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Level crossing analysis of the stock markets

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Abstract. We investigate the average frequency of positive slope ν_α^+ crossing for the returns of market prices. The method is based on stochastic processes in which no scaling feature is explicitly required. Using this method we define a new quantity to quantify the stage of development and activity of stock exchanges. We compare the Tehran and western stock markets and show that some, such as the Tehran (TEPIX) and New Zealand (NZX) stock exchanges, are emerging, and also that TEPIX is a non-active market and is financially motivated to absorb capital.

Keywords: stochastic processes

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Contents

1. Introduction

In recent years, financial markets have been at the focus of physicists' attempts to apply existing knowledge from statistical mechanics to economic problems [1]–[10]. Statistical properties of price fluctuations are very important in understanding and modelling financial market dynamics, which has long been a focus of economic research. These markets, though largely varying in details of trading rules and traded goods, are characterized by some generic features of their financial time series. The aim is to characterize the statistical properties of a given time series with the hope that a better understanding of the underlying stochastic dynamics could provide useful information to create new models able to reproduce experimental facts. An important aspect concerns the ability to define the concepts of activity and the degree of development of the markets. Acting on advantageous information moves the price such that the *a priori* gain is decreased or even destroyed by the feedback of the action on the price. This makes concrete the concept that prices are made random by the intelligent and informed actions of investors, as put forward by Bachelier, Samuelson, and many others [4]. In contrast, without informed traders, the profit opportunity remains, since the buying price is unchanged. Based on recent research for characterizing the stage of development of markets [11]–[13], it is well known that the Hurst exponent shows remarkable differences between developed and emerging markets.

Here we introduce a 'level crossing' to analyse these time series. It is based on stochastic processes that grasp the scale dependence of the time series [14]–[20], and no scaling feature is explicitly required. Also, this approach has turned out to be a promising tool for other systems with scale dependent complexity (see [15, 21] for its application to characterize the roughness of growing surfaces). Some authors have applied this method to study the fluctuations of velocity fields in Burgers turbulence [22], and in the Kardar–Parisi–Zhang equation in $d+1$-dimensions [23].

The level crossing analysis is very sensitive to correlation when the time series is shuffled and to probability density functions (PDFs) with fat tails when the time series is surrogated. The long-range correlations are destroyed by the shuffling procedure and in the surrogate method the phase of the discrete Fourier transform coefficients of the

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time series are replaced with a set of pseudo-independent distributed uniform $(-\pi, +\pi)$ quantities. The correlations in the surrogate series do not change, but the probability function changes to a Gaussian distribution [24]–[27].

The level crossing with detecting correlation is a useful tool to find the stage of development of markets, too. It is known that emerging markets have long-range correlation. This sensitivity of the level crossing to the market conditions provides a new and simple way of empirically characterizing the development of financial markets. This means that for mature markets the total number of level crossings is decreased under shuffling effectively, while the number in emerging markets is increased in agreement with the findings of Di Matteo *et al* [12, 13] which indicate that emerging markets have $H > 0.5$, while mature markets have $H \leq 0.5$; where H is the Hurst exponent. The level crossing analysis is a more simple calculation than other methods such as detrended fluctuation analysis (DFA) [28]–[34], detrended moving average (DMA) [35], wavelet transform modulus maxima (WTMM) [36], rescaled range analysis (R/S) [37, 38], and scaled windowed variance (SWV) [38]. It is well known that R/S, SWV and other non-detrending methods work well if the records are long and do not involve trends. Also in the detrending method one must pay attention to the fact that, in some cases, there exist one or more crossover (time) scales separating regimes with different scaling exponents [30, 31, 34]. In this case investigation of the scaling behaviour is more complicated and different scaling exponents are required for different parts of the series [32]. Therefore one needs a multitude of scaling exponents (multifractality) for a full description of the scaling behaviour. Crossover usually can arise either because of changes in the correlation properties of the signal at different time (space) scales, or it can often arise from trends in the data. The level crossing analysis does not require a modulus maxima procedure, in contrast with WTMM method, and hence does not require as much effort in writing computing code and computing time as the above methods. So the level crossing analysis is more suitable for short time series.

This paper is organized as follows. In section 2 we discuss the level crossing in detail. Data description and analysis based on this method for some stocks indices are given in section 3. Section 4 closes with a discussion of the present results.

2. Level crossing analysis

Let us consider a time series $\{p(t)\}$, of price index with length n, and the price returns $r(t)$ defined by $r(t) = \ln p(t+1) - \ln p(t)$. Here, we investigate the detrended log returns for different timescales.

