Machine learning

Reinforcement Learning

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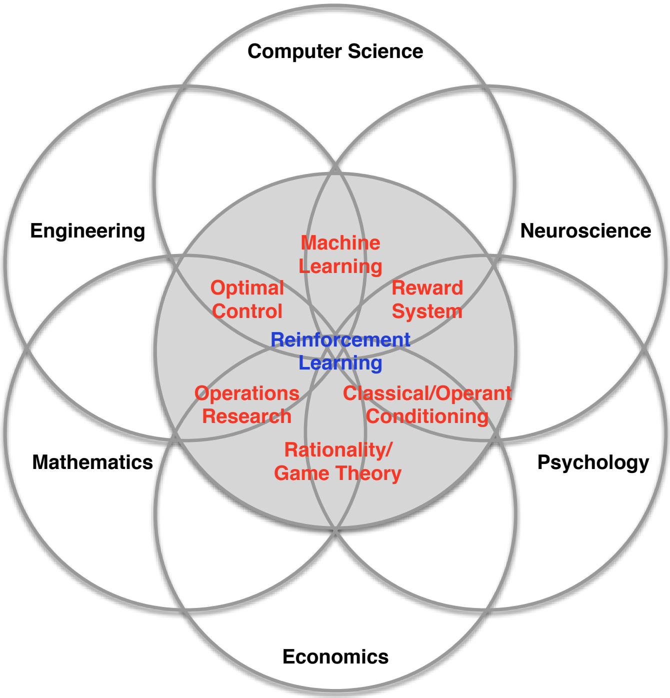
Introduction


Computer Science
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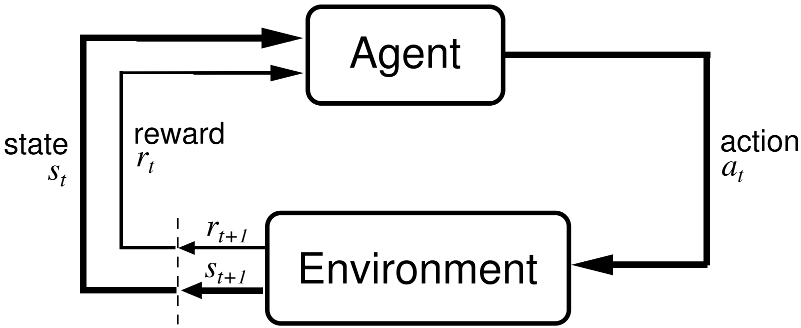
- Reinforcement learning is what to do (how to map situations to actions) so as to maximize a scalar reward/reinforcement signal
- The learner is not told which actions to take as in supervised learning, but discover which actions yield the most reward by trying them.
- The trial-and-error and delayed reward are the two most important feature of reinforcement learning.
- Reinforcement learning is defined not by characterizing learning algorithms, but by characterizing a learning problem.
- Any algorithm that is well suited for solving the given problem, we consider to be a reinforcement learning.
- One of the challenges that arises in reinforcement learning and other kinds of learning is tradeoff between exploration and exploitation.

- A key feature of reinforcement learning is that it explicitly considers the whole problem of a goal-directed agent interacting with an uncertain environment.



- Experience is a sequence of observations, actions, rewards.

