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# Feedback Systems

*An Introduction for Scientists and Engineers*

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## Preface

This book provides an introduction to the basic principles and tools for design and analysis of feedback systems. It is intended to serve a diverse audience of scientists and engineers who are interested in understanding and utilizing feedback in physical, biological, information and social systems. We have attempted to keep the mathematical prerequisites to a minimum while being careful not to sacrifice rigor in the process. We have also attempted to make use of examples from a variety of disciplines, illustrating the generality of many of the tools while at the same time showing how they can be applied in specific application domains.

This book was originally developed for use in an experimental course at Caltech involving students from a wide set of backgrounds. The course consisted of undergraduates at the junior and senior level in traditional engineering disciplines, as well as first and second year graduate students in engineering and science. This latter group included graduate students in biology, computer science and physics, requiring a broad approach that emphasized basic principles and did not focus on applications in any one given area. Over the course of several years, the text has been classroom tested at Caltech and at Lund University and the feedback from many students and colleagues has been incorporated to help improve the readability and accessibility of the material.

Because of its intended audience, this book is organized in a slightly unusual fashion compared to many other books on feedback and control. In particular, we introduce a number of concepts in the text that are normally reserved for second year courses on control and hence often not available to students who are not control systems majors. This has been done at the expense of certain traditional topics, which we felt that the astute student could learn independently and are often explored through the exercises. Examples of topics that we have included are nonlinear dynamics, Lyapunov stability, reachability and observability, and fundamental limits of performance and robustness. Topics that we have de-emphasized include root locus techniques, lead/lag compensation and detailed rules for generating Bode and Nyquist plots by hand.

Several features of the book are designed to facilitate its dual function as a basic engineering text and as an introduction for researchers in natural, information and social sciences. The bulk of the material is intended to be used regardless of the audience and covers the core principles and tools in the analysis and design of feedback systems. Advanced sections, marked by the “dangerous bend” symbol shown to the right, contain material that requires a slightly more technical background, of the sort that would be expected of senior undergraduates in engineering.



A few sections are marked by two dangerous bend symbols and are intended for readers with more specialized backgrounds, identified at the beginning of the section. To keep the length of the text down, several standard results and extensions are given in the exercises, with appropriate hints toward their solutions. Finally, we have included a glossary and a notation section at the end of the book in which we define some of the terminology and notation that may not be familiar to all readers.

To further augment the printed material contained here, a companion web site has been developed:

<http://www.cds.caltech.edu/~murray/amwiki>

The web site contains a database of frequently asked questions, supplemental examples and exercises, and lecture materials for courses based on this text. The material is organized by chapter and includes a summary of the major points in the text as well as links to external resources. The web site also contains the source code for many examples in the book, as well as utilities to implement the techniques described in the text. Most of the code was originally written using MATLAB M-files, but was also tested with LabVIEW MathScript to ensure compatibility with both packages. Many files can also be run using other scripting languages such as Octave, SciLab, SysQuake and Xmath. [Author's note: the web site is under construction as of this writing and some features described in the text may not yet be available.]

The first half of the book focuses almost exclusively on so-called “state-space” control systems. We begin in Chapter 2 with a description of modeling of physical, biological and information systems using ordinary differential equations and difference equations. Chapter 3 presents a number of examples in some detail, primarily as a reference for problems that will be used throughout the text. Following this, Chapter 4 looks at the dynamic behavior of models, including definitions of stability and more complicated nonlinear behavior. We provide advanced sections in this chapter on Lyapunov stability, because we find that it is useful in a broad array of applications (and is frequently a topic that is not introduced until later in ones studies).

The remaining three chapters of the first half of the book focus on linear systems, beginning with a description of input/output behavior in Chapter 5. In Chapter 6, we formally introduce feedback systems by demonstrating how state space control laws can be designed. This is followed in Chapter 7 by material on output feedback and estimators. Chapters 6 and 7 introduce the key concepts of reachability and observability, which give tremendous insight into the choice of actuators and sensors, whether for engineered or natural systems.

The second half of the book presents material that is often considered to be from the field of “classical control.” This includes the transfer function, introduced in Chapter 8, which is a fundamental tool for understanding feedback systems. Using transfer functions, one can begin to analyze the stability of feedback systems using frequency domain analysis, including the ability to reason about the closed



loop behavior of a system from its open loop characteristics. This is the subject of Chapter 9, which revolves around the Nyquist stability criterion.

In Chapters 10 and 11, we again look at the design problem, focusing first on proportional-integral-derivative (PID) controllers and then on the more general process of loop shaping. PID control is by far the most common design technique in control systems and a useful tool for any student. The chapter on frequency domain design introduces many of the ideas of modern control theory, including the sensitivity function. In Chapter 12, we pull together the results from the second half of the book to analyze some of the fundamental tradeoffs between robustness and performance. This is also a key chapter illustrating the power of the techniques that have been developed and serving as an introduction for more advanced studies.

The book is designed for use in a 10–15 week course in feedback systems that provides many of the key concepts needed in a variety of disciplines. For a 10 week course, Chapters 1–2, 4–6 and 8–11 can each be covered in a week’s time, with some dropping of topics from the final chapters. A more leisurely course, spread out over 14–15 weeks, could cover the entire book, with two weeks on modeling (Chapters 2 and 3)—particularly for students without much background in ordinary differential equations—and two weeks on robust performance (Chapter 12).

The mathematical pre-requisites for the book are modest and in keeping with our goal of providing an introduction that serves a broad audience. We assume familiarity with the basic tools of linear algebra, including matrices, vectors and eigenvalues. These are typically covered in a sophomore level course in the subject and the textbooks by Apostol [Apo69], Arnold [Arn87] or Strang [Str88] serve as good references. Similarly, we assume basic knowledge of differential equations, including the concepts of homogeneous and particular solutions for linear ordinary differential equations in one variable. Apostol [Apo69] or Boyce and DiPrima [BD04] cover this material well. Finally, we also make use of complex numbers and functions and, in some of the advanced sections, more detailed concepts in complex variables that are typically covered in a junior level engineering or physics course in mathematical methods. Apostol [Apo67] or Stewart [Ste02] can be used for the basic material, with Ahlfors [Ahl66], Marsden and Hoffman [MH99] or Saff and Snider [SS02] being good references for the more advanced material. We have chosen not to include appendices summarizing these various topics since there are a number of good books available and we believe that most readers will be familiar with material at this level.

One additional choice that we felt was important was the decision not to rely on knowledge of Laplace transforms in the book. While their use is by far the most common approach to teaching feedback systems in engineering, many students in the natural and information sciences may lack the necessary mathematical background. Since Laplace transforms are not required in any essential way, we have only included them in an advanced section intended to tie things together for students with that background. Of course, we make tremendous use of *transfer functions*, which we introduce through the notion of response to exponential

inputs, an approach we feel is more accessible to a broad array of scientists and engineers. For courses in which students have already had Laplace transforms, it should be quite natural to build on this background in the appropriate sections of the text.

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