Role of Cycle Inventory

- **Lot:** quantity that is produces or orders in a stage at a given time.
- **Cycle inventory:**
  - Average inventory that is built up because of a mismatch between production and demand.
  - $Q =$ lot or batch size of an order
  - $D =$ demand per unit time
- **Inventory profile**
  - Cycle inventory = $Q/2$
  - Average flow time = Avg inventory / Avg flow rate = $Q/(2D)$
Role of Cycle Inventory

- Q = 1000 units
- D = 100 units/day
- Cycle inventory = Q/2 = 1000/2 = 500
- Avg flow time = Q/2D = 1000/(2)(100) = 5 days

- Cycle inventory adds 5 days to the time a unit spends in the supply chain
- Lower cycle inventory is better because:
  - Average flow time is lower
  - Working capital requirements are lower
  - Lower inventory holding costs

Economies of Scale

- How do you decide whether to go shopping
  - a convenience store or Sam’s Club?
- Lot sizing for a single product (EOQ)
- Aggregating multiple products in a single order
- Lot sizing with multiple products or customers
  - Lots are ordered and delivered independently for each product
  - Lots are ordered and delivered jointly for all products
  - Lots are ordered and delivered jointly for a subset of products

Role of Cycle Inventory

- Cycle inventory is held primarily to
  - take advantage of economies of scale in the supply chain
- Supply chain costs influenced by lot size:
  - Material cost = C
  - Fixed ordering cost = S
  - Holding cost = H = hC
  - h = cost of holding $1 in inventory for one year
- Primary role of cycle inventory
  - to allow different stages to purchase product in lot sizes that minimize the sum of material, ordering, and holding costs
- Ideally, cycle inventory decisions should consider
  - costs across the entire supply chain,
  - but each stage generally makes its own supply chain decisions
  - this increases total cycle inventory and total costs in the supply chain

Economies of Scale

- Cost
- Total Cost
- Holding Cost
- Order Cost
- Material Cost
- Lot Size
Economies of Scale to Exploit Fixed Costs

- Annual demand = D
- Annual material cost = CD
- Number of orders per year = D/Q
- Annual order cost = (D/Q)S
- Annual holding cost = (Q/2)hC
- Total annual cost = TC = CD + (D/Q)S + (Q/2)hC

\[
\frac{d(TC)}{dQ} = -\frac{DS}{Q^2} + \frac{hC}{2} \quad Q = \sqrt{\frac{2DS}{hC}}
\]

Economic Order Quantity

\[H = hC\]
\[Q^* = \sqrt{\frac{2DS}{H}}\]
\[n^* = \frac{DH}{2S}\]

Example 1

- **Demand** for the Deskpro computer at Best Buy is 1000 units per month.
- Best Buy incurs a **fixed order placement**, transportation, and receiving cost of $4,000 each time an order is placed.
- Each computer costs Best Buy $500 and the retailer has a holding cost of 20 percent.
- Evaluate the number of computers that the store manager should order in each replenishment lot.

Example 1

- **Inputs:**
  - Demand D = 12,000 computers per year
  - d = 1000 computers/month
  - Unit cost, C = $500
  - Holding cost fraction, h = 0.2
  - Fixed cost, S = $4,000/order

  **Outputs:**
  - \[Q^* = \sqrt{\frac{(2)(12000)(4000)/(0.2)(500))^{0.5}} = 980\]
  - Cycle inventory = \[Q/2 = 490\]
  - Flow time = \[Q/2d = 980/(2)(1000) = 0.49\] month
  - Reorder interval, T = 0.98 month
Example 1

- Annual ordering and holding cost = \((12000/980)(4000) + (980/2)(0.2)(500)\) = $97,980

- Suppose lot size is reduced to Q=200, which would reduce flow time:
  - Annual ordering and holding cost = \((12000/200)(4000) + (200/2)(0.2)(500)\) = $250,000

Example 2

- If desired lot size = Q* = 200 units, what would S have to be?
  - D = 12000 units
  - C = $500
  - h = 0.2
  - Use EOQ equation and solve for S: \(Q = \sqrt{\frac{2DS}{hC}}\)
    - \(S = [\sqrt{bC(Q^*)^2}/2D] = [(0.2)(500)(200)^2]/(2)(12000) = 166.67\)$
  - To reduce optimal lot size by a factor of k, the fixed order cost must be reduced by a factor of \(k^2\)

Key Points from EOQ Model

- In deciding the optimal lot size, the tradeoff is between setup cost and holding cost. \(Q = \sqrt{\frac{2DS}{hC}}\)

- If demand increases by a factor of 4, it is optimal to increase batch size by a factor of 2.
  - *Cycle inventory should decrease as demand increases.*

- If lot size is to be reduced, one has to reduce fixed order cost.
  - To reduce lot size by a factor of 2, order cost has to be reduced by a factor of 4.