$$o_1, r_1, a_1, \ldots, a_{t-1}, o_t, r_t$$

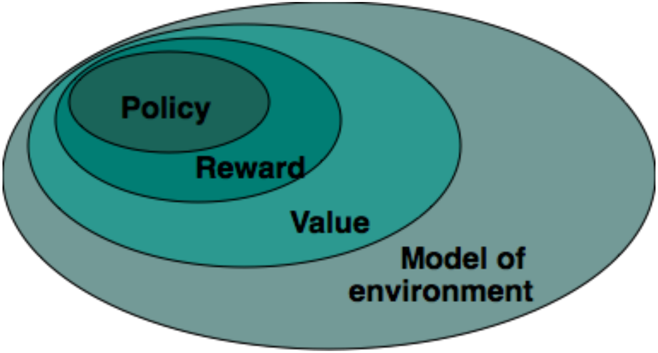
- The state is a summary of experience

$$s_t = f(o_1, r_1, a_1, \ldots, a_{t-1}, o_t, r_t)$$

- In a fully observed environment

$$s_t = f(o_t)$$

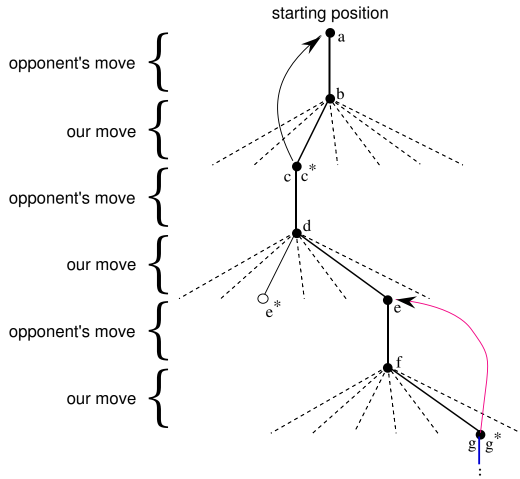
- Policy : A policy is a mapping from received states of the environment to actions to be taken (what to do?).
- Reward function: It defines the goal of RL problem. It maps each state-action pair to a single number called reinforcement signal, indicating the goodness of the action. (what is good?)
- Value : It specifies what is good in the long run. (what is good because it predicts reward?)
- Model of the environment (optional): This is something that mimics the behavior of the environment. (what follows what?)



An example : Tic-Tac-Toe

- Consider a two-playes game (Tic-Tac-Toe)

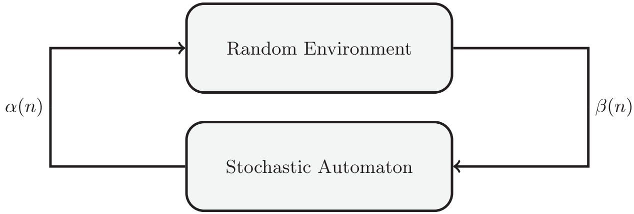
X	O	O
O	X	X
		X



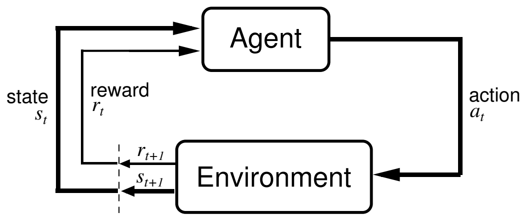
- Consider the following updating

$$V(s) \leftarrow V(s) + \alpha[V(s') - V(s)]$$

- Non-associative reinforcement learning : The learning method that does not involve learning to act in more than one state.



- Associative reinforcement learning : The learning method that involves learning to act in more than one state.



Non-associative reinforcement learning

- Consider that you are faced repeatedly with a choice among n different options or actions.
- After each choice, you receive a numerical reward chosen from a stationary probability distribution that depends on the action you selected.
- Your objective is to maximize the expected total reward over some time period.
- This is the original form of the $n-$armed bandit problem called a slot machine.

- Consider some simple methods for estimating the values of actions and then using the estimates to select actions.
- Let the true value of action a denoted as $Q^*(a)$ and its estimated value at t^{th} play as $Q_t(a)$.
- The true value of an action is the mean reward when that action is selected.
- One natural way to estimate this is by averaging the rewards actually received when the action was selected.
- In other words, if at the t^{th} play action a has been chosen k_a times prior to t, yielding rewards $r_1, r_2, \ldots, r_{k_a}$, then its value is estimated to be

$$Q_t(a) = \frac{r_1 + r_2 + \ldots + r_{k_a}}{k_a}$$

- Greedy action selection : This strategy selects the action with highest estimated action value.

$$a_t = \underset{a}{\text{argmax}}\ Q_t(a)$$

- $\epsilon-$greedy action selection : This strategy selects the action with highest estimated action value most of time but with small probability ϵ selects an action at random, uniformly, independently of the action-value estimates.
- Softmax action selection : This strategy selects actions using the action probabilities as a graded function of estimated value.

$$p_t(a) = \frac{\exp^{Q_t(a)/\tau}}{\sum_b \exp^{Q_t(b)/\tau}}$$

- Environment represented by a tuple $< \underline{\alpha}, \underline{\beta}, \underline{C} >$,
  1. $\underline{\alpha} = \{\alpha_1, \alpha_2, \ldots, \alpha_r\}$ shows a set of inputs,
  2. $\underline{\beta} = \{0, 1\}$ represents the set of values that the reinforcement signal can take,
  3. $\underline{C} = \{c_1, c_2, \ldots, c_r\}$ is the set of penalty probabilities, where $c_i = Prob[\beta(k) = 1|\alpha(k) = \alpha_i]$.
- A variable structure learning automaton is represented by triple $< \beta, \alpha, T >$,
  1. $\beta = \{0, 1\}$ is a set of inputs,
  2. $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_r\}$ is a set of actions,
  3. T is a learning algorithm used to modify action probability vector $\underline{p}$.

