EE 364a: Convex Optimization I — J. Duchi
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Name: ______________________ SUID: ______________________

Midterm Quiz

This is a 75 minute, closed notes, closed book midterm. Each question is worth 20 points. In questions 1–3, we will award 5 points per part for correct responses, and 2 points for parts left blank. For question 4, we'll award some credit to partially correct responses. No justification is required for any of these questions, however.

By taking this quiz you're agreeing to respect the honor code. Good luck!

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1. *Convexity of some sets.* Determine if each set below is convex.

   (a) $\left\{(x, y, z) \in \mathbf{R}^3 \mid \begin{bmatrix} x & y \\ y & z \end{bmatrix} \succeq 0\right\}$
   ☐ convex ☐ not convex

   (b) $\{(x, y, z) \in \mathbf{R}^3 \mid xz - y^2 \ge 0, x \ge 0, z \ge 0\}$
   ☐ convex ☐ not convex

   (c) $\{(x, y, z) \in \mathbf{R}^3 \mid xz - y^2 \ge 0\}$
   ☐ convex ☐ not convex

   (d) $\{(x, y, z) \in \mathbf{R}^3 \mid \frac{y^2}{z} \ge x, z < 0, x \le 0\}$
   ☐ convex ☐ not convex

2. *Curvature of some functions.* Determine the curvature of the functions below. For affine functions (which are both convex and concave), select only the 'affine' box. If a function is neither convex nor concave, select 'neither'.

   (a) $f(x) = \max\{2, x, 1/\sqrt{x}, x^3\}$, with $\mathbf{dom}\, f = \mathbf{R}_+$
   ☐ convex ☐ concave ☐ affine ☐ neither

   (b) $f(x, t) = \frac{\|x\|^{14}}{t^{13}}$ with $\mathbf{dom}\, f = \{x \in \mathbf{R}^n, t > 0\}$
   ☐ convex ☐ concave ☐ affine ☐ neither

   (c) $f(x) = (1/2)x^2 - (1/12)x^4$, with $\mathbf{dom}\, f = \mathbf{R}$
   ☐ convex ☐ concave ☐ affine ☐ neither

   (d) $f(x, y, z) = \log(y \log \frac{z}{y} - x) + \log(zy)$, with $\mathbf{dom}\, f = \{(x, y, z) \in \mathbf{R} \times \mathbf{R}^2_{++} \mid ye^{x/y} < z\}$
   ☐ convex ☐ concave ☐ affine ☐ neither

3. *Convexity of some sets of positive semidefinite matrices.* In each part of the question, n, k are fixed numbers with $k < n$. Determine if each set below is convex.

(a) $\{A \in \mathbf{S}_+^n \mid \mathbf{Rank}(A) \geq k\}$, where $k < n$.

☐ convex ☐ not convex

(b) $\{A \in \mathbf{S}_+^n \mid \mathbf{Rank}(A) \leq k\}$, where $k < n$.

☐ convex ☐ not convex

(c) $\{A \in \mathbf{S}_+^n \mid \mathbf{Rank}(A) = n\}$.

☐ convex ☐ not convex

(d) $\{C \in \mathbf{S}_{++}^n \mid A - B^T C^{-1} B \succeq 0\}$ where A, B are fixed matrices of appropriate size.

☐ convex ☐ not convex

4. *DCP rules.* The function

$$f(x) = \log\left(\exp\left(\frac{(a^T x)^2}{c^T x - d}\right) + \exp\left((c^T x - d)^{-1/2}\right)\right)$$

is convex in x over $\{x \in \mathbf{R}^n \mid c^T x - d > 0\}$. Express f using disciplined convex programming (DCP), limited to the following atoms:

`inv_pos(u)`, which is $1/u$, with domain $\mathbf{R}_{++}$

`square(u)`, which is u^2, with domain $\mathbf{R}$

`sqrt(u)`, which is $\sqrt{u}$, with domain $\mathbf{R}_+$

`geo_mean(u)`, which is $(\prod_{i=1}^n u_i)^{1/n}$, with domain $\mathbf{R}_+^n$

`quad_over_lin(u,v)`, which is u^2/v, with domain $\mathbf{R} \times \mathbf{R}_{++}$

`log_sum_exp(u)`, which is $\log(\sum_{i=1}^n \exp(u_i))$, with domain $\mathbf{R}^n$.

`log(u)`, which is $\log u$, with domain $\mathbf{R}_{++}$

`exp(u)`, which is e^u, with domain $\mathbf{R}$

You may also use addition, subtraction, scalar multiplication, and any constant functions. Assume that DCP is sign-sensitive, *e.g.*, `square(u)` increasing in u when $u \geq 0$. Please only write down your composition. *No justification is required.*