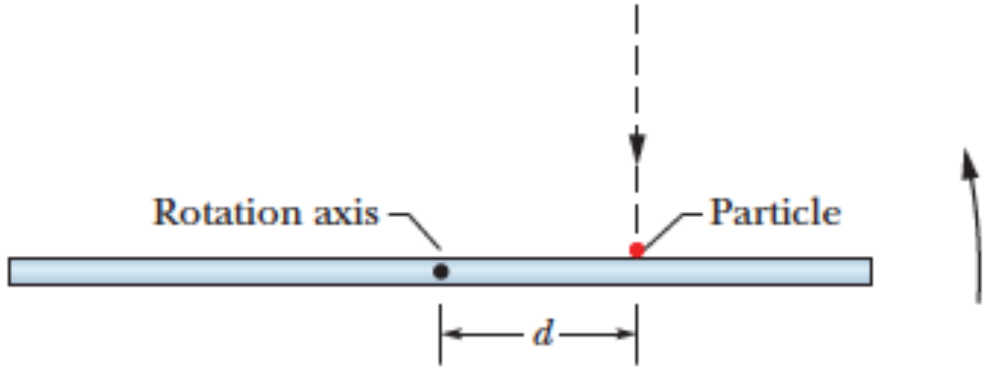


Figure 11-59 Problem 67.

Example:

•••66 GO In Fig. 11-58, a small 50 g block slides down a frictionless surface through height $h = 20$ cm and then sticks to a uniform rod of mass 100 g and length 40 cm. The rod pivots about point O through angle θ before momentarily stopping. Find θ.

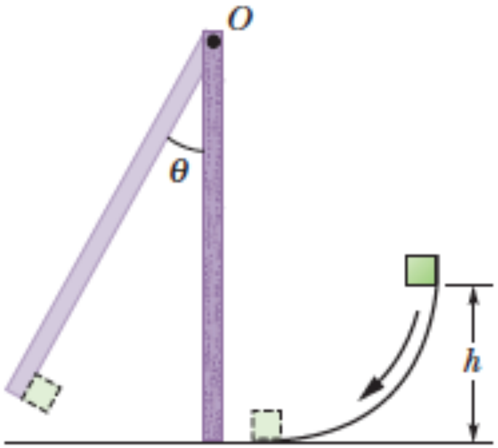


Figure 11-58 Problem 66.

Example:

••60 In Fig. 11-53, a 1.0 g bullet is fired into a 0.50 kg block attached to the end of a 0.60 m nonuniform rod of mass 0.50 kg. The block–rod–bullet system then rotates in the plane of the figure, about a fixed axis at A. The rotational inertia of the rod alone about that axis at A is $0.060\ \text{kg}\cdot\text{m}^2$. Treat the block as a particle. (a) What then is the rotational inertia of the block–rod–bullet system about point A? (b) If the angular speed of the system about A just after impact is 4.5 rad/s, what is the bullet's speed just before impact?

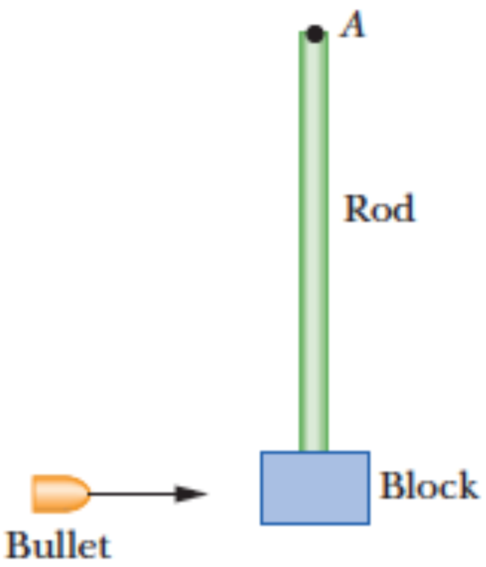


Figure 11-53 Problem 60.

Example:

71 SSM In Fig. 11-60, a constant horizontal force $\vec{F}_{app}$ of magnitude 12 N is applied to a uniform solid cylinder by fishing line wrapped around the cylinder. The mass of the cylinder is 10 kg, its radius is 0.10 m, and the cylinder rolls smoothly on the horizontal surface. (a) What is the magnitude of the acceleration of the center of mass of the cylinder? (b) What is the magnitude of the angular acceleration of the cylinder about the center of mass? (c) In unit-vector notation, what is the frictional force acting on the cylinder?