- In linear reward-ϵpenalty algorithm ($L_{R-\epsilon P}$) updating rule for p is defined as

$$p_j(k+1) = \begin{cases} p_j(k) + a \times [1 - p_j(k)] & \text{if} \quad i = j \\ p_j(k) - a \times p_j(k) & \text{if} \quad i \neq j \end{cases}$$

when $\beta(k) = 0$ and

$$p_j(k+1) = \begin{cases} p_j(k) \times (1 - b) & \text{if} \quad i = j \\ \frac{b}{r-1} + p_j(k)(1 - b) & \text{if} \quad i \neq j \end{cases}$$

when $\beta(k) = 1$.

- Parameters $0 < b \ll a < 1$ represent step lengths.
- When $a = b$, we call it linear reward penalty(L_{R-P}) algorithm.
- When $b = 0$, we call it linear reward inaction(L_{R-I}) algorithm.

- In stationary environments, average penalty received by automaton is

$$M(k) = E[\beta(k)|p(k)] = Prob[\beta(k) = 1|p(k)] = \sum_{i=1}^{r} c_i p_i(k).$$

- A learning automaton is called expedient if

$$\lim_{k\to\infty} E[M(k)] < M(0)$$

- A learning automaton is called optimal if

$$\lim_{k\to\infty} E[M(k)] = \min_i c_i$$

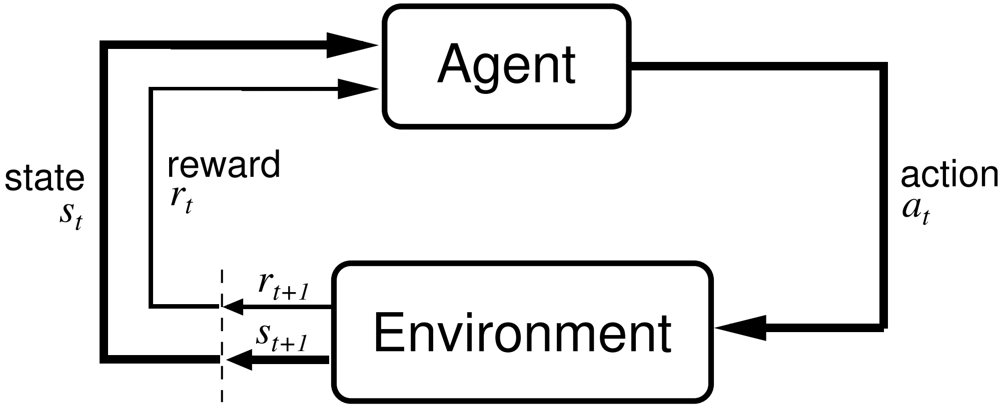
- A learning automaton is called $\epsilon-$optimal if

$$\lim_{k\to\infty} E[M(k)] < \min_i c_i + \epsilon$$

for arbitrary $\epsilon > 0$

Associative reinforcement learning

The learning method that involves learning to act in more than one state.



Goals,rewards, and returns

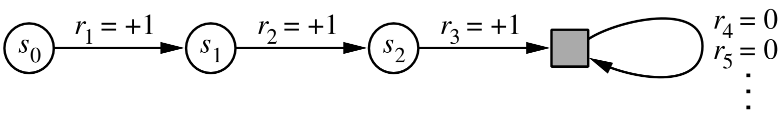
- In reinforcement learning, the goal of the agent is formalized in terms of a special reward signal passing from the environment to the agent.
- The agent's goal is to maximize the total amount of reward it receives. This means maximizing not immediate reward, but cumulative reward in the long run.
- How might the goal be formally defined?
- In episodic tasks the return, R_t, is defined as

