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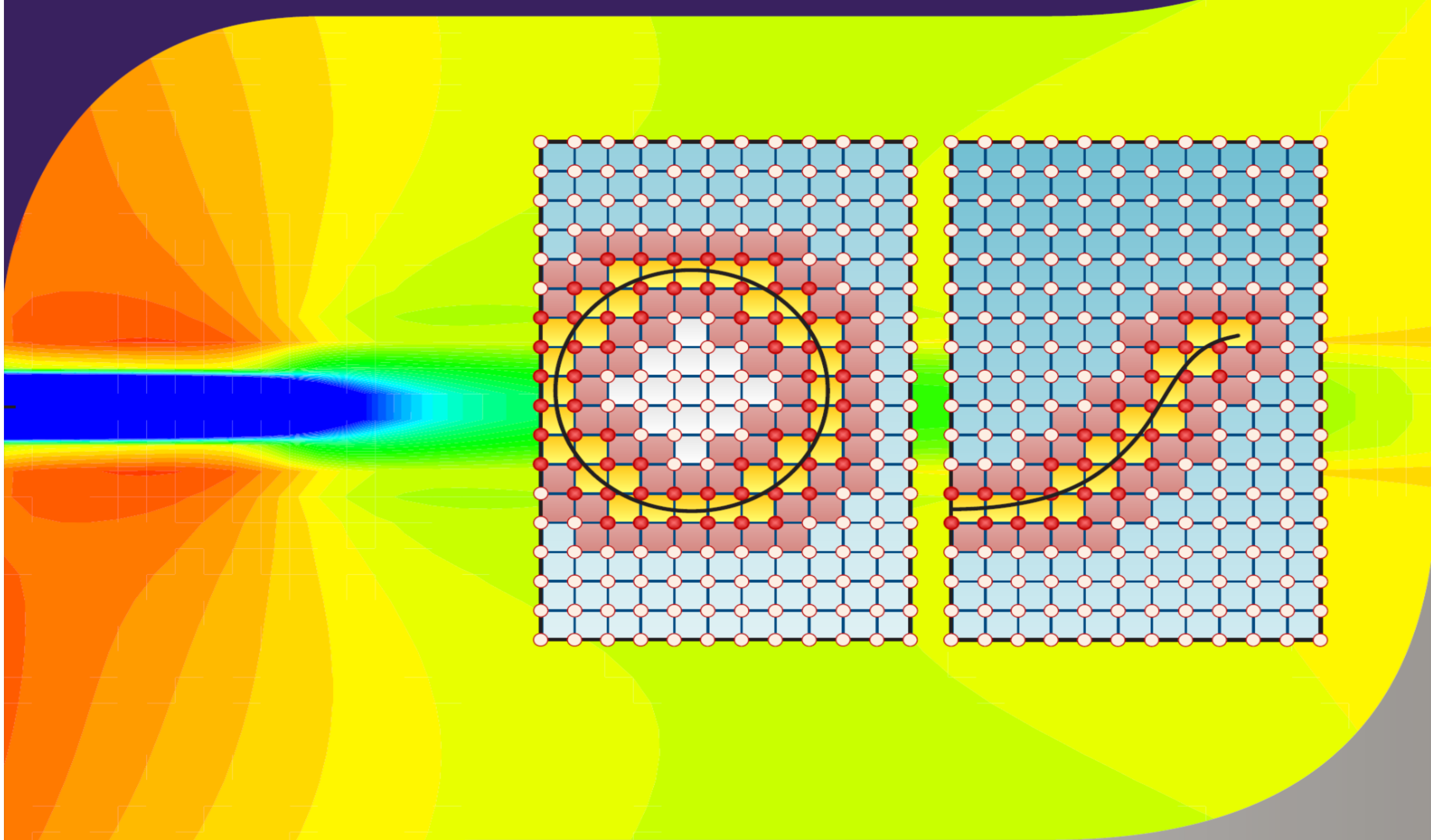
Extended Finite Element Method

Theory and Applications

Amir R. Khoei



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EXTENDED FINITE ELEMENT METHOD