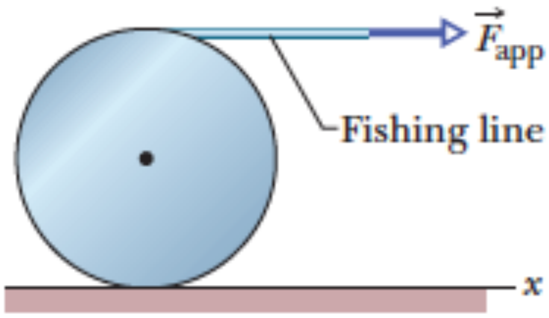


Figure 11-60 Problem 71.

Example:

81 SSM A uniform wheel of mass 10.0 kg and radius 0.400 m is mounted rigidly on a massless axle through its center (Fig. 11-62). The radius of the axle is 0.200 m, and the rotational inertia of the wheel–axle combination about its central axis is 0.600 kg·m². The wheel is initially at rest at the top of a surface that is inclined at angle $\theta = 30.0°$ with the horizontal; the axle rests on the surface while the wheel extends into a groove in the surface without touching the surface. Once released, the axle rolls down along the surface smoothly and without slipping. When the wheel–axle combination has moved down the surface by 2.00 m, what are (a) its rotational kinetic energy and (b) its translational kinetic energy?

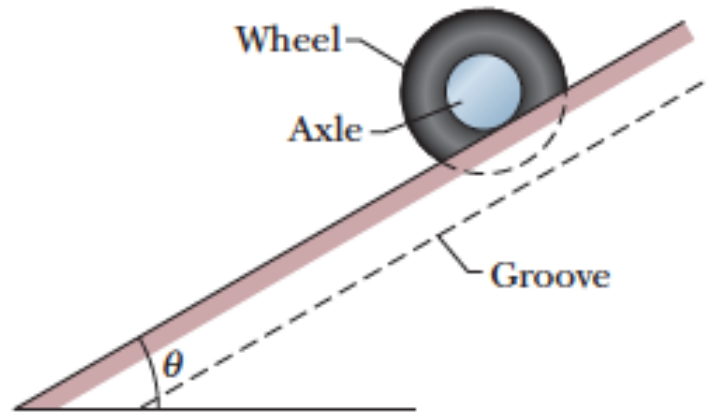


Figure 11-62 Problem 81.

Example:

58. (II) A ball of mass M and radius r_1 on the end of a thin massless rod is rotated in a horizontal circle of radius R_0 about an axis of rotation AB, as shown in Fig. 10–58. (*a*) Considering the mass of the ball to be concentrated at its center of mass, calculate its moment of inertia about AB. (*b*) Using the parallel-axis theorem and considering the finite radius of the ball, calculate the moment of inertia of the ball about AB. (*c*) Calculate the percentage error introduced by the point mass approximation for $r_1 = 9.0\text{ cm}$ and $R_0 = 1.0\text{ m}$.

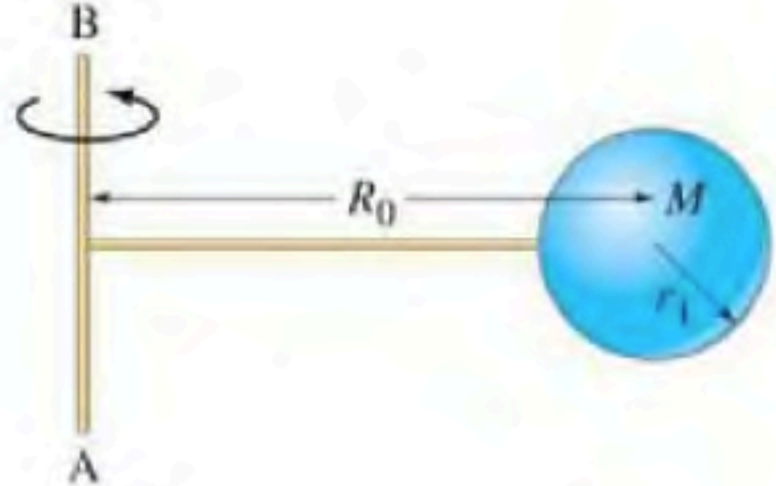


FIGURE 10–58
Problem 58.

