Logistics and Supply Chain Management

Demand Forecasting

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Outline

- The role of forecasting in a supply chain
- Characteristics of forecasts
- Components of forecasts and forecasting methods
- Basic approach to demand forecasting
- Time series forecasting methods
- Measures of forecast error

Role of Forecasting

- The basis for all strategic and planning decisions in a supply chain
- Used for both push and pull processes
- Examples:
  - Production: scheduling, inventory, aggregate planning
  - Marketing: sales force allocation, promotions, new production introduction
  - Finance: plant/equipment investment, budgetary planning
  - Personnel: workforce planning, hiring, layoffs
- All of these decisions are interrelated

Characteristics of Forecasts

- Forecasts are always wrong. Should include expected value and measure of error.
- Long-term forecasts are less accurate than short-term forecasts (forecast horizon is important)
- Aggregate forecasts are more accurate than disaggregate forecasts
- Bullwhip Effect: The farther up the supply chain a company is, the greater is the distortion of information it receives.
### Factors Related to Demand

- Past demand
- Lead time of product
- Planned advertising or marketing efforts
- State of the economy
- Planned price discounts
- Actions that competitors have taken

### Basic Steps to Demand Forecasting

- Understand the objectives of forecasting
- Integrate demand planning and forecasting
- Identify major factors that influence demand
- Identify customer segments
- Determine the appropriate forecasting technique
- Establish performance and error measures for the forecast

### Forecasting Methods

1. Qualitative: primarily subjective; rely on judgment and opinion
2. Time Series: use historical demand only
   - Static
   - Adaptive
3. Causal: use the relationship between demand and some other factor to develop forecast
4. Simulation
   - Imitate consumer choices that give rise to demand
   - Can combine time series and causal methods

### Components of an Observation

Observed demand \( (O) = \) Systematic component \( (S) + \) Random component \( (R) \)

- **Level** (current deseasonalized demand)
- **Trend** (growth or decline in demand)
- **Seasonality** (predictable seasonal fluctuation)

- Systematic component: Expected value of demand
- Random component: The part of the forecast that deviates from the systematic component
- Forecast error: difference between forecast and actual demand
Forecast demand for the next four quarters.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Demand D_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>II, 1998</td>
<td>8000</td>
</tr>
<tr>
<td>III, 1998</td>
<td>13000</td>
</tr>
<tr>
<td>IV, 1998</td>
<td>23000</td>
</tr>
<tr>
<td>I, 1999</td>
<td>34000</td>
</tr>
<tr>
<td>II, 1999</td>
<td>10000</td>
</tr>
<tr>
<td>III, 1999</td>
<td>18000</td>
</tr>
<tr>
<td>IV, 1999</td>
<td>23000</td>
</tr>
<tr>
<td>I, 2000</td>
<td>38000</td>
</tr>
<tr>
<td>II, 2000</td>
<td>12000</td>
</tr>
<tr>
<td>III, 2000</td>
<td>13000</td>
</tr>
<tr>
<td>IV, 2000</td>
<td>32000</td>
</tr>
<tr>
<td>I, 2001</td>
<td>41000</td>
</tr>
</tbody>
</table>

Goal is to predict systematic component of demand

- Multiplicative: (level)(trend)(seasonal factor)
- Additive: level + trend + seasonal factor
- Mixed: (level + trend)(seasonal factor)

- Static methods
- Adaptive forecasting
Static Methods

- Assume a mixed model:
  Systematic component = (level + trend)(seasonal factor)

\[ F_{t+l} = [L + (t + l)T]S_{t+l} \]

= forecast in period \( t \) for demand in period \( t + l \)

- \( L \) = estimate of level for period 0
- \( T \) = estimate of trend
- \( S_t \) = estimate of seasonal factor for period \( t \)
- \( D_t \) = actual demand in period \( t \)
- \( F_t \) = forecast of demand in period \( t \)

