Turbulencelike Behavior of Seismic Time Series

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We report on a stochastic analysis of Earth's vertical velocity time series by using methods originally developed for complex hierarchical systems and, in particular, for turbulent flows. Analysis of the fluctuations of the detrended increments of the series reveals a pronounced transition in their probability density function from Gaussian to non-Gaussian. The transition occurs 5–10 hours prior to a moderate or large earthquake, hence representing a new and reliable precursor for detecting such earthquakes.

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A grand challenge in geophysics is developing methods for predicting when earthquakes may occur. Although there are many known precursors, experience over the past several decades indicates that reliable and quantitative methods for analyzing seismic data are still lacking [1]. Several concepts and ideas have been advanced in order to explain important aspects of such seismic time series and what they imply for earthquakes [2–5]. There has also been much interest in investigating the precursors to, and the predictability of, extreme increments in the time series [6] associated with disparate phenomena, ranging from earthquakes [7,8] to epileptic seizures [9] and stock market crashes [10–12].

In this Letter, we provide compelling evidence for the existence of a novel transition in the probability density function (PDF) of the detrended increments of the stochastic fluctuations of Earth's vertical velocity V_z, collected by broadband stations (resolution 100 Hz). Most importantly, we demonstrate that there is a well-defined transition from a Gaussian to a non-Gaussian PDF of the detrended increments as an earthquake is approached.

We analyze in detail the data obtained from Spain's and California's broadband networks for three earthquakes: the May 21, 2003, $M = 7.1$ event in Oran-Argel, detected in Ibiza (Balearic Islands); the 2004, $M = 6.1$ event in Alhucemas, and the $M = 5.4$ earthquake in California on April 30, 2008. The results for other earthquakes are also described briefly. Because of localization of elastic waves in rock [13,14], we analyze the data from stations that are at a distance $d \leq 300$ km from the epicenters. The distance 300 km is *not* universal and depends on the geology but is of the correct order of magnitude.

The data are first detrended in order to remove the possible trends in the time series $x(t) \equiv V_z(t)$. To do so, $x(t)$ is divided into semioverlapping subintervals $[1 + s(k-1), s(k+1)]$ of length $2s$ and labeled by $k \geq 1$. $x(t)$ is then fitted to a third-order polynomial [15–17] to detrend the original series in the corresponding time window. The detrended increments on scale s are defined by $Z_s(t) = x^*(t+s) - x^*(t)$, where $t \in [1 + s(k-1), sk]$, with $x^*(t)$ being the detrended series, i.e., the deviation of $x(t)$ from its fitted value.

We then develop a new approach, originally proposed for fully developed turbulence [18–21], in order to describe the cascading process that determines how the fluctuations in the series evolve, as one passes from the coarse to the fine scales. For a fixed t, the fluctuations at scales s and λs are related through the cascading rule

$$Z_{\lambda s}(t) = W_\lambda Z_s(t), \qquad \forall\ s, \qquad \lambda > 0, \tag{1}$$

where $\ln(W_\lambda)$ is a random variable. Iterating Eq. (1) forces implicitly the random variable W_λ to follow a log infinitely divisible law [22]. One of the simplest candidates for such processes is [17] $Z_s(t) = \zeta_s(t)\exp[\omega_s(t)]$, where ζ_s and $\omega_s(t)$ are independent Gaussian variables with zero mean and variances σ_ζ^2 and σ_ω^2. The PDF of $Z_s(t)$ has fat tails that depend on the variance of ω_s and is expressed by [21]

$$P_s(Z_s) = \int F_s\left(\frac{Z_s}{\sigma}\right)\frac{1}{\sigma}G_s(\ln\sigma)d\ln\sigma, \tag{2}$$

where F_s and G_s are both Gaussian with zero mean and variances σ_s^2 and λ_s^2, respectively, e.g., $G_s(\ln\sigma) = 1/(\sqrt{2\pi}\lambda_s)\exp(-\ln^2\sigma/2\lambda_s^2)$. In this case, $P_s(Z_s)$ is expressed by Eq. (2) and converges to a Gaussian distribution as $\lambda_s^2 \to 0$. Although Eq. (2) is equivalent to that for a log-normal cascade model for fully developed turbulence [23,24], it also describes approximately the non-Gaussian PDFs observed in such phenomena and systems as the foreign exchange markets [17,24,25] and heartbeat interval fluctuations [17,18] (see also [26–28]).