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THEORY AND APPLICATIONS

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To Azadeh and Arsalan

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Series Preface

The series on *Computational Mechanics* is a conveniently identifiable set of books covering interrelated subjects that have received much attention in recent years, and need a place in senior undergraduate and graduate school curricula and engineering practice. The subjects of titles in the series cover applications and methods. They range from biomechanics to fluid-structure interactions to multiscale mechanics, and from computational geometry to meshfree techniques to parallel and iterative computing methods. Application areas are across the board in a wide range of industries, including civil, mechanical, aerospace, automotive, environmental, and biomedical engineering. Practicing engineers, researchers, and software developers at universities, in industry, government laboratories, and graduate students will find this series to be an indispensable source of new engineering approaches, interdisciplinary research, and a comprehensive learning experience in computational mechanics.

Since its conception in the late 1990s by Ted Belytschko, the eXtended Finite Element Method (XFEM) has become one of the most widely used numerical methods for simulating fracture. The method is highly versatile and has been applied to a variety of crack models, including linear elastic fracture mechanics and cohesive zone approaches, to shear banding and dislocations, as well as to other problems that involve discontinuities. *Extended Finite Element Method: Theory and Applications*, written by a leading expert in the field, is the most comprehensive book written to date on this important subject in computational mechanics. The book covers many aspects and application areas of the XFEM. It comes with detailed derivations and explanations, and an exhaustive bibliography that guides the reader into further developments in the field. Its engineering approach and standard notation make the book easy to read.

Preface

The finite element method is one of the most common numerical tools for obtaining approximate solutions of partial differential equations; the technique has been applied successfully in many areas of engineering sciences to study, model, and predict the behavior of structures. The area ranges from aeronautical and aerospace engineering, the automobile industry, mechanical engineering, civil engineering, biomechanics, geomechanics, material sciences, and many more. Despite its popularity, the finite element method suffers from certain drawbacks when the solution contains a non-smooth behavior, such as high gradients or singularities in the stress and strain fields, and/or strong discontinuities in the displacement field; then it becomes computationally expensive to get optimal convergence. In order to overcome such difficulties, the extended finite element method (X-FEM) has been developed to facilitate the modeling of arbitrary discontinuities such as jumps, kinks, singularities, and other non-smooth features within elements. The technique provides a powerful tool for enriching solution spaces with information from asymptotic solutions and other knowledge of the physics of the problem. The main purpose of this book is to present the theory and applications of the X-FEM in linear and nonlinear problems of continua, structures, and geomechanics.

There are a number of excellent books published on the finite element method, however, there are only three books released on the X-FEM that are geared to a specific audience. This book is aimed to provide a comprehensive study on the extended finite element modeling of continua, structures, and geomechanics that should appeal to a relatively wide audience. During the last two decades, the X-FEM has moved from purely research topic into mainstream day-to-day analysis in engineering problems. It is therefore necessary for both practicing engineers and students to become familiar with the subject. Since there is no comprehensive book explaining the X-FEM in various engineering problems, this book aims to rectify this situation and bring a comprehensive easy to follow introduction to the subject to researchers in the fields of civil, mechanical, materials, and aerospace engineering.

The book begins with an overview of the extended finite element method in Chapter 1, in which an emphasis is given on various applications of the technique in materials modeling problems. The mathematical formulation of the X-FEM is presented in Chapter 2 with special reference to solid mechanics problems. It includes the introduction of partition of unity method, enrichment functions, blending elements, the X-FEM discretization, and the numerical integration of X-FEM formulation. In this chapter, numerical implementation is presented for the linear and higher order quadrilateral elements in X-FEM modeling of linear and curved interfaces. Chapter 3 presents an overview of various X-FEM enrichment functions used in a wide variety of problems, such as bimaterials, cracks, dislocations, fluid-structure interactions, shear bands, convection-diffusion, thermo-mechanical, deformable porous media, piezoelectric, magneto-electro-elastic, topology optimization, rigid particles in Stokes flow, solidification, and so on. In Chapter 4, the problems of convergence rate and condition number within the X-FEM are discussed, and various remedies that are available in the literature are introduced for these issues. In Chapter 5, the X-FEM is developed for nonlinear behavior of materials in large deformations; it is first presented in the framework of a Lagrangian large plasticity deformation formulation, and is then described in the framework of an arbitrary Lagrangian–Eulerian method. In Chapter 6, the X-FEM method is presented for modeling frictional contact problems on the basis of the penalty method, Lagrange multipliers technique, and augmented Lagrange multipliers approach.