Example:

96. If a billiard ball is hit in just the right way by a cue stick, the ball will roll without slipping immediately after losing contact with the stick. Consider a billiard ball (radius r, mass M) at rest on a horizontal pool table. A cue stick exerts a constant horizontal force F on the ball for a time t at a point that is a height h above the table's surface (see Fig. 10–68). Assume that the coefficient of kinetic friction between the ball and table is μ_k. Determine the value for h so that the ball will roll without slipping immediately after losing contact with the stick.

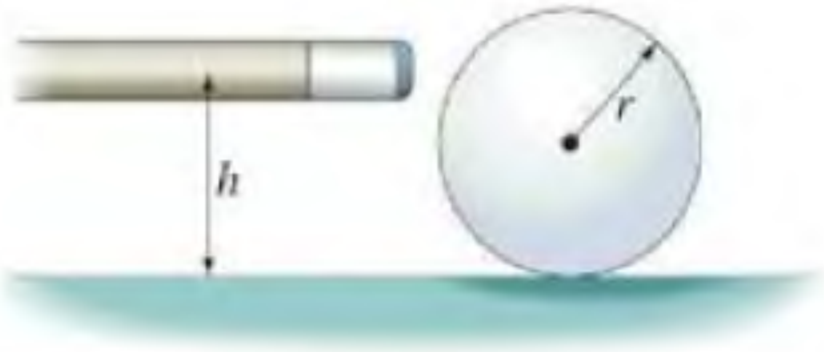


FIGURE 10–68
Problem 96.

Example:

102. A crucial part of a piece of machinery starts as a flat uniform cylindrical disk of radius R_0 and mass M. It then has a circular hole of radius R_1 drilled into it (Fig. 10–73). The hole's center is a distance h from the center of the disk. Find the moment of inertia of this disk (with off-center hole) when rotated about its center, C. [*Hint*: Consider a solid disk and "subtract" the hole; use the parallel-axis theorem.]

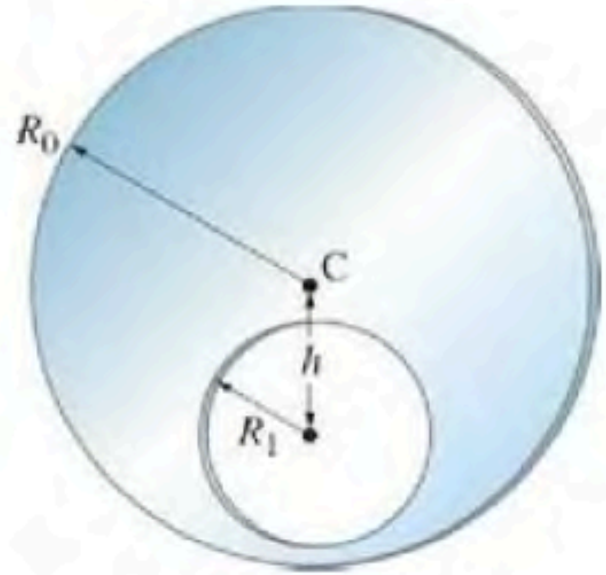
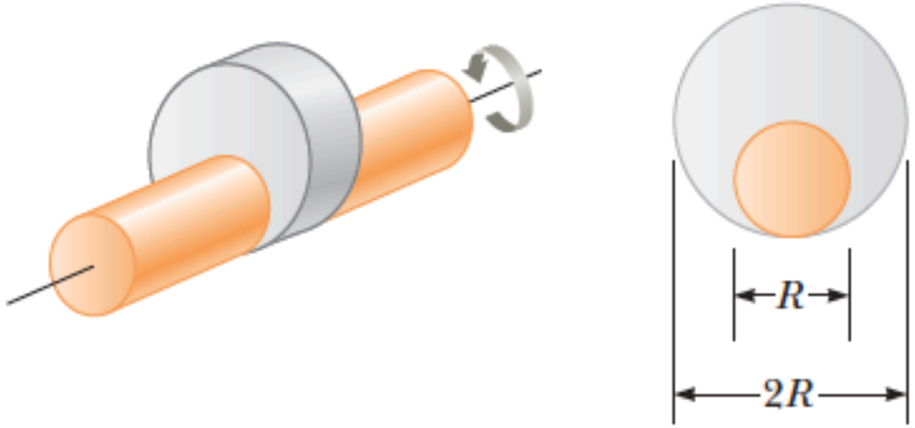


FIGURE 10–73
Problem 102.