For time interval T, let ν_α^+ denote the number of positive slope crossings $r(t) - \bar{r} = \alpha$ (see figure 1) and also let the mean value for all the samples be $N_\alpha^+(L)$, where

$$N_\alpha^+(T) = E[n_\alpha^+(T)]. \tag{1}$$

Since after detrending $r(t)$ is stationary (i.e. the averaged variance saturates to a certain value), if we take a second consecutive time interval T we obtain the same result, and therefore for the two intervals together we have

$$N_\alpha^+(2T) = 2N_\alpha^+(T), \tag{2}$$

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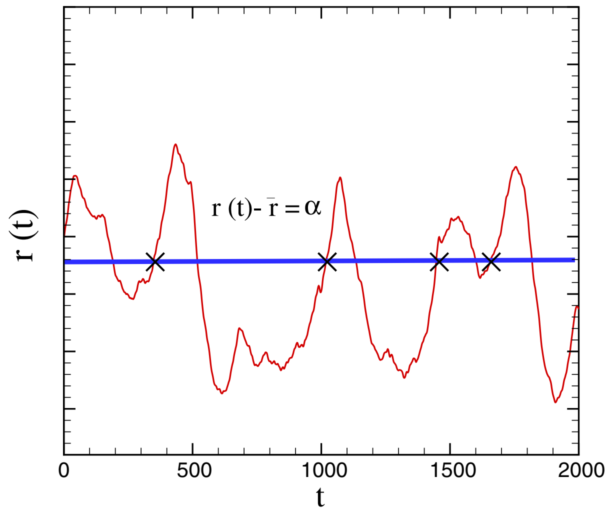

Figure 1. Positive slope crossing in a fixed α level.

from which it follows that, for stationary process, the average number of crossings is proportional to the space interval T. Hence

$$N_\alpha^+(T) \propto T, \tag{3}$$

or

$$N_\alpha^+(T) = \nu_\alpha^+ T \tag{4}$$

where ν_α^+ is the average frequency of positive slope crossing of the level $r(t) - \bar{r} = \alpha$. We show how the frequency parameter ν_α^+ can be deduced from the underlying probability distribution function (PDF) for $r(t)-\bar{r}$. In the time interval Δt the sample can only cross $r(t) - \bar{r} = \alpha$ with positive difference if it has the property $r(t) - \bar{r} < \alpha$ at the beginning of this time interval. Furthermore, there is a minimum difference at time t if the level $r(t) - \bar{r} = \alpha$ is to be crossed in interval Δt depending on the value of $r(t) - \bar{r}$ at time t. So there will be a positive crossing of $r(t) - \bar{r} = \alpha$ in the next time interval Δt if, at time t,

$$r(t) - \bar{r} < \alpha \qquad \text{and} \qquad \frac{\Delta\left[r(t) - \bar{r}\right]}{\Delta t} > \frac{\alpha - \left[r(t) - \bar{r}\right]}{\Delta t}. \tag{5}$$

As shown in [15], the frequency ν_α^+ can be written in terms of a joint PDF of $p(\alpha, y')$ as follows:

$$\nu_\alpha^+ = \int_0^\infty p(\alpha, y') y' \,\mathrm{d}y', \tag{6}$$

where $y' = (r(t + \Delta t) - r(t))/\Delta t$. Here we put $\Delta t = 1$.

Let us also introduce the quantity $N_{\mathrm{tot}}^+(q)$ as

$$N_{\mathrm{tot}}^+(q) = \int_{-\infty}^{+\infty} \nu_\alpha^+ |\alpha - \bar{\alpha}|^q \,\mathrm{d}\alpha \tag{7}$$

where zero moment (with respect to ν_α^+) $q = 0$ shows the total number of crossings for a return price with positive slope. The moments $q < 1$ will give information about the frequent events while moments $q > 1$ are sensitive for the tail of events.

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3. Application on stock market

Investments in the stock market are based on a quite straightforward rule: if you expect the market to go up in the future, you should buy (this is referred to as being long in the market) and hold the stock until you expect the trend to change direction; if you expect the market to go down, you should stay out of it, sell if you can (this is referred to as being short of the market) by borrowing a stock and giving it back later by buying it at a cheaper price in the future. It is difficult, to say the least, to predict future directions of stock market prices even if we are considering timescales of the order of decades, for which one could hope for a negligible influence of noise.

