

Journal of Contaminant Hydrology 56 (2002) 175-192

JOURNAL OF Contaminant Hydrology

www.elsevier.com/locate/jconhyd

# Effects of heterogeneities on capillary pressure-saturation-relative permeability relationships

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Received 21 May 2001; received in revised form 21 September 2001; accepted 12 November 2001

#### Abstract

In theories of multiphase flow through porous media, capillary pressure-saturation and relative permeability-saturation curves are assumed to be intrinsic properties of the medium. Moreover, relative permeability is assumed to be a scalar property. However, numerous theoretical and experimental works have shown that these basic assumptions may not be valid. For example, relative permeability is known to be affected by the flow velocity (or pressure gradient) at which the measurements are carried out. In this article, it is suggested that the nonuniqueness of capillary pressure-relative permeability-saturation relationships is due to the presence of microheterogeneities within a laboratory sample. In order to investigate this hypothesis, a large number of "numerical experiments" are carried out. A numerical multiphase flow model is used to simulate the procedures that are commonly used in the laboratory for the measurement of capillary pressure and relative permeability curves. The dimensions of the simulation domain are similar to those of a typical laboratory sample (a few centimeters in each direction). Various combinations of boundary conditions and soil heterogeneity are simulated and average capillary pressure, saturation, and relative permeability for the "soil sample" are obtained. It is found that the irreducible water saturation is a function of the capillary number; the smaller the capillary number, the larger the irreducible water saturation. Both drainage and imbibition capillary pressure curves are found to be strongly affected by heterogeneities and boundary conditions. Relative permeability is also found to be affected by the boundary conditions; this is especially true about the nonaqueous phase

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permeability. Our results reveal that there is much need for laboratory experiments aimed at investigating the interplay of boundary conditions and microheterogeneities and their effect on capillary pressure and relative permeability. © 2002 Elsevier Science B.V. All rights reserved.

*Keywords:* Two-phase flow; Heterogeneity; Porous media; Capillary pressure; Saturation; Relative permeability; Upscaling; Effective parameters

#### 1. Introduction

Quantitative description and modelling of two-phase flow in porous media requires specification of functional relationships between capillary pressure,  $P^c$ , saturation, S, and relative permeability,  $k_r$ . These relationships are commonly determined from laboratory experiments on soil samples. However, sometimes only  $P^c-S$  curves are measured and then used to obtain  $k_r-S$  relations by means of empirical formulas (see, e.g. Demond and Roberts, 1993; Honarour et al., 1986; van Genuchten, 1980; Mualem, 1976). An evaluation of various formulas proposed in the literature is provided in Chen et al. (1999).

Capillary pressure-saturation and relative permeability-saturation relationships are highly nonlinear, and their determination is often a difficult task. This task can be made even more difficult by the presence of microheterogeneities within a sample of porous medium, and by the possibility that dynamic effects can influence the measured quantities. Usually, the  $P^{c}-S-k_{r}$  curves are considered to be intrinsic properties of the soil and independent of experimental conditions under which they have been determined. This might be true under certain conditions, but many authors have shown that it is not true under all conditions. For example, various authors have shown, both experimentally and by means of computational results, that both the capillary pressure function and the relative permeability function can depend on the capillary number, which measures the relative importance of viscous to capillary forces (see, e.g. Delshad et al., 1987; Fulcher et al., 1985; Blunt and King, 1991; Avraam and Payatakes, 1995, 1999; Blom, 1999; Henderson et al., 2000). In general, changes in these functions occur in the range of high capillary numbers, corresponding to high flow rates and corresponding high-pressure gradients. While previous studies have observed these dependencies, and some have modeled them in various ways, the role that material heterogeneity plays in this functional dependence has not been explored. In particular, the so-called microheterogeneities, which occur at length scales below those of typical laboratory measurement devices, can play a significant role in the resulting functional relationships. Herein we focus on these heterogeneities, which occur within a laboratory sample that might be on the order of  $100 - 1000 \text{ cm}^3$ .