Example:

32. **S** Many machines employ cams for various purposes, such as opening and closing valves. In Figure P10.32, the cam is a circular disk of radius R with a hole of diameter R cut through it. As shown in the figure, the hole does not pass through the center of the disk. The cam with the hole cut out has mass M. The cam is mounted on a uniform, solid, cylindrical shaft of diameter R and also of mass M. What is the kinetic energy of the cam–shaft combination when it is rotating with angular speed ω about the shaft's axis?



Example:

53. **S** A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim (Fig. P10.53). If the disk is released from rest in the position shown by the copper-colored circle, (a) what is the speed of its center of mass when the disk reaches the position indicated by the dashed circle? (b) What is the speed of the lowest point on the disk in the dashed position? (c) **What If?** Repeat part (a) using a uniform hoop.

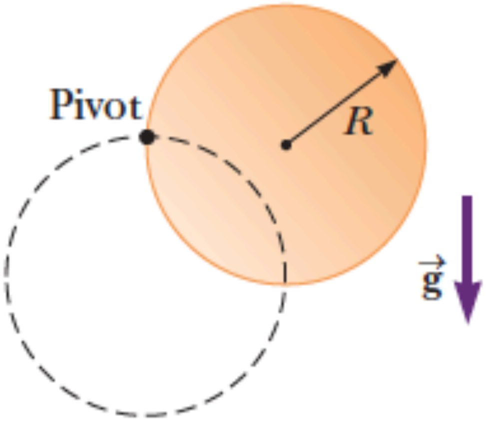
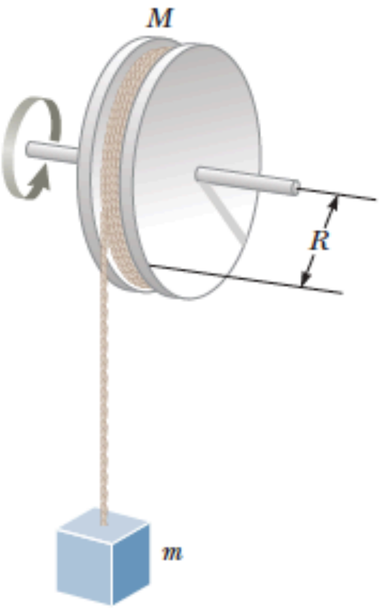


Figure P10.53

Example:

51. **M** **Review.** An object with a mass of $m = 5.10$ kg is attached to the free end of a light string wrapped around a reel of radius $R = 0.250$ m and mass $M = 3.00$ kg. The reel is a solid disk, free to rotate in a vertical plane about the horizontal axis passing through its center as shown in Figure P10.51. The suspended object is released from rest 6.00 m above the floor. Determine (a) the tension in the string, (b) the acceleration of the object, and (c) the speed with which the object hits the floor. (d) Verify your answer to part (c) by using the isolated system (energy) model.



Example:

67. **S** A long, uniform rod of length L and mass M is pivoted about a frictionless, horizontal pin through one end. The rod is nudged from rest in a vertical position as shown in Figure P10.67. At the instant the rod is horizontal, find (a) its angular speed, (b) the magnitude of its angular acceleration, (c) the x and y components of the acceleration of its center of mass, and (d) the components of the reaction force at the pivot.

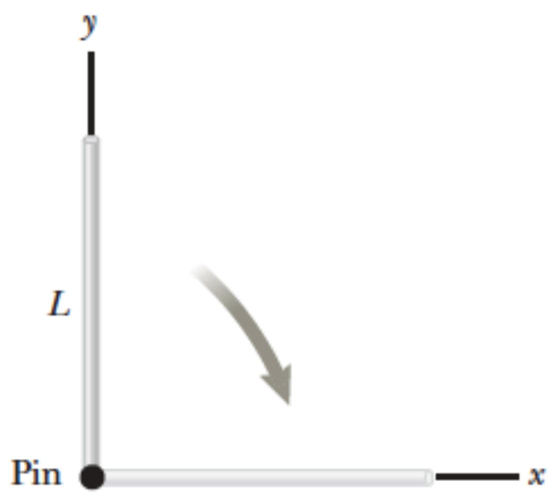


Figure P10.67

Example:

72. **S** The reel shown in Figure P10.72 has radius R and moment of inertia I. One end of the block of mass m is connected to a spring of force constant k, and the other end is fastened to a cord wrapped around the reel. The reel axle and the incline are frictionless. The reel is wound counterclockwise so that the spring stretches a distance d from its unstretched position and the reel is then released from rest. Find the angular speed of the reel when the spring is again unstretched.