Aggregating Multiple Products

- **Transportation** is a significant contributor to cost.

- Combine different products from the same supplier?
  - same overall fixed cost
  - shared over more than one product
  - effective fixed cost is reduced for each product
  - lot size for each product can be reduced

- A single delivery from multiple suppliers?
- A single truck delivering to multiple retailers?
Aggregating Multiple Products

- Aggregating across products, retailers, suppliers

- Allows for a reduction in lot size because fixed ordering and transportation costs are now spread across multiple products, retailers, or suppliers

Example 3

- Suppose there are 4 computer products: Deskpro, Litepro, Medpro, and Heavpro
- Demand for each is 1000 units per month
- If each product is ordered separately:
  - $Q^* = 980$ units for each product
  - Total cycle inventory $= 4(Q/2) = (4)(980)/2 = 1960$ units
- Aggregate orders of all four products:
  - Combined $Q^* = 1960$ units
  - For each product: $Q^* = 1960/4 = 490$
  - Cycle inventory for each product is reduced to $490/2 = 245$
  - Total cycle inventory $= 1960/2 = 980$ units
  - Average flow time, inventory holding costs will be reduced

Multiple Products or Customers

- In practice, the fixed ordering cost is dependent at least in part on the variety associated with an order of multiple models
  - A portion of the cost is related to transportation (independent of variety)
  - A portion of the cost is related to loading and receiving (not independent of variety)
- Three scenarios:
  - Lots are ordered and delivered independently for each product
  - Lots are ordered and delivered jointly for all three models
  - Lots are ordered and delivered jointly for a selected subset of models

Example 4

- Best Buy sells three models of computers: the Litepro, the Medpro, and the Heavypro.
- Annual demands:
  - $O_L = 12000$, $O_M = 1200$, and $O_H = 120$
- Costs:
  - Each model costs Best Buy $500
  - Fixed transportation cost: $4000
  - Product specific order cost: $1000
  - Best Buy incurs a holding cost of 20 percent.
### No Aggregation

<table>
<thead>
<tr>
<th></th>
<th>Litepro</th>
<th>Medpro</th>
<th>Heavypro</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand per year</td>
<td>12,000</td>
<td>1,200</td>
<td>120</td>
</tr>
<tr>
<td>Fixed cost/order</td>
<td>$5,000</td>
<td>$3,000</td>
<td>$5,000</td>
</tr>
<tr>
<td>Optimal order size</td>
<td>1,095</td>
<td>346</td>
<td>110</td>
</tr>
<tr>
<td>Cycle inventory</td>
<td>548</td>
<td>173</td>
<td>55</td>
</tr>
<tr>
<td>Annual holding cost</td>
<td>$54,772</td>
<td>$17,321</td>
<td>$5,477</td>
</tr>
<tr>
<td>Order frequency</td>
<td>11.8/year</td>
<td>5.5/year</td>
<td>1.1/year</td>
</tr>
<tr>
<td>Annual ordering cost</td>
<td>$54,772</td>
<td>$17,321</td>
<td>$5,477</td>
</tr>
<tr>
<td>Average flow time</td>
<td>2.4 weeks</td>
<td>7.5 weeks</td>
<td>23.7 weeks</td>
</tr>
<tr>
<td>Annual cost</td>
<td>$109,544</td>
<td>$34,862</td>
<td>$10,954</td>
</tr>
</tbody>
</table>

Total cost = $155,140

### Delivery Options

- **No Aggregation**
  - Each product ordered separately

- **Complete Aggregation**
  - All products delivered on each truck

- **Tailored Aggregation**
  - Selected subsets of products on each truck

### Order All Products Jointly

\[ S^* = S_l + s_m + s_h = 4000 + 1000 + 1000 + 1000 = 7000 \]

\[ n^* = \sqrt{\frac{DH}{2S}} \]

\[ n^* = \sqrt{\frac{12,000 \times 100 + 1,200 \times 100 + 120 \times 100}{2 \times 7,000}} = 9.75 \]

- \( Q_L = D_l/n^* = 12000/9.75 = 1230 \)
- \( Q_M = D_m/n^* = 1200/9.75 = 123 \)
- \( Q_H = D_h/n^* = 120/9.75 = 12.3 \)

Annual order cost = 9.75 × $7,000 = $68,250

Annual total cost = $136,528
Tailored Aggregation

- The procedure we discuss
  - Does not provide the optimal solution.
  - It yields an ordering policy whose cost is close to optimal.