The reason why in very liquid markets of equities and currency exchanges correlations of returns are extremely small is because any significant correlation would lead to an arbitrage opportunity that is rapidly exploited and thus washed out. Indeed, the fact that there are almost no correlations between price variations in liquid markets can be understood from simple calculation by [4, 39]. In other words, liquidity and efficiency of markets control the degree of correlation, that is compatible with a near absence of arbitrage opportunity. It is important to consider that the more intelligent and hard working the investors, the more random is the sequence of price changes generated by such a market.

Acting on advantageous information moves the price such that the *a priori* gain is decreased or even destroyed by the feedback of the action on the price. This makes concrete the concept that prices are made random by the intelligent and informed actions of investors, as put forward by Bachelier, Samuelson, and many others [4]. In contrast, without informed traders, the profit opportunity remains, since the buying price is unchanged. Grossman and Stiglitz [40] argued that, perfectly informationally, efficient markets are an impossibility, for if markets are perfectly efficient, the return on gathering information is nil, in which case there would be little reason to trade and markets would eventually collapse. Alternatively, the degree of market inefficiency determines the effort investors are willing to expend to gather and trade on information; hence a non-degenerate market equilibrium will arise only when there are sufficient profit opportunities, that is, inefficiencies, to compensate investors for the costs of trading and information-gathering. The profits earned by these industrious investors may be viewed as economic rents that accrue to those willing to engage in such activities. Who are the providers of these rents? Noise traders, individuals who trade on what they think is information but is in fact merely noise. More generally, at any time there are always investors who trade for reasons other than information (for example, those with unexpected liquidity needs), and these investors are willing to pay for the privilege of executing their trades immediately.

For these purposes, we have analysed the level crossings of detrended log return time series, $r(t)$ for S&P500, Djindu, Biojen, 10ytsy, Composite, Amex, TEPIX and NZX. To have a good comparison, we have chosen the time series over the same time interval: 20 May 1994 to 18 March 2004, and data have been recorded each trading day. Consider a price time series with length n. Here, we investigate the detrended log returns on different timescales. To remove the trends present in the timescales $r(t)$ in each subinterval of length s, we fit $r(t)$ using a linear function, which represents the exponential trend of the original index in the corresponding time window. After this detrending procedure, we define detrended log returns, $r(t)$ is a deviation from the fitting function [41, 42].

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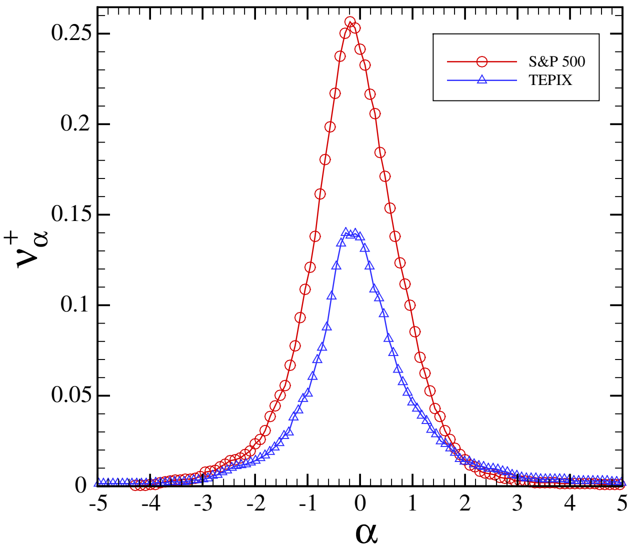

Figure 2. The positive difference crossing of return price for S&P500 and TEPIX markets in the same time interval.

According to equation (6), the level crossing, ν^+_α, is calculated for each index. Figure 2 shows a comparison of ν^+_α for TEPIX and S&P500 as a function of level α. It is clear that ν^+_α scales inversely with time, so $\tau(\alpha) = 1/\nu^+_\alpha$ is a time interval; within this time, the level crossing in average will be observed again. Table 1 shows the time interval in the high-frequency ($\tau(\alpha = 0)$) and the low-frequency (tails, $\tau(\alpha = \pm 3\sigma)$) regimes for some indices. The time interval $\tau(\alpha = 0)$ for TEPIX and S&P500 is 7.0 and 4.0 days, respectively. Still, the tails are comparable. Another difference between TEPIX and the other markets (except the Amex market) is seen in the time interval of the left ($\tau(\alpha = -3\sigma)$) and right ($\tau(\alpha = +3\sigma)$) tails. The time length in the left tail is larger than the time length in the right but also less than in other markets, and also in TEPIX and NZX the mean $\overline{\alpha}$ is 0.24 and 0.17, respectively. This means that TEPIX is financially motivated to absorb capital. It is clear that when we apply equation (7) for a small q regime, high frequency is more significant, whereas in the large q regime, low frequency (the tail) is more significant. Figure 3 shows that when $q < 2$, the value of $N^+_{\rm tot}$ for TEPIX is smaller than that of S&P500, while for $q > 2$, the value of $N^+_{\rm tot}$ for TEPIX becomes larger than that for the other markets. This is because for small q the low-frequency events of tails are more significant than the high-frequency peak. According to the last section and equation (7), the area under the level crossing curve, $N^+_{\rm tot}(q = 0)$, shows the total number of crossings. This means that the larger the area, the larger the activity. In essence, by comparing $N^+_{\rm tot}(q = 0)$, the activity of the index is obtained.