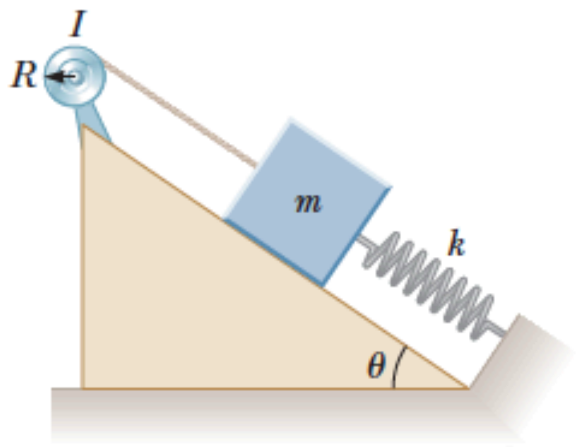


Figure P10.72

Example:

73. **S** **Review.** A string is wound around a uniform disk of radius R and mass M. The disk is released from rest with the string vertical and its top end tied to a fixed bar (Fig. P10.73). Show that (a) the tension in the string is one third of the weight of the disk, (b) the magnitude of the acceleration of the center of mass is $2g/3$, and (c) the speed of the center of mass is $(4gh/3)^{1/2}$ after the disk has descended through distance h. (d) Verify your answer to part (c) using the energy approach.

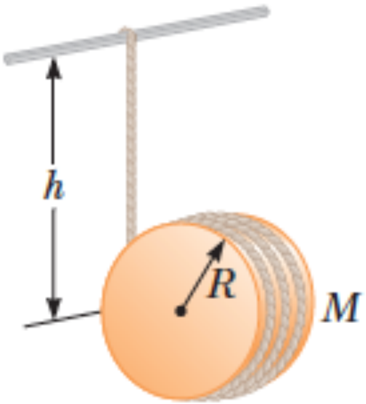


Figure P10.73

Example:

77. **S** A solid sphere of mass m and radius r rolls without slipping along the track shown in Figure P10.77. It starts from rest with the lowest point of the sphere at height h above the bottom of the loop of radius R, much larger than r. (a) What is the minimum value of h (in terms of R) such that the sphere completes the loop? (b) What are the force components on the sphere at the point P if $h = 3R$?

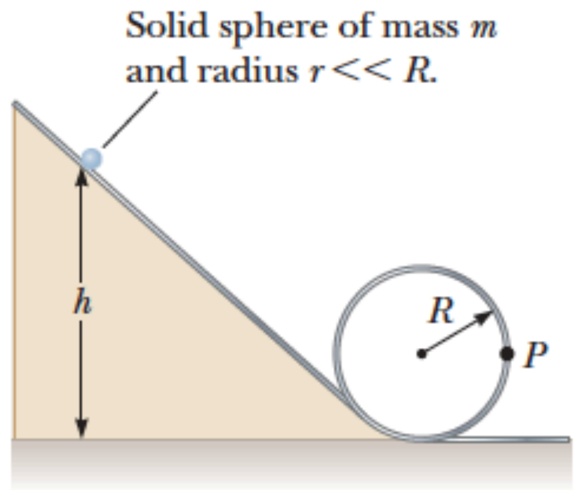


Figure P10.77

85. **S** A spool of thread consists of a cylinder of radius R_1 with end caps of radius R_2 as depicted in the end view shown in Figure P10.85. The mass of the spool, including the thread, is m, and its moment of inertia about an axis through its center is I. The spool is placed on a rough, horizontal surface so that it rolls without slipping when a force $\vec{\mathbf{T}}$ acting to the right is applied to the free end of the thread. (a) Show that the magnitude of the friction force exerted by the surface on the spool is given by

$$f = \left(\frac{I + mR_1R_2}{I + mR_2^{\,2}}\right)T$$

(b) Determine the direction of the force of friction.