- Step 1
  - Identify the most frequently ordered.
  - Assuming each product is ordered independently.

\[ n_i = \sqrt{\frac{hC_iD_i}{2(S + s_i)}} \]

- Step 2

- Step 3
  - Having the ordering frequency of each product, **recalculate** the ordering frequency of the most frequently ordered product.

\[ n = \sqrt{\frac{\sum hCim_iD_i}{2(S + \sum s_i/m_i)}} \]

- Step 4
  - For each product, evaluate an order frequency

\[ n_i = \frac{n}{m_i} \]

Tailored Aggregation

- Step 2
  - Order frequency for all other products
  - only the product-specific fixed cost

\[ \bar{n} = \sqrt{\frac{hC_iD_i}{2s_i}} \]

- Evaluate the frequency of product i relative to the most frequently ordered product

\[ m_i = \frac{n}{\bar{n}} \]

Example

- Consider the Best Buy data

- Step 1

\[ n_L = \sqrt{\frac{hC_DL}{2(S + s_L)}} = 11.0 \]

\[ n_M = 3.5, \text{ and } n_H = 1.1 \]

\[ \bar{n} = 11.0 \]
Example

Step 2

\[ \bar{n}_M = \sqrt{\frac{\bar{C}_M \bar{D}_M}{2\bar{p}_M}} \approx 7.7 \quad \text{and} \quad \bar{n}_H = 2.4 \]

\[ \bar{m}_M = \frac{\bar{n}_M}{\bar{n}_M} = \frac{11.0}{7.7} = 1.4 \quad \text{and} \quad \bar{m}_H = 4.5 \]

\[ \bar{m}_M = [1.4] = 2 \quad \text{and} \quad \bar{m}_H = [4.5] = 5 \]

Step 3

\[ n = 11.47 \]

Step 4

\[ n_c = 11.47/\text{year}, n_M = 5.74/\text{year}, \text{and} n_H = 11.47/5 = 2.29/\text{year} \]

Lessons from Aggregation

- Aggregation allows firm to
  - lower lot size without increasing cost

- Complete aggregation is effective if
  - product specific fixed cost is a small fraction of joint fixed cost

- Tailored aggregation is effective if
  - product specific fixed cost is a large fraction of joint fixed cost

Economies of Scale: Quantity Discounts

- All-unit quantity discounts
- Marginal unit quantity discounts
- Why quantity discounts?
  - Coordination in the supply chain
  - Price discrimination to maximize supplier profits

Quantity Discounts

- Lot size based
  - All units
  - Marginal unit
- Volume based

- How should buyer react?
- What are appropriate discounting schemes?
All-Unit Quantity Discounts

- Pricing schedule has specified quantity break points
  - $q_0$, $q_1$, ..., $q_r$, where $q_0 = 0$

- The objective is
  - To decide on a lot size that will minimize total costs

All-Unit Quantity Discount Procedure

1. Step 1
   - Calculate EOQ for the lowest price. If it is feasible, then stop.
   - This is the optimal lot size.

2. Step 2
   - If the EOQ is not feasible, calculate the TC for this price and the smallest quantity for that price.

3. Step 3
   - Calculate EOQ for next lowest price.
   - If it is feasible, stop and calculate TC for that price.

4. Step 4
   - Compare TC from Steps 2 and 3. Choose the cheapest.

5. Step 5
   - If EOQ in Step 3 is not feasible, repeat Steps 2, 3, and 4 until a feasible EOQ is found.

All-Unit Quantity Discounts: Example

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Unit Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5000</td>
<td>$3.00</td>
</tr>
<tr>
<td>5001-10000</td>
<td>$2.96</td>
</tr>
<tr>
<td>Over 10000</td>
<td>$2.92</td>
</tr>
</tbody>
</table>

$q_0 = 0$, $q_1 = 5000$, $q_2 = 10000$

$C_0 = $3.00, $C_1 = $2.96, $C_2 = $2.92

D = 120000 units/year,

S = $100/lot,

h = 0.2
**All-Unit Quantity Discount: Example**

- **Step 1**
  - Calculate $Q^*$
    - $Q^* = \sqrt{\frac{2DS}{hC_2}} = \sqrt{\frac{(2)(120000)(100)}{(0.2)(2.92)}} = 6410$
  - Not feasible ($6410 < 10001$)
  - Calculate $TC_2$
    - Using $C_2 = $2.92 and $q_2 = 10001$
    - $TC_2 = (\frac{120000}{10001})(100) + (\frac{10001}{2})(0.2)(2.92) + (120000)(2.92) = $354,520