From another point of view, based on recent research for characterizing the stage of development of markets [11]–[13], it is shown that the Hurst exponent is sensitive to the degree of development of the market. Emerging markets are associated with a high value of the Hurst exponent and developed markets are associated with a low value of the exponent. In particular, it is found that all emerging markets have Hurst exponents larger than 0.5 (strongly correlated) whereas all the developed markets have Hurst exponents near to or less than 0.5 (white noise or anti-correlated). Here we have shown that the

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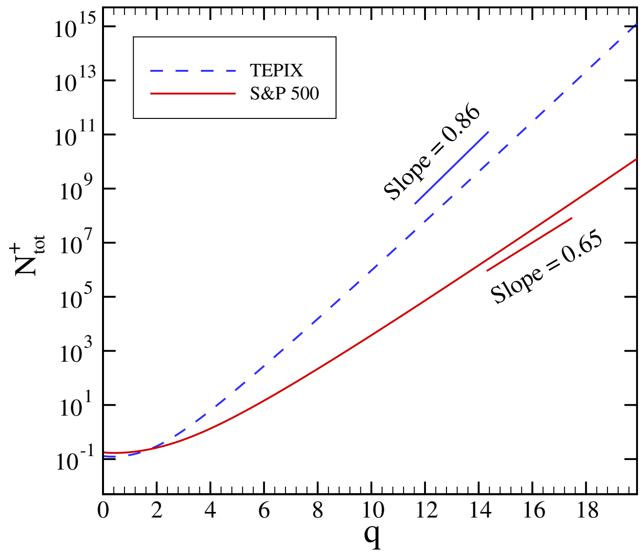

Figure 3. Generalized total number of crossings with positive slope $N^+_{\rm tot}$ for TEPIX and S&P 500 markets.

Table 1. The values of $\tau(\alpha)$ for different level, α.

Market	$\tau(\alpha = 0)$	$\tau(\alpha = 3\sigma)$	$\tau(\alpha = -3\sigma)$
S&P 500	4.0	218.4	115.1
Djindu	4.0	178.3	150.0
Biogen	4.0	179.1	113.8
10ytsy	4.2	656.3	98.2
Composit	4.3	178.6	140.1
Amex	4.6	115.1	178.4
NZX	5.3	135.3	120.3
TEPIX	7.0	102.8	114.2

level crossing has the ability to characterize the degree of development of markets. The sensitivity of the level crossing to the market conditions provides a new and more simple way of empirically characterizing the activity and development of financial markets.

Since $N^+_{\rm tot}(q = 0)$ is very sensitive to correlation, it increases when the time series is shuffled so that the correlation disappears. Thus, by comparing the change between $N^+_{\rm tot}(q = 0)$ and $N^+_{\rm sh}(q = 0)$ (shuffled), the stage of development of markets can be determined. Figure 4 shows ν^+_α as a function of α for original and shuffled data in TEPIX and S&P500. The relative changes of $N^+_{\rm tot}(q = 0)$ for TEPIX and S&P500 are 0.41 and 0.02 respectively. For the sake of comparison between various stock markets, the two parameters $N^+_{\rm tot}$ and $N^+_{\rm sh}$, the relative variation of $N^+_{\rm tot}(q = 0)$ and also The Hurst exponent which is obtained by using the detrended fluctuation analysis (DFA) method [28], are reported in table 2. We notice that TEPIX and NZX belong to the emerging markets category; each is far from an efficient and developed market. These results are comparable to the results reported in [11] and show that the Tehran stock exchange belongs to the category of emerging financial markets. The level crossing analysis is a more simple