The implementation of X-FEM technique in linear elastic fracture mechanics is presented in Chapter 7. The basis of linear elastic fracture mechanics is first introduced by defining the stress and displacement distributions around the crack tip and the stress intensity factors for different loading modes. The governing equation of a cracked body is then derived in the framework of an X-FEM. In Chapter 8, the X-FEM technique is utilized to simulate a cracked body combined with the cohesive crack model. Various cohesive crack growths are demonstrated in the framework of extended-FEM technique based on the stress criterion, the stress intensity factor criterion, and the cohesive segments method. In Chapter 9, the X-FEM technique is presented for crack growth simulation in ductile fracture problems. A non-local damage-plasticity model is employed to capture the fracture process zone within the X-FEM technique. The Lagrangian X-FEM formulation is utilized to model large deformation crack propagation and, the process of failure and crack propagation in dynamic and cyclic loading conditions is performed using dynamic large deformation X-FEM formulation. In Chapter 10, the X-FEM is developed to model the deformable porous media with weak and strong discontinuities. The fluid phase mass balance equation is applied together with the momentum balance of bulk and fluid phases to model hydraulic fracture propagation in porous media on the basis of a u–p X-FEM formulation. In Chapter 11, the X-FEM is proposed for the fully coupled hydro-mechanical analysis of deformable, progressively fracturing porous media interacting with the flow of two immiscible, compressible wetting and non-wetting pore fluids. The fluid flow within the crack is simulated using Darcy's Law in which the permeability variation with porosity due to the cracking of the solid skeleton is accounted. The cohesive crack model is integrated into the numerical modeling, in which the nonlinear fracture processes occurring along the fracture process zone are simulated. Finally, Chapter 12 is devoted to the implementation of the X-FEM technique in thermo-hydro-mechanical modeling of saturated porous media. The thermo-hydro-mechanical governing equations are derived by utilizing the momentum equilibrium equation, mass balance equation, and the energy conservation relation within the X-FEM framework.

Basically, the material presented in this book is a part of established X-FEM research articles; however, for the most parts of the book, the detailed derivations have not been reported in a single source. Thus, I am indebted to the authors of all books and journal papers listed in the bibliography. I wish to express my sincere gratitude to the pioneers of the X-FEM method, in particular Ted Belytschko, John Dolbow, Nicolas Moës, and Natarajan Sukumar, whose work formed the basis of new development reported here. I would like to express my sincere gratitude to my friend and colleague, Soheil Mohammadi, for the fruitful discussions held on many occasions over a long period of time. I wish to thank my former graduate students in the Department of Civil Engineering at Sharif University of Technology, who have contributed to the advances in the application of the X-FEM; M. Anahid, S.O.R. Biabanaki, T. Mohamadnejad, P. Broumand, M. Vahab, M. Hirmand, E. Haghighat, S. Moallemi, A. Shamloo, M. Nikbakht, K. Karimi, K. Shahim, S.M.T. Mousavi, M.R. Hajiabadi, N. Hosseini, H. Akhondzadeh, and E. Abedian. Moreover, I would like to express a special thank to my students who have had a major contribution in the preparation of this manuscript; in particular, M. Vahab in the first three chapters, P. Broumand in Chapters 4 and 9, M. Anahid and S.O.R. Biabanaki in Chapter 5, S. Moallemi in Chapters 7 and 12, T. Mohamadnejad in Chapters 8 and 11, E. Haghighat in Chapter 10 and M. Hirmand in Chapter 6 and the worked examples given in a companion website of the book.

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Amir R. Khoei
Tehran, April 2014