- **Step 2**
  - Calculate $Q^*$
    - $Q^* = \sqrt{\frac{2DS}{hC_1}} = \sqrt{\frac{(2)(120000)(100)}{(0.2)(2.96)}} = 6367$
  - Feasible ($5000 < 6367 < 10000$)  ➔  Stop
  - Calculate $TC_1$
    - $TC_1 = (\frac{120000}{6367})(100) + (\frac{6367}{2})(0.2)(2.96) + (120000)(2.96) = $358,969
  - $TC_2 < TC_1$
  - The optimal order quantity $Q^*$ is $q_2 = 10001$

**All-Unit Quantity Discounts**

- Suppose fixed order cost were reduced to $4
  - Without discount, $Q^*$ would be reduced to 1265 units
  - With discount, optimal lot size would still be 10001 units

- What is the effect of such a discount schedule?
  - Retailers are encouraged to increase the size of their orders
  - Average inventory in the supply chain is increased
  - Average flow time is increased
  - Is an all-unit quantity discount an advantage in the supply chain?

**Why Quantity Discounts?**

- Coordination in the supply chain
  - Commodity products
  - Products with demand curve
    - 2-part tariffs
    - Volume discounts
Coordination for Commodity Products

- $D = 120,000$ bottles/year
- $S_{Retailer} = $100, $h_R = 0.2$, $C_R = $3
- $S_{Supplier} = $250, $h_S = 0.2$, $C_S = $2

- Retailer’s optimal lot size = 6,324 bottles
- Retailer cost = $3,795
- Supplier cost = $6,009
- Supply chain cost = $9,804

Discounts: Firm Has Market Power

- No inventory related costs
- Demand curve
  
  $360,000 - 60,000p$
- What are the optimal prices and profits in the following situations?
  - The two stages coordinate the pricing decision
    - Price = $4, Profit = $240,000, Demand = 120,000
  - The two stages make the pricing decision independently
    - Price = $5, Profit = $180,000, Demand = 60,000

Coordination for Commodity Products

- What can the supplier do to decrease supply chain costs?
  - Coordinated lot size: 9,165; Retailer cost = $4,059; Supplier cost = $5,106; Supply chain cost = $9,165

- Effective pricing schemes
  - All-unit quantity discount
    - $3 for lots below 9,165
    - $2.99 for lots of 9,165 or more
  - Pass some fixed cost to retailer (enough that he raises order size from 6,324 to 9,165)

Two-Part Tariffs and Volume Discounts

- Design a two-part tariff that achieves the coordinated solution
- Design a volume discount scheme that achieves the coordinated solution
- Impact of inventory costs
  - Pass on some fixed costs with above pricing
Lessons from Discounting Schemes

- **Lot size** based discounts
  - increase lot size and cycle inventory
  - are justified to achieve coordination for **commodity products**
- **Volume** based discounts
  - with some fixed cost passed on to retailer
  - more effective in general
  - better over rolling horizon

Short-Term Discounting

- Trade promotions are price discounts for a limited period of time.
- Key goals from a manufacturer’s perspective:
  - Induce retailers to use price discounts, displays, advertising to **increase sales**.
  - **Shift inventory** to the retailer and customer
  - **Defend a brand** against competition

Short-Term Discounting

- What is the impact on behavior of retailer?
  - performance of supply chain?
- Retailer has two primary options:
  - Pass through some of the promotion to customers.
  - Purchase in greater quantity during promotion period to take advantage of temporary price reduction.

**Short Term Discounting**

\[ Q^d = \frac{dD}{(C - d)h} + \frac{CQ^*}{C - d} \]

- Assuming
  - Discount is offered once
  - Customer demand is unchanged.

\( Q^* \): Normal order quantity
\( C \): Normal unit cost
\( d \): Short term discount
\( D \): Annual demand
\( h \): Cost of holding $1 per year
\( Q^d \): Short term order quantity
Example: Forward Buying

- Normal order size, $Q^* = 6,324$ bottles
- Normal cost, $C = $3 per bottle
- Discount per tube, $d = $0.15
- Annual demand, $D = 120,000$
- Holding cost, $h = 0.2$

- $Q^d =$?
- Forward buy =?