Another potentially important issue involves the scalar nature of relative permeability. Bear et al. (1987) suggest that it is not appropriate to assume relative permeability to be scalar for an anisotropic medium. In fact, a number of studies indicate that the relationship between the saturation and effective permeability for a heterogeneous medium may depend on direction (Yeh et al., 1985; Mantoglou and Gelhar, 1987; Braun et al., 1998; Ataie-Ashtiani et al., 2001). In other words, relative permeability is in general not a scalar. Lake (1989) suggests that the combination of the intrinsic permeability and the relative permeability should be combined into a single tensorial quantity. Independent of how the expression for the effective permeability is ultimately written, the origin of the direction dependence, and quantification of its magnitude, require much additional study. While the stochastic approaches of, for example, Yeh et al. (1985) and Mantoglou and Gelhar (1987) focused on field-scale heterogeneity, herein we focus on microheterogeneities and their effect on measured relationships, including measures of anisotropy.

Ideally, to investigate these issues, one should carry out many sets of measurements at different initial and/or boundary conditions and with soils containing different kinds of heterogeneity in order to investigate the nonuniqueness of both  $P^{c}-S$  and  $k_{r}-S$  relationships. However, such measurements are tedious, time consuming and expensive. The next best alternative is to employ numerical models and perform "numerical experiments" that simulate laboratory experiments. Of course, one cannot replace reality with a computer model, but it is possible to use carefully designed simulations to gain insight into the effects of various factors on processes of interest. Moreover, "numerical experiments" help to identify major issues and determine important processes. The insight obtained can help to improve designs for actual laboratory experiments. For example, pore-scale network models and percolation models have been used successfully in order to enhance our understanding of multiphase flow problems. Ferrand and Celia (1992) developed a static pore-scale network model to study the effect of heterogeneity on the drainage capillary pressure-saturation relationship. However, in their study, the effect of heterogeneity on  $k_r - S$  relations were not investigated, and imbibition processes were not addressed. Moreover, pore-scale models are typically limited in the size of the domain they can simulate. In this article, we examine the effects of heterogeneities present within a laboratory sample. The modelling domain, corresponding to a laboratory sample, is composed of sands of different characteristics. Each sandy region is treated as a continuum with its own two-phase flow properties.

The objective of this work is to investigate the effects of microheterogeneities on the  $P^c-S-k_r$  relationships for a laboratory-scale sample. This is done by simulating the experimental procedures that are usually used to determine capillary pressure–saturation–relative permeability relationships. Thus, we carry out a large number of numerical experiments in order to show how the interplay of heterogeneities and boundary conditions can affect capillary pressure and relative permeability curves. Simulations are carried out on a soil block with dimensions similar to those used in laboratory experiments. Numerical simulations are conducted with the water–oil mode of the multi-fluid flow simulator STOMP (White and Oostrom, 1997). Processes modeled in STOMP and its numerical features are described in Section 2. Characteristics of the porous medium block and the employed boundary conditions are presented in Section 3. In Section 4, results of the numerical simulations are analyzed and discussed. Finally, conclusions are presented in Section 5.

# 2. Numerical model

The two-phase flow simulations are carried out using the simulation package STOMP (Subsurface Transport Over Multiple Phases). STOMP is a package consisting of 11

modes for simulation of nonisothermal multiphase flow and multi-component transport in a heterogeneous medium in three dimensions. This fully implicit, integrated finite difference code has been used to simulate a variety of multi-fluid systems (Oostrom and Lenhard, 1998; Schroth et al., 1998; Oostrom et al., 1997; Ataie-Ashtiani et al., 2001). The simulations discussed in this paper were conducted with a nonhysteretic two-phase version of the water–oil mode of STOMP. In this mode, the following mass balance equations for the phases water (superscript w) and oil (superscript o) are solved:

$$\frac{\partial}{\partial t}(n\rho_{\gamma}S_{\gamma}) = -\nabla \cdot (\rho_{\gamma}\mathbf{q}_{\gamma}) \quad \text{for } \gamma = \mathbf{w},\mathbf{o}$$
(1)

where

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$$\mathbf{q}_{\gamma} = -\frac{k_{r\gamma}\mathbf{k}}{\mu_{\gamma}} \cdot (\nabla P_{\gamma} + \rho_{\gamma}g\mathbf{e}_{g}) \quad \text{for } \gamma = \text{w,o}$$
<sup>(2)</sup>