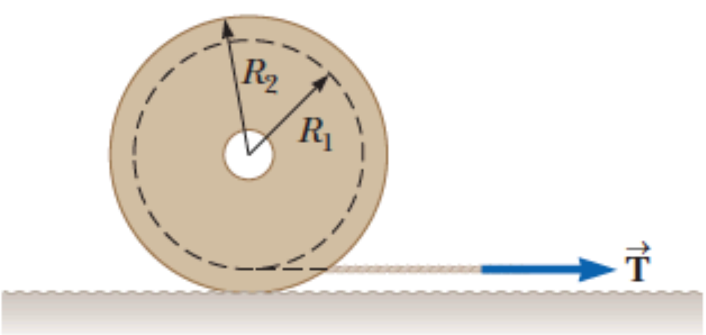


Figure P10.85

88. A plank with a mass $M = 6.00$ kg rests on top of two identical, solid, cylindrical rollers that have $R = 5.00$ cm and $m = 2.00$ kg (Fig. P10.88). The plank is pulled by a constant horizontal force $\vec{\mathbf{F}}$ of magnitude 6.00 N applied to the end of the plank and perpendicular to the axes of the cylinders (which are parallel). The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the plank. (a) Find the initial acceleration of the plank at the moment the rollers are equidistant from the ends of the plank. (b) Find the acceleration of the rollers at this moment. (c) What friction forces are acting at this moment?

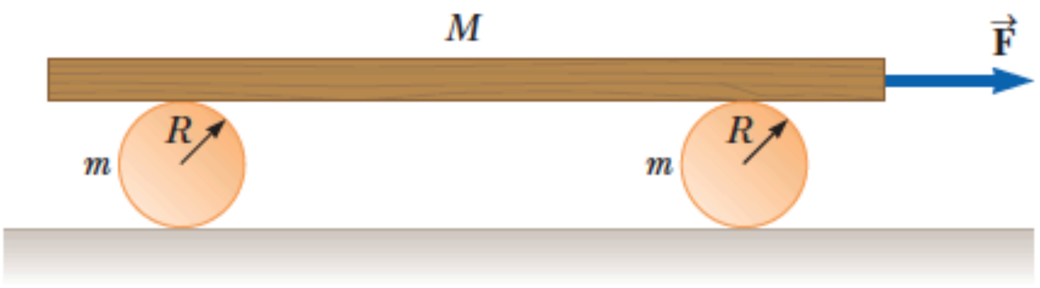


Figure P10.88

71 SSM In Fig. 11-60, a constant horizontal force $\vec{F}_{app}$ of magnitude 12 N is applied to a uniform solid cylinder by fishing line wrapped around the cylinder. The mass of the cylinder is 10 kg, its radius is 0.10 m, and the cylinder rolls smoothly on the horizontal surface. (a) What is the magnitude of the acceleration of the center of mass of the cylinder? (b) What is the magnitude of the angular acceleration of the cylinder about the center of mass? (c) In unit-vector notation, what is the frictional force acting on the cylinder?

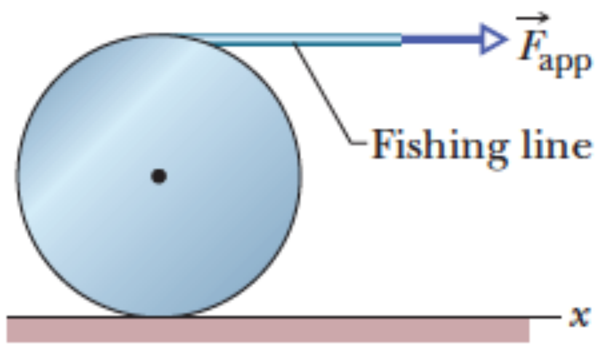


Figure 11-60 Problem 71.

88. A small mass m attached to the end of a string revolves in a circle on a frictionless tabletop. The other end of the string passes through a hole in the table (Fig. 8–62). Initially, the mass revolves with a speed $v_1 = 2.4\ \text{m/s}$ in a circle of radius $r_1 = 0.80\ \text{m}$. The string is then pulled slowly through the hole so that the radius is reduced to $r_2 = 0.48\ \text{m}$. What is the speed, v_2, of the mass now?