Estimating Level and Trend

- Before estimating level and trend, demand data must be deseasonalized
  - Deseasonalized demand = demand that would have been observed in the absence of seasonal fluctuations
- Periodicity (\( p \))
  - the number of periods after which the seasonal cycle repeats itself
  - for demand at Tahoe Salt \( p = 4 \)

Example: Tahoe Salt

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>Period t</th>
<th>Demand ( D_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>6,000</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>13,000</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td>23,000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>34,000</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5</td>
<td>10,000</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>18,000</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>7</td>
<td>23,000</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>8</td>
<td>38,000</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>9</td>
<td>12,000</td>
</tr>
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<td>3</td>
<td>3</td>
<td>10</td>
<td>13,000</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>11</td>
<td>32,000</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>12</td>
<td>41,000</td>
</tr>
</tbody>
</table>
For the example, \( p = 4 \) is even

\[
\overline{D}_3 = \frac{D_1 + D_5 + \sum_{i=2}^{4} [2D_i]}{8} = \frac{8000 + 10000 + [(2)(13000)+(2)(23000)+(2)(34000)]}{8} = 19750
\]

\[
\overline{D}_4 = \frac{D_2 + D_6 + \sum_{i=3}^{5} [2D_i]}{8} = \frac{13000 + 18000 + [(2)(23000)+(2)(34000)+(2)(10000)]}{8} = 20625
\]

Then include trend: \( \overline{D}_t = \overline{D}_0 + tT \)

- \( \overline{D}_t \) = deseasonalized demand in period \( t \)
- \( \overline{D}_0 \) = level (deseasonalized demand at period 0)
- \( T \) = trend (rate of growth of deseasonalized demand)

- Trend is determined by linear regression:
  - Dependent variable: deseasonalized demand
  - Independent variable: period

- In the example, \( L = 18,439 \) and \( T = 524 \)
Estimating Seasonal Factors

- Calculate deseasonalized demand for each period:
  \[ S_t = \frac{D_t}{\bar{D}_t} \] where \( S_t \) is seasonal factor for period \( t \)

- In the example,
  - \( \bar{D}_2 = 18439 + (524)(2) = 19487 \)
  - \( D_2 = 13000 \)
  - \( S_2 = \frac{13000}{19487} = 0.67 \)
  - Other seasonal factors are calculated similarly

Estimating the Forecast

- Forecast the next four periods:

\[
\begin{align*}
F_{13} &= (L+13T)S_1 = [18439+(13)(524)](0.47) = 11868 \\
F_{14} &= (L+14T)S_2 = [18439+(14)(524)](0.67) = 17527 \\
F_{15} &= (L+15T)S_3 = [18439+(15)(524)](1.17) = 30770 \\
F_{16} &= (L+16T)S_4 = [18439+(16)(524)](1.67) = 44794
\end{align*}
\]
Adaptive Forecasting

- The estimates of level, trend, and seasonality are adjusted after each demand observation
  - Moving average
  - Simple exponential smoothing
  - Trend-corrected exponential smoothing (Holt’s model)
  - Trend- and seasonality-corrected exponential smoothing (Winter’s model)

Basic Formulas

\[ F_{t+1} = (L_t + T_t)S_{t+1} \]

- \( L_t \) = Estimate of level at the end of period \( t \)
- \( T_t \) = Estimate of trend at the end of period \( t \)
- \( S_t \) = Estimate of seasonal factor for period \( t \)
- \( F_t \) = Forecast of demand for period \( t \)
- \( D_t \) = Actual demand observed in period \( t \)
- \( E_t \) = Forecast error in period \( t \)
- \( A_t \) = Absolute deviation for period \( t = |E_t| \)
- \( \text{MAD} = \text{Mean Absolute Deviation} = \text{average value of } A_t \)

General Steps

- Initialize
  - Compute initial estimates of level (\( L_0 \)), trend (\( T_0 \)), and seasonal factors (\( S_1, \ldots, S_p \)).
  - As in static forecasting.
- Forecast
  - Forecast demand for period \( t+1 \)
- Estimate error
  - Compute error \( E_{t+1} = F_{t+1} - D_{t+1} \)
- Modify estimates
  - Modify the estimates of level (\( L_{t+1} \)), trend (\( T_{t+1} \)), and seasonal factor (\( S_{t+p+1} \)), given the error \( E_{t+1} \) in the forecast
- Repeat steps 2, 3, and 4 for each subsequent period