To carry out a quantitative analysis of the seismic times series, we focus on the deviations of the detrended increments' PDF from a Gaussian distribution and the dependence of the correlations in the increments on the scale parameter s. Consider the time series for the $M = 7.1$ event over two distinct time intervals: (i) data set (I)—the background fluctuations far from the event's time—and (ii) data set (II)—close (<5 h) to the earthquake. To fit the increments' PDF to Eq. (2), we estimate the variance $\lambda_s^2(s)$, using the least-squares method, with the error bars estimated by the goodness of the fit method. Deviation of $\lambda_s^2(s)$ from zero is a possible indicator of non-Gaussian statistics. As shown in Fig. 1, we find an accurate parametrization of the PDFs by $\lambda_s^2(s)$ for both data sets. Moreover, the PDF of Z_s for data set (I) becomes essentially Gaussian as s increases to 800 ms, whereas it deviates from the Gaussian distribution for data set (II). The time scale $s = 800$ ms for λ_s^2 within a moving window was estimated by plotting λ_s^2 vs s for data set (I) and selecting s such that $\lambda_s \to 0$ (see also below).

The scale dependence of λ_s^2 is shown in Fig. 2. For data set (I) of the $M = 7.1$ earthquake, shown in Fig. 2(a), and times 200 ms $< s <$ 500 ms, we find $\lambda_s^2 \propto \log s$. For data set (II), the logarithmic regime extends to 300 ms $< s <$ 2000 ms. Figure 2(b) presents similar behavior for the $M = 6.1$ earthquake. Note that, for data set (II) of the $M = 6.1$ earthquake, there is a crossover time at which λ_s^2 changes from a $\sim \log(s)$ behavior to having a finite value $\simeq 0.3$.

The importance of the results shown in Fig. 2 is that they indicate that the increments' PDFs for $s > 2000$ ms and $s > 1500$ ms are almost Gaussian ($\lambda_s^2 \to 0$) for the $M = 7.1$ and $M = 6.1$ earthquakes [for data set (II)], respectively. Transforming the time scales to length scales via the velocity of the elastic waves in Earth, ~5000 m/sec, the corresponding length scales are about 10 and 7.5 km, for the same earthquakes, respectively, implying that larger earthquakes have larger characteristic length scales and that for the $M = 6.1$ event the active part in the fault is smaller. As one moves down the cascade process from the large to the small scales, one expects the statistics to