$$R_t = r_1 + r_2 + \ldots + r_T$$

- In continuous tasks the return, R_t, is defined as

$$R_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}$$

- The unified approach

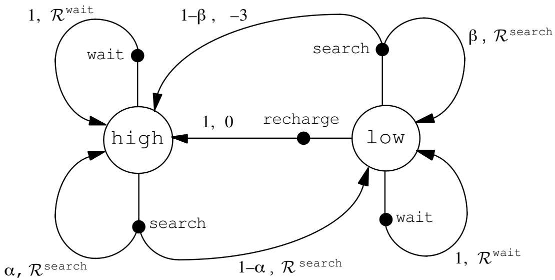


Markov decision process

- A RL task satisfing the Markov property is called a Markov decision process (MDP).
- If the state and action spaces are finite, then it is called a finite MDP.
- A particular finite MDP is defined by its state and action sets and by the one-step dynamics of the environment.

$$
\begin{aligned}
P^a_{ss'} &= Prob\{s_{t+1} = s' | s_t = s, a_t = a\} \\
\mathcal{R}^a_{ss'} &= E[r_{t+1} | s_t = s, a_t = a, s_{t+1} = s']
\end{aligned}
$$

- Recycling Robot MDP



- Let in state s action a is selected with probability of $\pi(s,a)$.
- Value of state s under a policy π is the expected return when starting in s and following π thereafter.

$$
\begin{aligned}
V^{\pi}(s) &= E_{\pi}\{R_t|s_t = s\} = E_{\pi}\left\{\sum_{k=0}^{\infty}\gamma^k r_{t+k+1}\middle| s_t = s\right\} \\
&= \sum_{\pi}\pi(s,a)\sum_{s'}P^a_{ss'}\left[\mathcal{R}^a_{ss'} + \gamma V^{\pi}(s')\right].
\end{aligned}
$$

- Value of action a in state s under a policy π is the expected return when starting in s taking action a and following π thereafter.

$$
Q^{\pi}(s,a) = E_{\pi}\{R_t|s_t = s, a_t = a\} = E_{\pi}\left\{\sum_{k=0}^{\infty}\gamma^k r_{t+k+1}\middle| s_t = s, a_t = a\right\}
$$

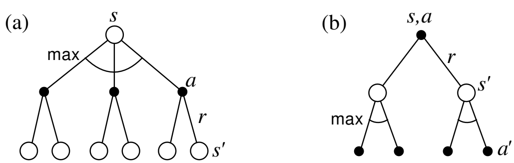
- Policy π is better than or equal of π' iff for all s $V^{\pi}(s) \geq V^{\pi'}(s)$.
- There is always at least one policy that is better than or equal to all other policies. This is an optimal policy.
- Value of state s under the optimal policy ($V^*(s)$) equals

$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

- Value of action a in state s under the optimal policy ($Q^*(s,a)$ equals

$$Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a)$$

- Backup diagram for V^* and Q^*



1. Model-based RL
    1.1 Build a model of the environment.
    1.2 Plan (e.g. by lookahead) using model.
2. Value-based RL
    2.1 Estimate the optimal value function $Q^*(s, a)$
    2.2 This is the maximum value achievable under any policy
3. Policy-based RL
    3.1 Search directly for the optimal policy π^*.
    3.2 This is the policy achieving maximum future reward.

Model based methods

- The key idea of DP is the use of value functions to organize and structure the search for good policies.
- We can easily obtain optimal policies once we have found the optimal value functions, or , which satisfy the Bellman optimality equations:

$$\begin{aligned} V^*(s) &= \max_a E\{r_{t+1} + \gamma V^*(s_{t+1}) | s_t = s, a_t = a\} \\ &= \max_a \sum_{s'} P^a_{ss'} \left[\mathcal{R}^a_{ss'} + \gamma V^*(s')\right]. \end{aligned}$$

- Value of action a in state s under a policy π is the expected return when starting in s taking action a and following π thereafter.

$$\begin{aligned} Q^*(s,a) &= E\{r_{t+1} + \gamma \max_{a'} Q^*(s_{t+1}, a') | s_t = s, a_t = a\} \\ &= \sum_{s'} P^a_{ss'} \left[\mathcal{R}^a_{ss'} + \gamma \max_{a'} Q^*(s', a')\right]. \end{aligned}$$