Moving Average

- Used when demand has no observable trend or seasonality
- Systematic component of demand = level
- Level in period \( t \) is the average over last \( N \) periods
- Current forecast for all future periods is the same:
  \[ L_t = (D_1 + D_2 + \ldots + D_{t-N}) / N \]
  \[ F_{t+1} = L_t \text{ and } F_{t+n} = L_t \]
- After observing the demand for period \( t+1 \), revise:
  \[ L_{t+1} = (D_{t+1} + D_t + \ldots + D_{t+N}) / N \]
  \[ F_{t+2} = L_{t+1} \]
Example

In the Tahoe Salt example, what is the forecast demand for periods 5 through 8 using a 4-period moving average?

- \( L_4 = \frac{(D_4+D_3+D_2+D_1)}{4} \)
  - \( = \frac{(34000+23000+13000+8000)}{4} = 19500 \)
- \( F_5 = 19500 = F_6 = F_7 = F_8 \)
- Observe demand in period 5 to be \( D_5 = 10000 \)
- Forecast error in period 5: \( E_5 = 19500 - 10000 = 9500 \)
- Revise estimate of level in period 5:
  - \( L_5 = \frac{(D_5+D_4+D_3+D_2)}{4} = 20000 \)
- \( L_5 = \frac{(D_5+D_4+D_3+D_2)}{4} = 20000 \)
- \( F_6 = L_5 = 20000 \)

Example

- Tahoe Salt data
  - Forecast demand using exponential smoothing:
    - Period 1:
      - \( L_0 = \text{average of all periods} = \frac{\sum_{i=1}^{12} D_i}{12} = 22083 \)
      - \( F_1 = L_0 = 22083 \)
      - Observed demand for period 1 = \( D_1 = 8000 \)
      - Forecast error for period 1: \( E_1 = F_1 - D_1 = 14083 \)
    - Assuming \( \alpha = 0.1 \), revised the estimates:
      - \( L_1 = \alpha D_1 + (1-\alpha)L_0 = (0.1)(8000) + (0.9)(22083) = 20675 \)
      - \( F_2 = L_1 = 20675 \)

Simple Exponential Smoothing

- Used when demand has no observable trend or seasonality
- Systematic component of demand = level
- Initial estimate of level
  - average of all demand data: \( L_0 = \frac{\sum_{i=1}^{n} D_i}{n} \)
- Current forecast
  - current estimate of the level: \( F_{t+n} = L_t \)
- Revise, after observing demand \( D_{t+1} \)
  - \( L_{t+1} = \alpha D_{t+1} + (1-\alpha)L_t \)
  - \( \alpha \): Smoothing constant

Holt’s Model

- Trend-Corrected Exponential Smoothing
- When demand is assumed to have no seasonality
- Obtain initial estimate of level and trend by running a linear regression of the following form:
  - \( D_t = at + b \)
  - \( T_0 = a \)
  - \( L_0 = b \)
- In period \( t \), the forecast for future periods is:
  - \( F_{t+1} = L_t + T_t \)
  - \( F_{t+n} = L_t + nT_t \)
Holt’s Model

- Revise the estimates after observing demand for period t:
  - \( L_{t+1} = \alpha D_{t+1} + (1-\alpha)(L_t + T_t) \)
  - \( T_{t+1} = \beta (L_{t+1} - L_t) + (1-\beta)T_t \)
  - \( \alpha = \) smoothing constant for level
  - \( \beta = \) smoothing constant for trend