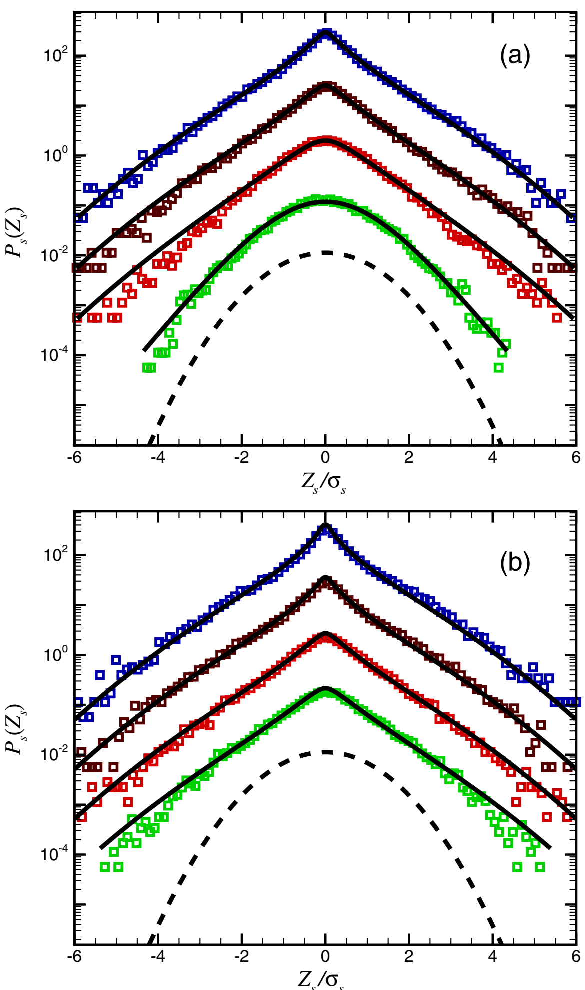

FIG. 1 (color online). Continuous deformation of the increments' PDFs for the $M = 7.1$ earthquake for, from top to bottom, $s = 200$, 400, 600, and 800 ms and (a) far from and (b) close to the earthquake. Solid curves are the PDFs based on Eq. (2), while dashed curves are the Gaussian PDF.

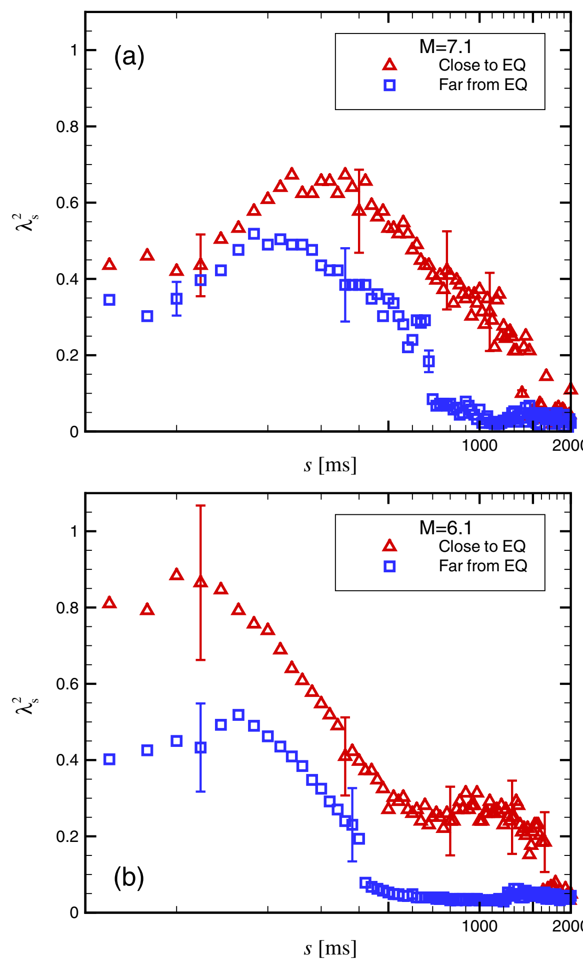

FIG. 2 (color online). Scale dependence of λ_s^2 vs $\log s$. (a) The $M = 7.1$ event, far from [data set (I)] and close to [data set (II)] the earthquake. For data set (I) and $s > 700$ ms, $\lambda_s^2 \to 0$, implying that the increments' PDF is Gaussian, but, for data set (II), λ_s^2 deviates strongly from 0 for 700 ms $< s <$ 1500 ms. (b) The same as in (a), but for the $M = 6.1$ event. When $\lambda_s \to 0$ the error bars are about the same size as the symbols.

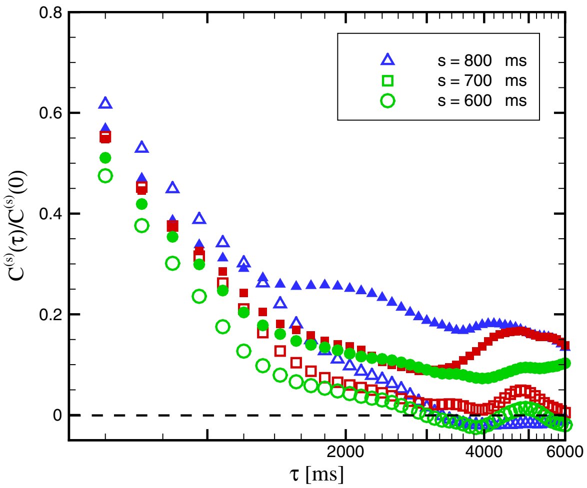

FIG. 3 (color online). The correlation function $C^{(s)}(\tau)$ for the data; bold symbols, data set (I) far from, and open symbols, set (II) close to, the $M = 7.1$ event.

increasingly deviate from Gaussianity, in order to arrive at Eq. (2). Note that a non-Gaussian PDF with fat tails on small scales indicates an increased probability of occurrence of short-time *extreme* seismic fluctuations.