In these equations, subscripts  $\gamma$  may be 'w' for water phase or 'o' for nonaqueous (oil) phase, *n* is porosity,  $\rho$  is density (ML<sup>-3</sup>), *S* is saturation, **q** is fluid flow velocity (LT<sup>-1</sup>),  $k_r$  is relative permeability, **k** is the intrinsic permeability tensor (L<sup>2</sup>),  $\mu$  is viscosity (ML<sup>-1</sup> T<sup>-1</sup>), *P* is pressure (ML<sup>-1</sup> T<sup>-2</sup>), *g* is the magnitude of gravitational acceleration (LT<sup>-2</sup>), and  $\mathbf{e}_g$  is the unit vector in the vertical direction.

In addition to the well-known van Genuchten (1980) and Brooks and Corey (1964) capillary pressure–saturation relations, this version of the simulator also allows for tabular input of fluid saturations and pressures. In the present article, the Brooks–Corey formula for capillary pressure is used:

$$S_{\rm e} = \left(\frac{P^{\rm c}}{P_{\rm d}}\right)^{-\lambda} \quad \text{for } P^{\rm c} \ge P_{\rm d} \tag{3}$$

where

$$S_{\rm e} = \left(\frac{S-S_{\rm r}}{1-S_{\rm r}}\right), \quad 0 \le S_{\rm e} \le 1 \tag{4}$$

In these equations,  $P^c$  is capillary pressure (ML<sup>-1</sup> T<sup>-2</sup>),  $P_d$  is displacement pressure (ML<sup>-1</sup> T<sup>-2</sup>),  $S_e$  is effective saturation (-),  $S_r$  is irreducible water saturation (-), and  $\lambda$  is the pore size distribution index.

The relative permeability of the medium to the aqueous phase,  $k_{rw}$ , and nonaqueous phase,  $k_{rn}$ , are given by the Brooks–Corey–Burdine formula (Brooks and Corey, 1964):

$$k_{\rm rw} = S_{\rm e}^{\left[(2+3\lambda)/\lambda\right]} \tag{5}$$

$$k_{\rm m} = (1 - S_{\rm e})^2 (1 - S_{\rm e}^{[(2+\lambda)/\lambda]})$$
(6)

Also, for later use, we define the capillary number,  $N_c$ , in terms of the pressure gradient across a soil block:

$$N_{\rm c} = \frac{k\Delta P_0}{\sigma L} \tag{7}$$

where k (L<sup>2</sup>) is the saturated permeability of the main material in the block,  $\sigma$  (M L<sup>2</sup>) is interfacial tension, L (L) is the block length, and  $\Delta P_0$  (M L<sup>-1</sup> T<sup>-2</sup>) is the pressure difference across the block. The capillary number is a dimensionless number that indicates the relative importance of viscous forces as compared to capillary forces.

#### 3. Description of numerical experiments

In this section, we define properties of our "numerical soil sample" and describe the procedure for our numerical "experiments". We consider a laboratory-scale porous medium  $(10 \times 10 \text{ cm})$  containing heterogeneities as illustrated in Fig. 1. Two rather simple patterns are used here: a straight pattern and a staggered pattern. We realize that the results of simulations will be affected by the choice of the heterogeneity pattern. Our main goal is to illustrate the combined influence of various types of boundary conditions and



Fig. 1. Hypothetical soil samples with heterogeneity patterns (a) P1 and (b) P2.