Example

- Forecast Tahoe Salt demand using Holt’s model
  - Using linear regression,
    - \( L_0 = 12015 \) (linear intercept)
    - \( T_0 = 1549 \) (linear slope)
  - Forecast for period 1:
    - \( F_1 = L_0 + T_0 = 12015 + 1549 = 13564 \)
    - Observed demand for period 1 = \( D_1 = 8000 \)
    - \( E_1 = F_1 - D_1 = 13564 - 8000 = 5564 \)
    - Assume \( \alpha = 0.1, \beta = 0.2 \)
    - \( L_1 = \alpha D_1 + (1-\alpha)(L_0 + T_0) = (0.1)(8000) + (0.9)(13564) = 13008 \)
    - \( T_1 = \beta (L_1 - L_0) + (1-\beta)T_0 = (0.2)(13008 - 12015) + (0.8)(1549) = 1438 \)
    - \( F_1 = L_1 + T_1 = 13008 + 1438 = 14446 \)
    - \( F_5 = L_1 + 4T_1 = 13008 + (4)(1438) = 18760 \)

Winter’s Model

- Trend- and Seasonality-Corrected Exponential Smoothing
  - Systematic component of demand is assumed to have a level, trend, and seasonal factor
  - Systematic component = (level + trend)(seasonal factor)
  - Assume periodicity \( p \)
  - Obtain initial estimates
    - using procedure for static forecasting
  - Forecast in period \( t \):
    - \( F_{t+1} = (L_{t+1} + T_{t+1})(S_{t+1}) \) and \( F_{t+n} = (L_t + nT_t)S_{t+n} \)

Winter’s Model

- Revise estimates
  - Observing demand for period \( t+1 \),
    - \( L_{t+1} = \alpha \frac{D_{t+1}}{S_{t+1}} + (1-\alpha)(L_t + T_t) \)
      - \( \alpha = \) smoothing constant for level
    - \( T_{t+1} = \beta (L_{t+1} - L_t) + (1-\beta)T_t \)
      - \( \beta = \) smoothing constant for trend
    - \( S_{t+p+1} = \gamma \frac{D_{t+1}}{L_{t+1}} + (1-\gamma)S_{t+1} \)
      - \( \gamma = \) smoothing constant for seasonal factor
Example

- Forecast Tahoe Salt demand using Winter’s model.
- Initial estimates:
  - $L_0 = 18439$, $T_0 = 524$
  - $S_1=0.47$, $S_2=0.68$, $S_3=1.17$, $S_4=1.67$
  - $F_1 = (L_0 + T_0)S_1 = (18439+524)(0.47) = 8913$
- Forecast error
  - $D_1 = 8000$
  - $E_1 = F_1-D_1 = 8913 - 8000 = 913$
- Revise estimates, assume $\alpha = 0.1$, $\beta=0.2$, $\gamma=0.1$
  - $L_1 = \alpha(D_1/S_1)+(1-\alpha)(L_0+T_0) = (0.1)(8000/0.47)+(0.9)(18439+524)=18769$
  - $T_1 = \beta(L_1-L_0)+(1-\beta)T_0 = (0.2)(18769-18439)+(0.8)(524) = 485$
  - $S_5 = \gamma(D_1/L_1)+(1-\gamma)S_1 = (0.1)(8000/18769)+(0.9)(0.47) = 0.47$
  - $F_2 = (L_1+T_1)S_2 = (18769 + 485)(0.68) = 13093$

Measures of Forecast Error

- Forecast error $= E_t = F_t - D_t$
- Mean squared error (MSE)
  - $MSE_n = (\text{Sum}_{t=1}^{n}[E_t^2])/n$
- Absolute deviation $= A_t = |E_t|$
- Mean absolute deviation (MAD)
  - $MAD_n = (\text{Sum}_{t=1}^{n}[A_t])/n$
- Bias
  - Consistently under- or overestimate demand
  - $bias_n = \text{Sum}_{t=1}^{n}[E_t]$
- Tracking signal
  - $TS_t = bias / MAD_t$
  - Should be within the range of $\pm6$
  - Otherwise, possibly use a new forecasting method

Homework 3

Chapter 7

- Exercises:
  - 2
  - 3
- Due on 3 November.