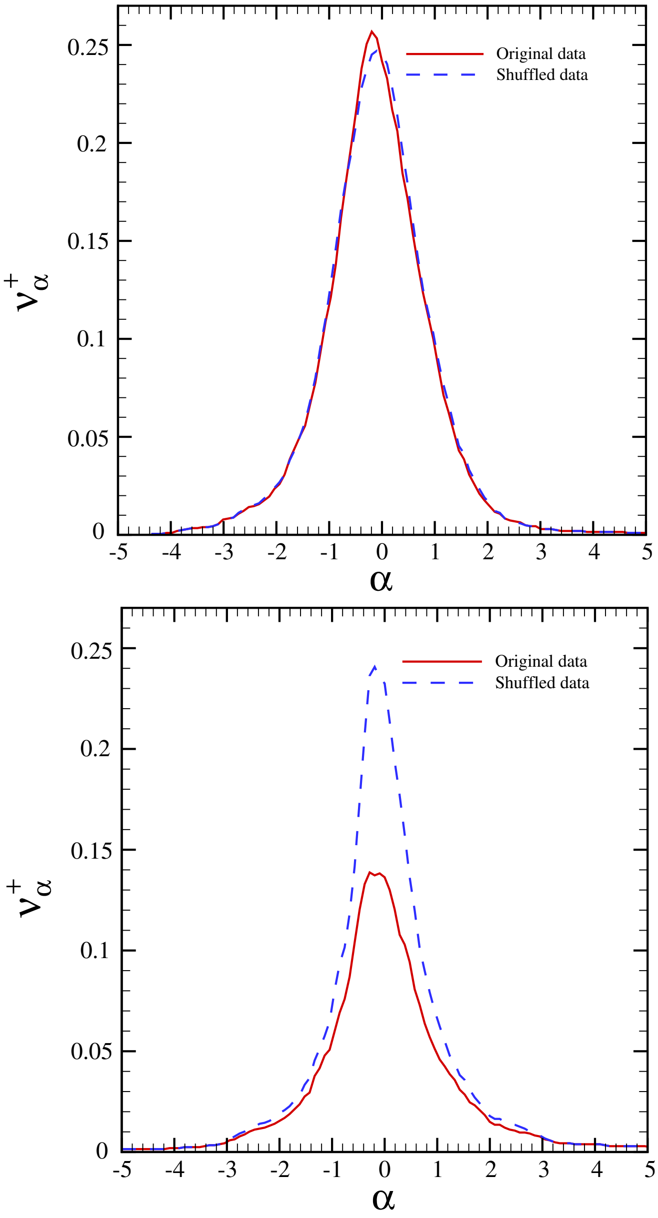

Figure 4. Comparison of the positive slope crossing of return price between original and shuffled data for the S&P 500 (upper panel) and TEPIX (lower panel) markets in the same time interval.

calculation than the other methods such as the generalized Hurst exponent approach, DFA, rescaled range analysis (R/S), and wavelet technique (WT). Also, in short time series these methods are not stable.

4. Conclusion

In this paper the concept of level crossing analysis has been applied to several stock market indices. It is shown that the level crossing is able to detect the activity of markets. This

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Table 2. The values of $N^{+}_{tot}(q=0)$, $N^{+}_{sh}(q=0)$ and Hurst exponent for some markets over the same period.

Market	N^{+}_{tot}	N^{+}_{sh}	$\vert N^{+}_{sh} - N^{+}_{tot}\vert / N^{+}_{tot}$	H
S&P 500	0.52	0.53	0.02	0.44 ± 0.01
Djindu	0.51	0.52	0.02	0.46 ± 0.01
10ytsy	0.50	0.52	0.04	0.47 ± 0.01
Biogen	0.48	0.51	0.06	0.51 ± 0.01
Composit	0.50	0.52	0.04	0.45 ± 0.01
Amex	0.45	0.50	0.10	0.51 ± 0.01
NZX	0.40	0.52	0.30	0.61 ± 0.01
TEPIX	0.32	0.45	0.41	0.74 ± 0.01

method is based on stochastic processes which should grasp the scale dependence of any time series in a most general way. No scaling feature is explicitly required. Based on the recent research for characterizing the stage of development of markets [11]–[13], it is shown that the level crossing is sensitive to the degree of development of the market, too. This sensitivity of the level crossing to market conditions provides a new and simple way of empirically characterizing the activity and development of financial markets. Considering all of the above discussions and results, we notice that Tehran Stock Exchange belongs to the emerging markets category. It is far from an efficient and developed market, and also we have found that it is financially motivated to absorb capital. Using this method we classify the activity and stage of development of some markets.

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