heterogeneity on soil characteristic curves. If we can show the significance of these effects for the heterogeneity patterns considered here, such effects are probably important for other types of heterogeneity. Therefore, we prefer to avoid complications that may be introduced by an intricate heterogeneity pattern. These two patterns were studied recently in a numerical investigation of the upscaling of two-phase flow in a heterogeneous medium (Ataie-Ashtiani et al., 2001). Moreover, very similar patterns have been considered in other studies (Amazine et al., 1991; Durlofsky, 1991; Bourgeat and Hidani, 1995). The white background in Fig. 1 is coarse-grained sand while the black areas comprise fine-grained sand. As the nonaqueous phase fluid, we choose the common groundwater contaminant tetrachloroethylene (PCE). Porous medium properties, including permeability and the associated Brooks-Corey parameters for the PCE-water main drainage capillary pressure curve, are listed in Table 1. The PCE has a density of 1630 kg m<sup>-3</sup> and a viscosity of  $0.9 \times 10^{-3}$  kg m<sup>-1</sup> s<sup>-1</sup>. The two heterogeneity patterns of Fig. 1 are designated by P1 and P2 (see the figure). Compared to the coarse-grained sands, the fine-grained sands have lower saturated permeabilities, higher entry pressures, a different pore size distribution index, and higher irreducible water saturation. The only difference between the two coarse-grained sands is the index  $\lambda$  that results in differences in the slope of  $P^{c}-S$  curves.

In our numerical experiments, the domain was discretized into a uniform mesh with a fine, constant nodal spacing of 2.5 mm in both horizontal and vertical directions. First, we calculated the average saturated permeability of the block. This was done by establishing steady-state flow of a single fluid at a given  $\Delta P_0$  and from the ratio of flow rate to  $\Delta P_0/L\mu$  (see Ataie-Ashtiani et al., 2000). Obviously, average permeability tensor will be anisotropic for an equivalent homogeneous block with its components being dependent on the pattern and type of heterogeneity. Next, we simulated measurements of  $P^c - S - k_r$  curves. There are two main methods for such measurements: a flow-through experiment and a pressure-cell experiment. Our simulations closely followed the corresponding procedure for each of these two methods. In a flow-through experiment, a unidirectional steady-state flow of both water and PCE was established through the sample block. This was achieved by applying a constant pressure difference for both fluids,  $\Delta P^w = \Delta P^{nw} = \Delta P_0$ , across the block. This means that the capillary pressures at the two opposing faces of the sample were equal. On the block faces parallel to the flow, no-flow boundary conditions were imposed.

Table 1 Properties of sands used in simulations

Property	Units	Coarse-grained sand C1	Coarse-grained sand C2	Fine-grained sand F1	Fine-grained sand F2
Permeability, k	[m <sup>2</sup> ]	$5  imes 10^{-10}$	$5  imes 10^{-10}$	$5  imes 10^{-11}$	$15  imes 10^{-11}$
Porosity, n	[-]	0.4	0.4	0.4	0.4
Displacement pressure, $P_d$	[Pa]	370	370	1325	764
Pore size distribution index, $\lambda$	[-]	3.86	1.25	2.49	2.49
Irreducible water saturation, $S_{\rm r}$	[-]	0.078	0.078	0.098	0.098

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Initially, the block was assumed to be saturated with the aqueous phase. Once the imposed  $P^c$  became larger than the displacement pressure of sand, PCE started to invade the block. For that given  $P^c$ , simulations were carried out until apparent equilibrium (i.e. steady state) was reached. This was determined by calculating fluxes of both water and PCE; at steady state, there was no change in these fluxes. Then, average saturation was calculated and plotted versus the  $P^c$  imposed at the block faces. This provided one point for the  $P^c-S$  curve. Next,  $P^c$  was increased by a small amount by increasing  $P^{nw}$  at both faces, keeping  $\Delta P^w = \Delta P^{nw} = \Delta P_0$ . Again, simulations were carried out until steady state was reached. This procedure was repeated whereby  $P^c$  was increased incrementally up to 10,000 Pa (holding  $\Delta P^w = \Delta P^{nw} = \Delta P_0$ ). This way, a plot of  $P^c$  versus saturation was constructed, using data points taken at equilibrium conditions. The procedure was carried out for flow in both vertical and horizontal directions.

Three series of simulations were carried out: low-pressure difference with  $\Delta P_0 = 100$  Pa (denoted as PGL), medium pressure difference with  $\Delta P_0 = 1000$  Pa (denoted as PGM) and high-pressure difference with  $\Delta P_0 = 10,000$  Pa (denoted as PGH). For each steady-state situation, relative permeability for each phase was obtained from the ratio of flow rate of that phase to the term  $k\Delta P_0/\mu L$ . The total simulation period for each experiment was 420 days.