- Policy iteration is an iterative process

$$\pi_0 \xrightarrow{E} V^{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} V^{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} \ldots\ldots \xrightarrow{I} \pi^* \xrightarrow{E} V^*$$

- Policy iteration has two phases : policy evaluation and improvement.
- In policy evaluation, we compute state or state-action value functions

$$\begin{aligned} V^\pi(s) &= E_\pi\{R_t | s_t = s\} = E_\pi\left\{\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} \middle| s_t = s\right\} \\ &= \sum_\pi \pi(s,a) \sum_{s'} P^a_{ss'}\left[\mathcal{R}^a_{ss'} + \gamma V^\pi(s')\right]. \end{aligned}$$

- In policy improvement, we change the policy to obtain a better policy

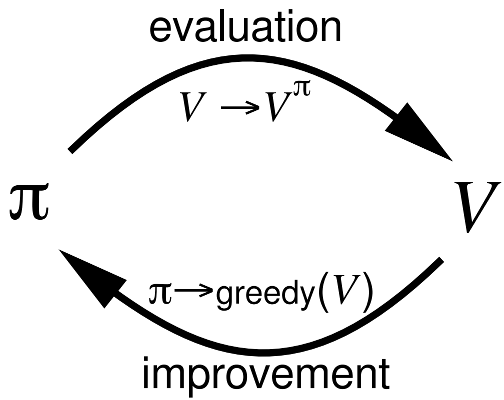
$$\begin{aligned} \pi'(s) &= \operatorname*{argmax}_a Q^\pi(s,a) \\ &= \operatorname*{argmax}_a \sum_{s'} P^a_{ss'}\left[\mathcal{R}^a_{ss'} + \gamma V^\pi(s')\right]. \end{aligned}$$

Value and generalized policy iteration

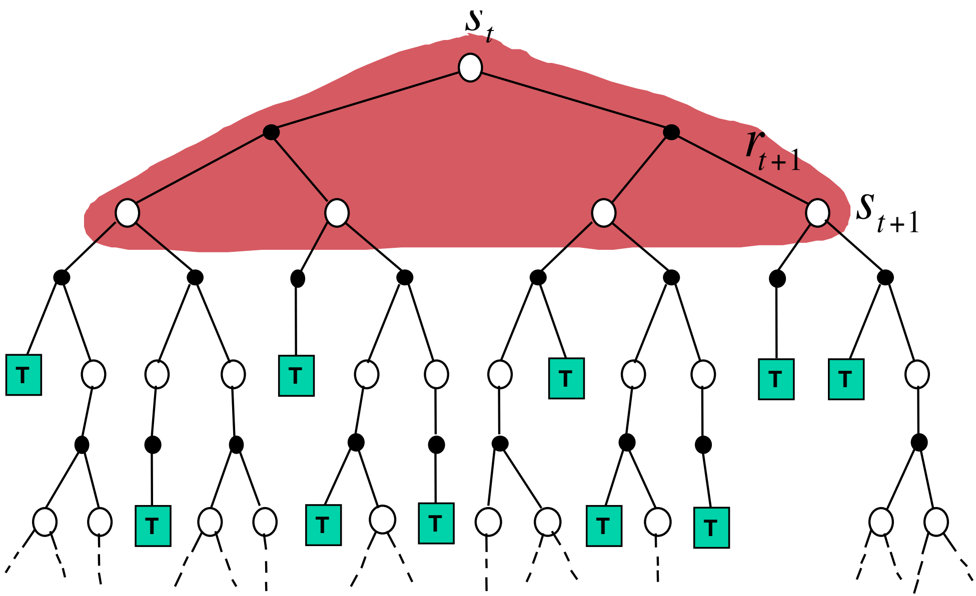
- In value iteration we have

$$\begin{aligned} V_{k+1}(s) &= \max_a E\{r_{t+1} + \gamma V_k(s_{t+1}) | s_t = s, a_t = a\} \\ &= \max_a \sum_{s'} P^a_{ss'} \left[\mathcal{R}^a_{ss'} + \gamma V_k^{(}s') \right]. \end{aligned}$$