From the point of view of the increments' PDF, the non-Gaussian noise with uncorrelated ω_s in the process $Z_s(t) = \zeta_s(t)\exp[\omega_s(t)]$ and a multifractal formulation are indistinguishable, because their one-point statistics at any given scale may be identical. To gain a deeper understanding of the non-Gaussian fluctuations, we explore the correlation properties of ω_s, by using an alternative method for studying the correlation functions of the local fluctuations [20]. We define the magnitude of local variance over a scale s by $\sigma_s^2(t) = n_s^{-1}\sum_{k=-n_s/2}^{n_s/2} Z_s(t + k\Delta t)^2$ and $\bar{\omega}_s(i) = \frac{1}{2} \times \log\sigma_s^2(i)$, respectively. Here Δt is the sampling interval and $n_s \equiv s/\Delta t$. The magnitude of the correlation function of $\bar{\omega}_s$ is then defined by

$$C^{(s)}(\tau) = \langle[\bar{\omega}_s(t) - \langle\bar{\omega}_s\rangle][\bar{\omega}_s(t + \tau) - \langle\bar{\omega}_s\rangle]\rangle, \quad (3)$$

where $\langle\cdot\rangle$ indicates a statistical average. Figure 3 shows the results for the two data sets. The correlation function decays sharply for data set (I)—far from the earthquakes—whereas it is of long-range type for set (II) close to the earthquakes for which the PDF deviates from being Gaussian even for $s > 800$ ms. Although one might argue that the deviations might be due to an underlying Lévy statistics, this possibility is ruled out due to the deduced hierarchical structures that imply that the increments for different scales are *not* independent; see Fig. 3.

The above analysis provides a new precursor for detecting an impending earthquake. A window containing 1 h of data is selected and moved with $\Delta t = 15$ min to determine the temporal dependence of λ_s^2. Guided by Fig. 2, the local temporal variations of λ_s^2 for $s = 800$ ms are investigated. According to Fig. 2, for $s \simeq 800$ ms, the difference between the values of λ^2 is large enough for the background data and the data set near the earthquakes. Hence, such a time scale may be used as the characteristic time for the dynamics of the non-Gaussian indicator λ_s^2. Figures 4(a) and 4(b) display well-pronounced, systematic increases in λ_s^2 as the earthquakes are approached. Taking into account the estimated error of λ_s^2 for the background fluctuations in Fig. 4, we see that about 7 and 5 h before the earthquakes values of λ_s^2 are larger, by more than 2 standard deviations, than those for the background.

Defining λ_s entails assuming a log-normal PDF for the increments, which may induce errors in estimating λ. An unbiased quantity for estimating the deviation from a Gaussian PDF is flatness, which needs no specific PDF functional form. In Figs. 4(c) and 4(d), we present the flatness of the time series in the same windows (see above) for the time scale $s \simeq 800$ ms. Consistent with λ_s^2, the flatness also yields a clear alert for an impending earthquake.

Because of the localization of elastic waves in Earth [13], stations that are far from an earthquake epicenter cannot provide any clue to the occurrence of the earthquakes. We checked this for several earthquakes. Shown in Fig. 5 are the results for the $M = 5.4$ California earth-