The reverse procedure was applied to obtain imbibition curves. Both primary and main imbibition curves were obtained. In the case of primary curve, the sample was considered to be initially fully saturated with the nonaqueous phase. We started with a large capillary pressure of 10,000 Pa imposed at the two opposing faces with  $\Delta P^{nw} = \Delta P^w = \Delta P_0$ . After reaching equilibrium, the average saturation was calculated. The full imbibition curve was constructed by decreasing capillary pressure incrementally (by decreasing PCE pressure), all at constant  $\Delta P_0$ . The procedure was carried out until zero capillary pressure was reached. The procedure for the main imbibition was the same except that the initial saturations of the two phases were given by the final saturation state of a drainage curve.

In the pressure cell experiments, the soil sample was assumed to be saturated with water and placed inside a reservoir filled with the nonaqueous phase. The sample was assumed to rest on a hydrophobic porous plate that would allow water to drain out of the sample into a water reservoir, but had a very low displacement pressure for the nonaqueous phase. So, the sample was surrounded by the nonaqueous phase on three sides (for these two-dimensional domains) and was in contact with the water reservoir at the bottom boundary. The water reservoir phase pressure was kept constant and the nonaqueous phase reservoir pressure was increased incrementally, each time waiting until equilibrium was reached. Capillary pressure was calculated from the difference in reservoir pressures of the nonaqueous and aqueous phases. Saturation was determined from the amount of water leaving the sample. The pressure cell procedure was simulated by applying, at the right, left and top boundaries, a constant pressure  $P^{nw}$ for PCE and no-flow boundary condition for water. At the lower boundary, constant water pressure  $P^{w}$  for the aqueous phase and a no-flow boundary condition for the nonaqueous phase were applied. The rest of procedure is the same as in the previous case. This series of simulations is denoted by PCL (for pressure cell). All simulations carried out in the horizontal plane and therefore the, gravity was neglected. This ensured that any differences in results for horizontal and vertical direction was intrinsic to the presence of heterogeneities and not due to presence/absence of gravity.

# 4. Results

In this section, results of various simulations are presented and discussed. First, we introduce a nomenclature to denote individual simulations. As mentioned earlier, there are two heterogeneity patterns P1 and P2, as illustrated in Fig. 1. Four different boundary conditions, as described in the previous section, are imposed. These are summarized in Table 2, where the corresponding capillary numbers are also given.

In the case of flow-through boundary conditions, the flow can be either vertical (denoted by V) or horizontal (denoted by H). Four different soil types are used whose properties are given in Table 1: two coarse sands C1 and C2, and two fine sands F1 and F2. In most cases, the block is assumed to be composed of coarse sand C1 and fine sand F1. A simulation denoted by P1-PGL-V indicates vertical flow through sample P1 (with coarse sand C1 and fine sand F1) at  $\Delta P_0 = 100$  Pa. In some simulations, coarse sand C2 and fine sand F1 are used; this is indicated by adding C2. In some other simulations, coarse sand C1 and fine sand F2 are used; this is indicated by adding F2. Thus, for example, the simulation P2-F2-PCL indicates a pressure cell experiment involving pattern P2 with coarse sand C1 and fine sand F2.

#### 4.1. Effect of boundary conditions

Simulated capillary pressure saturation curves for block P1 are shown in Fig. 2. Also the original  $P^c-S$  curves for coarse sand C1 and fine sand F1 are shown. One might have expected that the average  $P^c-S$  curve for the block would lie between the original  $P^c-S$ curves of the two sands. It is evident from Fig. 2 that this does not happen except for the high-pressure gradient (P1-PGH-V) case. In fact, most curves remain close to the  $P^c-S$ curve for the coarse sand, for saturations greater than about 0.40. The average irreducible water saturation  $S_{ir}$ , however, is not only much higher than that of the coarse sand but also higher than that of the fine sand. It is found that  $S_{ir}$  is higher for lower capillary number.