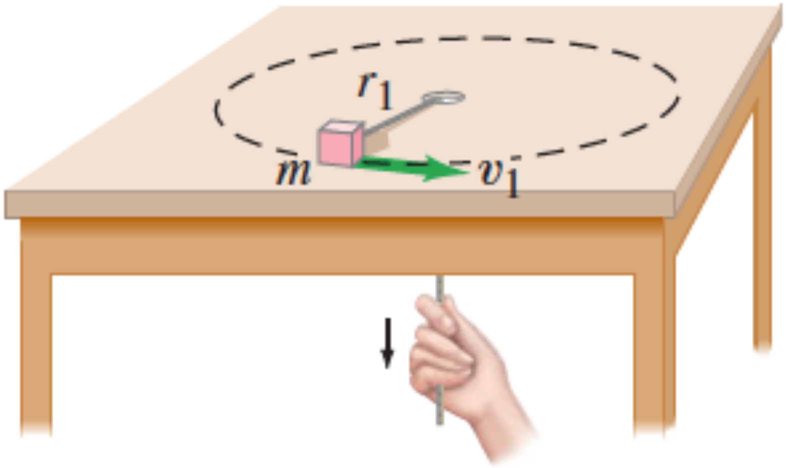


FIGURE 8–62
Problem 88.

91. A large spool of rope rolls on the ground with the end of the rope lying on the top edge of the spool. A person grabs the end of the rope and walks a distance ℓ, holding onto it, Fig. 8–64. The spool rolls behind the person without slipping. What length of rope unwinds from the spool? How far does the spool's center of mass move?

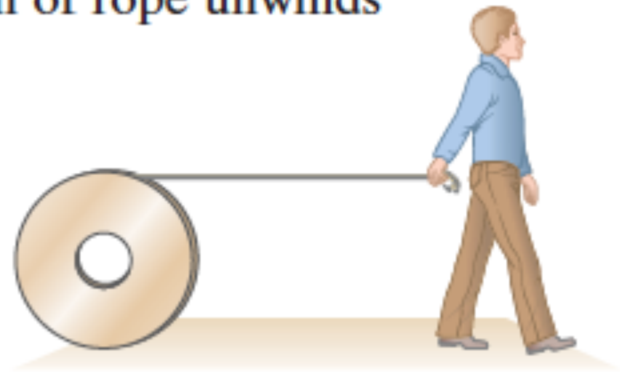
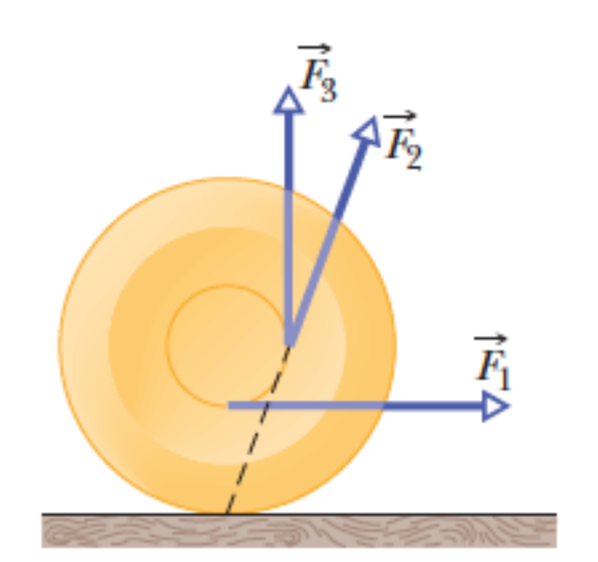


FIGURE 8–64
Problem 91.

3 What happens to the initially stationary yo-yo in Fig. 11-25 if you pull it via its string with (a) force $\vec{F}_2$ (the line of action passes through the point of contact on the table, as indicated), (b) force $\vec{F}_1$ (the line of action passes above the point of contact), and (c) force $\vec{F}_3$ (the line of action passes to the right of the point of contact)?



Example 11-2: Clutch.

A simple clutch consists of two cylindrical plates that can be pressed together to connect two sections of an axle, as needed, in a piece of machinery. The two plates have masses M_A = 6.0 kg and M_B = 9.0 kg, with equal radii R_0 = 0.60 m. They are initially separated. Plate M_A is accelerated from rest to an angular velocity ω_1 = 7.2 rad/s in time Δt = 2.0 s. Calculate (a) the angular momentum of M_A, and (b) the torque required to have accelerated M_A from rest to ω_1. (c) Next, plate M_B, initially at rest but free to rotate without friction, is placed in firm contact with freely rotating plate M_A, and the two plates both rotate at a constant angular velocity ω_2, which is considerably less than ω_1. Why does this happen, and what is ω_2?