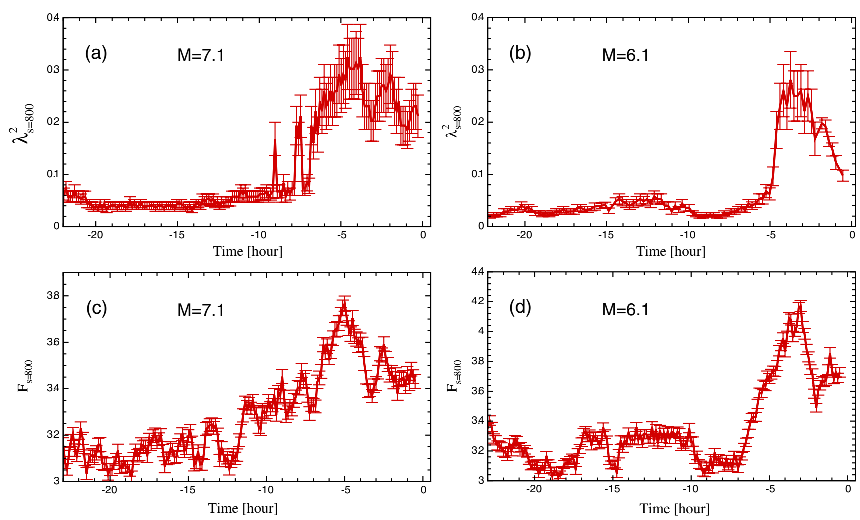

FIG. 4 (color online). The local temporal dependence of λ_s^2 and the flatness for $s = 800$ ms, over a one-hour period, for the $M = 7.1$ and $M = 6.1$ events, indicating a gradual, systematic increase on approaching the earthquakes.

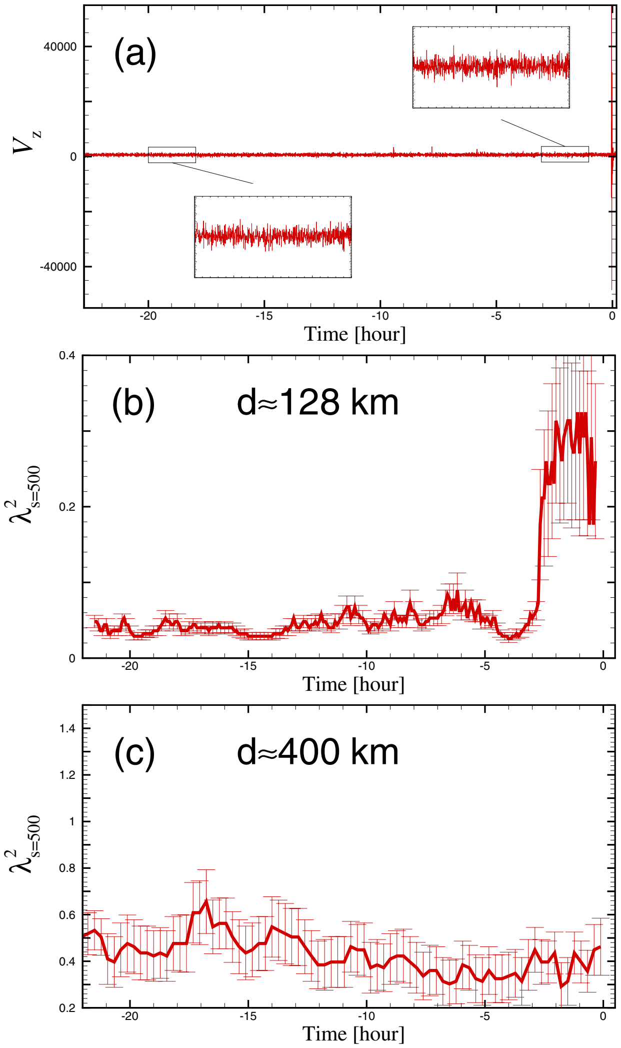

FIG. 5 (color online). (a) The data for the $M = 5.4$ earthquake in California. (b) and (c) show the local temporal dependence of λ_s^2 for $s = 500$ ms, collected at stations with a distance d from the epicenter. The station at $d = 128$ km provides the alert, whereas the other station does not.

quake, occurred at (40.837 N, 123.499 W). The station at a distance $d \simeq 128$ km from the epicenter does provide an alert of about 3 h for the earthquake, whereas that at $d \simeq 400$ km does not.

We also analyzed several other earthquakes of various magnitudes. As expected, for events with $M \leq 5$ the increase in λ_s^2 is not large, even if the data are collected in stations as close as 100 km from the epicenters. When the data for large earthquakes in Pakistan and Iran were analyzed, they exhibited the same types of trends as those presented above. For example, for the $M = 7.6$ earthquake that occurred on August 10, 2005, in Pakistan, the transition in the value of λ_s^2 occurred about 10 h before the event, and, for the $M = 6.3$ earthquake that occurred in northern Iran on May 28, 2004, it happened about 4 h before the earthquake.