- Generalized policy iteration



$$V(S_t) \leftarrow E_\pi[R_{t+1} + \gamma V(S_{t+1})]$$



Value-based methods

- These methods lean policy function implicitly.
- These methods first learn a value function $Q(s, a)$.
- Then infer policy $\pi(s, a)$ from $Q(s, a)$.
- Examples
  - Monte-carlo methods
  - Q-learning
  - SARSA
  - TD(λ)

Value-based methods

Monte Carlo methods

- MC methods learn directly from episodes of experience.
- MC is model-free: no knowledge of MDP transitions / rewards
- MC learns from complete episodes
- MC uses the simplest possible idea: value = mean return
- Goal: learn V_π from episodes of experience under policy π

$$S_1 \xrightarrow[R_1]{\alpha_1} S_2 \xrightarrow[R_2]{\alpha_2} S_3 \xrightarrow[R_3]{\alpha_3} S_4 \ldots \xrightarrow[R_{k-1}]{\alpha_{k-1}} S_k$$

- The return is the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$$

- The value function is the expected return:

$$V_\pi(s) = E_\pi[G_t | S_t = s]$$

- Monte-Carlo policy evaluation uses empirical mean return instead of expected return

First-Visit Monte-Carlo Policy Evaluation

- To evaluate state s
- The first time-step t that state s is visited in an episode, Increment counter

$$N(s) \leftarrow N(s) + 1$$

- Increment total return

$$S(s) \leftarrow S(s) + G_t$$

- Value is estimated by mean return

$$V(s) = \frac{S(s)}{N(s)}$$

- By law of large numbers,

$$V(s) \rightarrow v_\pi(s)$$

as

$$N(s) \rightarrow \infty$$

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- Every time-step t that state s is visited in an episode, Increment counter

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$$S(s) \leftarrow S(s) + G_t$$