These results show that a variety of capillary pressure-saturation curves may be obtained for the same soil sample, depending on the boundary condition. Although these are numerical results, we believe they have roots in real physical processes and, thus,

Table 2 Types of boundary conditions

• 1			
PGL	Flow through	$\Delta P_0 = 100 \text{ Pa}$	$N_{\rm c} = 1.126 {\rm e} - 5$
PGM	Flow through	$\Delta P_0 = 1000$ Pa	$N_{\rm c} = 1.126 {\rm e} - 4$
PGH	Flow through	$\Delta P_0 = 10,000 \text{ Pa}$	$N_{\rm c} = 1.126 {\rm e} - 3$
PCL	Pressure cell	No flow	-



Fig. 2. P<sup>c</sup>-S drainage curves for P1 block with various pressure gradients imposed at boundaries.

similar results may be obtained in practice. Support for this contention can be found by examining the reasons for obtaining different  $P^c-S$  curves for the same "soil sample". The explanations for these results should be sought in the detailed saturation distribution within the block. In Fig. 3, steady-state saturation distributions at five different  $P^c$ 's for the four different boundary conditions are given.

The results of P1-PGL-V case show that even when the  $P^{c}$  imposed at the boundaries is 1500 Pa, which is more than the entry pressure of fine sand (i.e. 1325 Pa), PCE cannot penetrate the fine sand region. This is because the course sand is already fully invaded by PCE (i.e. water is at irreducible saturation) and, thus, its relative permeability to water is reduced to zero. As a result, water cannot exit the fine sand and it remains trapped in the fine-grained material. A similar situation occurs in the pressure cell case (P1-PCL). However, for the high-pressure gradient case (P1-PGH-V), PCE penetrates the fine sand even though the imposed  $P^{c}$  is only 450 Pa, which is much smaller than the entry pressure of fine sand. This is because a small pool of PCE is formed on top of the fine sand (see Fig. 3) so that PCE pressure exceeds the entry pressure of fine sand and penetrates it. This is confirmed by the plot of PCE saturation, PCE pressure, and water pressure at  $P^{c} = 450$ Pa at a vertical cross-section in the middle of sample given in Fig. 4. It is evident that the PCE saturation at the top boundary of fine sand is significantly higher in the high-gradient case (P1-PGH-V) than in the low-gradient case (P1-PGL-V). Also, the capillary pressure at the top of fine sand reaches 1720 Pa for the high-pressure gradient case (P1-PGH-V), which exceeds the entry pressure of fine sand.

Notice that this kind of macroscopic trapping should be expected whenever a discrepancy exists between the local  $P^c-S$  curves, such that residual saturation of the background material is reached prior to a complete drainage of the imbedded lenses. This general principle applies to the upscaled  $P^c-S$  relationships for these types of heterogeneities, despite the obvious effects that flow rate has on the specific results. In the examples used here, the relatively flat shape of the local  $P^c-S$  curves makes this



Fig. 3. Saturation distribution within the P1 block for different capillary pressures for boundary conditions, PGL-V, PGM-V, PGH-V, and PCL for drainage case.

phenomenon very clear, and allows influences of boundary conditions and associated flow rates to be illustrated clearly.

#### 4.2. Effects of heterogeneity on relative permeability-saturation curves

Average relative permeability of the sample can be calculated at any imposed capillary pressure (and thus at a given average saturation). This is obtained from the ratio of the steady-state flux through the sample to  $k\Delta P_0/\mu L$ . This calculation could not be done for the PCL boundary condition, as there is no pressure gradient and flow between the faces in this case.

Alternatively, one may construct a relative permeability curve on the basis of the average  $P^{c}-S$  curve. To do so, first the Brooks and Corey formula is fitted to one of the average capillary pressure curves in Fig. 2 and an effective value of the pore size index,  $\lambda$ , for the sample is obtained. Then, Brooks–Corey–Burdine equation (Eqs. (5) and (6)) is used to obtain the corresponding relative permeability curve.



Fig. 4. Variation of PCE saturation, PCE pressure and water pressure at 450-Pa capillary pressure for; (a) P1-PGL-V and (b) P1-PGH-V cases at a vertical cross-section in the middle of sample.