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[1] *Modelling Critical and Catastrophic Phenomena in Geoscience: A Statistical Physics Approach*, edited by P. Bhattacharyya and B. K. Chakrabarti, Lect. Notes Phys. Vol. 705 (Springer, Berlin, 2006).
[2] B. Gutenberg and R. F. Richter, *Seismicity of the Earth* (Hafner Publishing, New York, 1965).
[3] T. Utsu *et al.*, J. Phys. Earth **43**, 1 (1995).
[4] P. Bak and C. Tang, J. Geophys. Res. **94**, 15 635 (1989).
[5] M. Sahimi *et al.*, Phys. Rev. Lett. **70**, 2186 (1993); M. Sahimi and S. Arbabi, *ibid.* **77**, 3689 (1996); H. Nakanishi *et al.*, J. Phys. I **3**, 733 (1993); M. C. Robertson *et al.*, J. Geophys. Res. B **100**, 609 (1995).
[6] E. G. Altmann *et al.*, Phys. Rev. E **75**, 016706 (2007); S. Hallerberg and H. Kantz, *ibid.* **77**, 011108 (2008).
[7] D. D. Jackson, Proc. Natl. Acad. Sci. U.S.A. **93**, 3772 (1996).
[8] M. R. Rahimi Tabar *et al.*, in Ref. [1], p. 281.
[9] F. Mormann *et al.*, Clin. Neurophysiol. **116**, 569 (2005).
[10] A. Johansen and D. Sornette, Eur. Phys. J. B **1**, 141 (1998).
[11] N. Vandewalle *et al.*, Eur. Phys. J. B **4**, 139 (1998).
[12] D. Sornette, Proc. Natl. Acad. Sci. U.S.A. **99**, 2522 (2002).
[13] E. Larose *et al.*, Phys. Rev. Lett. **93**, 048501 (2004); N. M. Shapiro *et al.*, Science **307**, 1615 (2005).
[14] F. Shahbazi, A. Bahraminasab, S. M. Vaez Allaei, M. Sahimi, and M. R. Rahimi Tabar, Phys. Rev. Lett. **94**, 165505 (2005); S. M. Vaez Allaei and M. Sahimi, *ibid.* **96**, 075507 (2006); A. Bahraminasab *et al.*, Phys. Rev. B **75**, 064301 (2007); R. Sepehrinia *et al.*, *ibid.* **77**, 014203 (2008); S. M. Vaez Allaei *et al.*, J. Stat. Mech. (2008) P03016.
[15] K. Kiyono *et al.*, Phys. Rev. Lett. **93**, 178103 (2004).
[16] K. Kiyono *et al.*, Phys. Rev. Lett. **95**, 058101 (2005).
[17] K. Kiyono *et al.*, Phys. Rev. Lett. **96**, 068701 (2006); Phys. Rev. E **76**, 041113 (2007); G. R. Jafari *et al.*, Int. J. Mod. Phys. C **18**, 1689 (2007).
[18] U. Frisch and D. Sornette, J. Phys. I (France) **7**, 1155 (1997).
[19] J. F. Muzy *et al.*, Eur. Phys. J. B **17**, 537 (2000).
[20] A. Arneodo *et al.*, Phys. Rev. Lett. **80**, 708 (1998).
[21] B. Castaing *et al.*, Physica (Amsterdam) **46D**, 177 (1990).
[22] B. Dubrulle, Phys. Rev. Lett. **73**, 959 (1994); Z.-S. She and E. C. Waymire, *ibid.* **74**, 262 (1995).
[23] B. Chabaud *et al.*, Phys. Rev. Lett. **73**, 3227 (1994).
[24] H. E. Stanley and V. Plerou, Quantitative Finance **1**, 563 (2001).
[25] S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner, and Y. Dodge, Nature (London) **381**, 767 (1996).
[26] P. A. Varotsos *et al.*, Phys. Rev. E **72**, 041103 (2005).
[27] M. De Menech and A. L. Stella, Physica (Amsterdam) **309A**, 289 (2002).
[28] F. Caruso *et al.*, Phys. Rev. E **75**, 055101(R) (2007).