- Value is estimated by mean return

$$V(s) = \frac{S(s)}{N(s)}$$

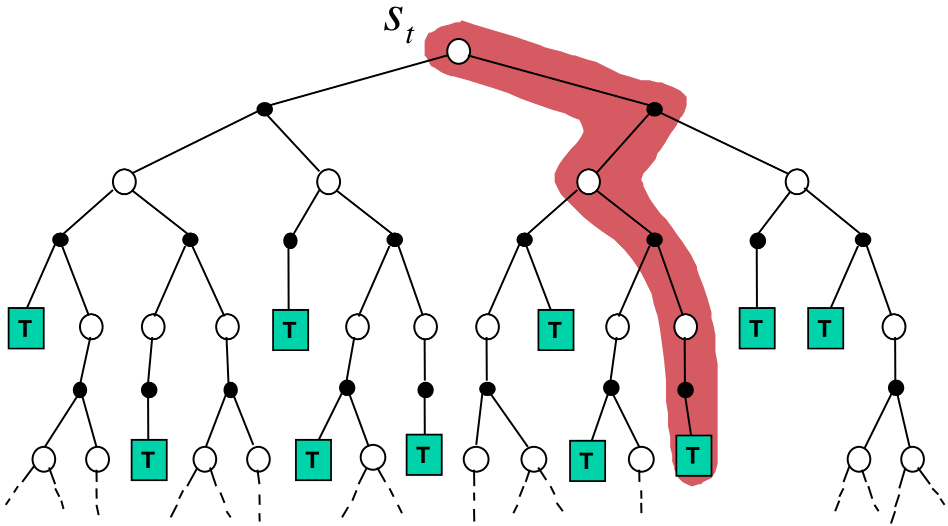
- By law of large numbers,

$$V(s) \rightarrow v_\pi(s)$$

as

$$N(s) \rightarrow \infty$$

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$



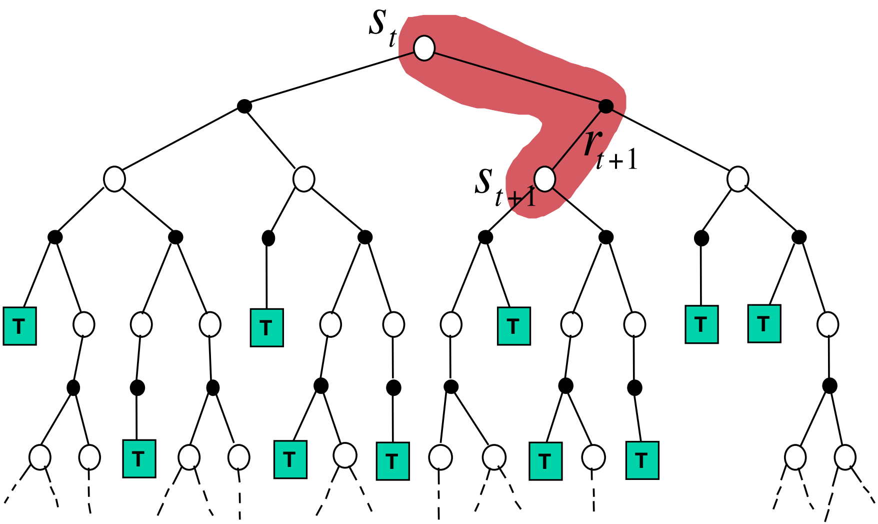
Value-based methods

Temporal-difference methods

- TD learning is a combination of Monte Carlo ideas and dynamic programming (DP) ideas.
- Like Monte Carlo methods, TD methods can learn directly from raw experience without a model of the environment's dynamics.
- Like DP, TD methods update estimates based in part on other learned estimates, without waiting for a final outcome (they bootstrap).
- Monte Carlo methods wait until the return following the visit is known, then use that return as a target for $V(s_t)$ while TD methods need wait only until the next time step.
- The simplest TD method, known as TD(0), is

$$V(s_t) \leftarrow V(s_t) + \alpha \left[r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \right]$$

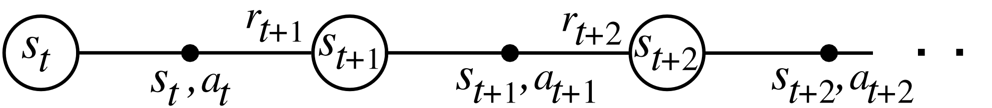
$$V(s_t) \leftarrow V(s_t) + \alpha \left[r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \right]$$



- Algorithm for TD(0)

Initialize $V(s)$ arbitrarily, π to the policy to be evaluated
Repeat (for each episode):
.   Initialize s
.   Repeat (for each step of episode):
.   . $a \leftarrow$ action given by π for s
.   .   Take action a; observe reward, r, and next state, s'
.   . $V(s) \leftarrow V(s) + \alpha \left[r + \gamma V(s') - V(s)\right]$
.   . $s \leftarrow s'$
.   until s is terminal

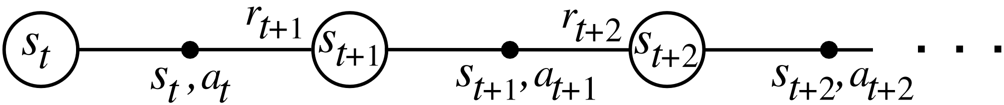
- An episode consists of an alternating sequence of states and state-action pairs:



- SARSA, which is an on policy, updates values using

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)\right]$$

- An episode consists of an alternating sequence of states and state-action pairs:



- Q-learning, which is an off policy, updates values using

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

Policy-based methods

- In policy-based learning, there is no value function.
- The policy $\pi(s, a)$ is parametrized by vector θ ($\pi(s, a; \theta)$).
- Explicitly learn policy $\pi(s, a; \theta)$ that implicitly maximize reward over all policies.
- Given policy $\pi(s, a; \theta)$ with parameters θ, find best θ.
- How do we measure the quality of a policy $\pi(s, a; \theta)$?
- Let objective function be $J(\theta)$.
- Find policy parameters θ that maximize $J(\theta)$.
- Sample algorithm: REINFORCE

- Advantages of policy-based methods over value-based methods
  - Usually, computing Q-values is harder than picking optimal actions
  - Better convergence properties
  - Effective in high dimensional or continuous action spaces
  - Can benefit from demonstrations
  - Policy subspace can be chosen according to the task
  - Exploration can be directly controlled
  - Can learn stochastic policies
- Disadvantages of policy-based methods over value-based methods
  - Typically converge to a local optimum rather than a global optimum
  - Evaluating a policy is typically data inefficient and high variance

Reading

Readings

1. Chapters 1-6 of Reinforcement Learning: An Introduction (Sutton and Barto 2018).

Sutton, Richard S. and Andrew G. Barto (2018). *Reinforcement Learning: An Introduction*. Second edition. The MIT Press.

Questions?