Fig. 5 shows the calculated and constructed relative permeability curves of aqueous and nonaqueous phases. Also, relative permeability curves for coarse sand C1 and fine sand F1 are given. An interesting result is that the relative permeability curves constructed from capillary pressure curves agree reasonably well with the calculated relative permeability curves. However, they are different from the coarse sand relative permeabilities and



Fig. 5. Calculated and constructed relative permeability-saturation curves under drainage for (a) P1-PGL-V case, (b) P1-PGH-V case and (c) P1-PCL case.

somewhat closer to the fine sand curve. Also, the imposed pressure gradient seems to have a significant effect on the result; there is a drastic difference between the relative permeabilities obtained for a sample under different pressure gradients.

# 4.3. Effect of pressure gradient on imbibition capillary pressure curves

As mentioned earlier, in one set of simulations, block P1 was assumed to be originally at irreducible water saturation and nonaqueous phase was forced to invade the sample. This way, imbibition capillary curves were obtained. In these calculations,  $P^c-S$  curves for individual sands were the same as the one used in drainage simulations. Of course, one could have chosen to work with imbibition curves of coarse and fine sands. However, this was not done in order to focus on the effect of boundary conditions only. Fig. 6 shows drainage and primary and main imbibition  $P^c-S$  curves for block P1 for both low- and high-pressure gradient cases. The most interesting result is that the imposed flow conditions have little effect on primary imbibition curves. The curves are close to the original  $P^c-S$  curves of the coarse and fine sands. The conditions simulated here correspond to the waterflooding of oil reservoirs. Thus, one may conclude that in the case of waterflooding in a formation containing fine-grained microheterogeneities, the effective capillary pressure curves lie in between the imbibition curves of sands constituting the medium.

The main imbibition curve for the low-pressure gradient case is also shown (P1-PGLimb-main) in Fig. 6. As seen, despite the fact that  $P^c-S$  curves for individual sands are not changed, the main imbibition curve is different from the main drainage curve. This is a hysteresis that is purely caused by the presence of fine-grained heterogeneities. This can be



Fig. 6.  $P^{c}-S$  curves for P1 block calculated for both drainage and imbibition processes, under low- and highpressure gradient conditions.

explained because the saturation of PCE inside the fine sand will not decrease from its maximum value, which occurs at  $P^c = 10,000$  Pa, until a decrease in the saturation of coarse sand is observed. Therefore, the average water saturation for the sample stays almost the same as the minimum value of its drainage curve (in fact it decreases slightly) until water invades the neighborhood of the fine sand.

#### 4.4. Directional dependency of capillary pressure curve

In this section, results of  $P^c-S$  calculations for block P1 using different flow directions are presented. The effective capillary pressure curves, as well as the curves for the two sands C1 and F1, are given in Fig. 7. The curves are obtained for both low- and high-pressure gradients and for flow in both vertical and horizontal directions.

The main observation is that averaged  $P^c-S$  curves do not show a significant directional dependency, and for the most part, there is little directional dependence. This holds for both low- and high-pressure gradient cases.

# 4.5. Effects of hydraulic characteristics of sands

In order to examine the effects of hydraulic characteristics of fine and coarse sand on the  $P^c-S$  curves, two sets of simulations with block P1 are carried out. In one set, coarse sand C1 and fine sand F2 (which has a lower entry pressure than sand F1) are chosen. The calculated curves for the medium-pressure-gradient case,  $\Delta P_0 = 1000$  Pa (case P1-F2-PGM-V) and for the pressure-cell case (P1-F2-PCL-V) are compared with corresponding curves for sands C1 and F1 in Fig. 8. Compared to sand F1, sand F2 has less contrast in hydraulic properties (saturated permeability and entry pressure) with sand C1. As a result, as is evident in Fig. 6, effective  $P^c-S$  curves lie rather close to the  $P^c-S$  curves for the two sands.



Fig. 7. Effect of flow direction on  $P^c-S$  drainage curves for P1 block, calculated for both vertical and horizontal directions for drainage case.



