

# Takehome Exam

## Graph II

Start: 94-11-6 9:00 am

End: 94-11-8 13:00 pm

1. Given a graph  $G$  whose girth is greater than 10 provide an algorithm that can estimate the connectivity of  $G$  (i.e. given an extra input  $k$  the algorithm decides whether the graph is  $k$ -connected or not). Discuss the efficiency of your algorithm.
2. Does there exist a homomorphism from the Coxeter graph to the Petersen graph? (why?)
3. Construct a sequence of 3-regular graphs  $G_n$  such that  $\lim_{n \rightarrow \infty} |V(G_n)| = \infty$  and the second eigenvalue of the Laplacian is greater than 0.05 for all  $n$ .
4. Compute the Tutte polynomial of the  $n$  dimensional cube  $Q_n$ . (Any information about the Tutte polynomial has credits for this question.)
5. Provide a definition that can capture the concept of a *center* of a tree. Provide an algorithm that can find such a vertex for a given tree and discuss the efficiency of your algorithm.
6. What can you say about the chromatic number and the flow number of a graph  $G$  that does not have  $K_{4,4}$  as a minor?