Fig. 8. Effect of displacement pressure of fine sand on  $P^{c}-S$  drainage curves for block P1 (for cases of medium pressure gradient and pressure cell).

In another set of simulations, coarse sand C2 and fine sand F1 are assigned to block P1. Coarse sand C2 has a smaller pore size index,  $\lambda$ , which means its  $P^c-S$  curve has a steeper slope between entry pressure and irreducible saturation. The results of this case for medium-pressure-gradient and pressure cell are shown in Fig. 9. Interestingly, in this case, no significant effects of the imposed boundary conditions are observed. In fact, calculated curves for both the medium-pressure-gradient case (P1-C2-PGM-V) and the pressure-cell case (P1-C2-PCL) lie very closely to the  $P^c-S$  curve for sand C2. In summary, it is evident that the weaker the heterogeneity, the less is the influence of boundary conditions on effective  $P^c-S$  curves.



Fig. 9. Effect of pore size index of coarse sand on  $P^{c}-S$  drainage curve for P1 block (for cases of medium pressure gradient and pressure cell).



Fig. 10. Effect of heterogeneity pattern on  $P^{c}-S$  drainage curve for P1 and P2 blocks.

# 4.6. Effects of heterogeneity patterns

In order to investigate the effects of heterogeneity configurations on the  $P^c-S$  curves, some simulations were repeated for pattern P2 and were compared to those for pattern P1. Fig. 10 shows the calculated curves for the four cases: low-pressure-gradient (P1-PGL-V and P2-PGL-V) and high-pressure-gradient (P1-PGH-V and P2-PGH-V). It is evident that there is very little difference in the calculated  $P^c-S$  curves for the two patterns. Therefore, it appears that the results obtained for the pattern P1 in the previous section, hold equally well for pattern P2 and similar patterns. However, these results cannot be generalized to other cases such as layered media (see Braun et al., 1998).

# 5. Summary and conclusions

The presence of microheterogeneities can lead to complex functional dependencies in constitutive relationships between capillary pressure, saturation, and relative permeability, where each of these three variables is defined as an average over a laboratory sample. Careful numerical experiments were used to demonstrate a number of the effects that microheterogeneities have on the resulting relationships. These effects include directional dependencies as well as enhanced hysteresis. Microheterogeneities also influence the response to different boundary conditions, in particular to those that lead to different capillary numbers. While previous researchers have identified changes in functional relationships as a function of capillary number, this is the first time that the influence of heterogeneities have been combined with the influences of flow rates and capillary number changes. In the present work, a large number of numerical experiments were carried out in order to show how the interplay between heterogeneities and boundary conditions can affect capillary pressure and relative permeability curves. The dimensions of the simulation

domain were similar to those of a typical laboratory sample. It was found that the irreducible water saturation is a function of the capillary number; the smaller the capillary number, the larger the irreducible water saturation. Both drainage and imbibition capillary pressure curves were found to be strongly affected by heterogeneities and boundary conditions. Relative permeability was also found to be affected by the boundary conditions; this is especially true for the nonaqueous phase permeability. In general, we observe that the weaker the heterogeneity, the less is the influence of boundary conditions on effective  $P^c-S$  curves.

Our results reveal that equilibrium relationships between capillary pressure, saturation, and relative permeability are influenced in complex ways by both boundary condition specification and subscale heterogeneity. There is much need for laboratory experiments aimed at investigating the interplay of boundary conditions and microheterogeneities and their effect on capillary pressure and relative permeability. In addition, there may be information in the transient responses of these systems that can be used to identify underlying structures of heterogeneity, or other important properties of the system. Herein we used only equilibrium information, but future work needs to incorporate dynamics into the overall relationships between capillary pressure, saturation, and relative permeability.

# Acknowledgements

This research has been carried out in the framework of the TRIAS project "Upscaling microheterogeneities in two phase flow in porous media," Delft Cluster Project no. 5.3.1. Partial support to M.A. Celia was provided by the Delft University of Technology through a Visiting Faculty appointment. Stimulating discussions with Mart Oostrom (Battelle-PNL) are gratefully acknowledged. Critical comments by two anonymous referees helped to improve the manuscript.

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