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# Simulation of compressible flow in high pressure buried gas pipelines

Ali Nouri-Borujerdi, Masoud Ziaei-Rad *

*School of Mechanical Engineering, Sharif University of Technology, P.O. Box 11365-9567, Azadi Avenue, Tehran, Iran*

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ABSTRACT

The aim of this work is to analyze the gas flow in high pressure buried pipelines subjected to wall friction and heat transfer. The governing equations for one-dimensional compressible pipe flow are derived and solved numerically. The effects of friction, heat transfer from the wall and inlet temperature on various parameters such as pressure, temperature, Mach number and mass flow rate of the gas are investigated. The numerical scheme and numerical solution was confirmed by some previous numerical studies and available experimental data. The results show that the rate of heat transfer has not a considerable effect on inflow Mach number, but it can reduce the choking length in larger $f_D L/D$ values. The temperature loss will also increase in this case, if smaller pressure drop is desired along the pipe. The results also indicate that for $f_D L/D = 150$, decreasing the rate of heat transfer from the pipe wall, indicated here by Biot number from 100 to 0.001, will cause an increase of about 7% in the rate of mass flow carried by the pipeline, while for $f_D L/D = 50$, the change in the rate of mass flow has not a considerable effect. Furthermore, the mass flow rate of choked flow could be increased if the gas flow is cooled before entrance to the pipe.


## 1. Introduction

There are many published books and articles about the viscous compressible pipe flows under two limiting cases, adiabatic and isothermal [1–5]. In the isothermal flow conditions, the gas temperature is held constant. Such a flow occurs in long pipes where sufficient time is available for heat transfer to occur, therefore, it is acceptable to assume that the temperature may remain constant. On the contrary, in an adiabatic model, the pipe is insulated or the available time for heat transfer is short and it is usually occurred in short lines. Most real situations behave somehow between these two extreme cases.

An analytical solution to the case of frictionless compressible pipe flow subjected to heat addition was derived by Rayleigh in the late nineteen century then it was followed by Fanno for the case of pure friction in the early twenty century. Cochran [6] analyzed simple and practical methods for sizing pipelines and considering compressible flows that are rigorous enough to handle in most industrial situations, but simple enough to be easily programmed by personal computers. Keith and Crowl [7] presented a detailed review of sonic gas flow in pipelines, particularly in long pipelines. They found that the mass flow rate asymptotically approaches a constant value as the velocity head loss increases. The asymptotic value is identical for both the adiabatic and isothermal conditions and it is close to the maximum value.

Some attempts have been also made on the combination of Rayleigh and Fanno solutions to find out a general solution when a pipe flow is subjected to both heat transfer and wall friction. Shafeie [8] presented two different solution techniques to involve friction and heat transfer in the pipe flow for a wide velocity ranges. Toplak [9] solved the pipe flow with constant heat flux and constant friction factor. Both differential equations were solved by the same ODE solver but the solutions contained different number of points. The approaches of Shafeie [8] and Toplak [9] produced similar solutions, but with some differences. The Toplak's equation was solved faster than Shafeie's equation even though the solution contained a larger number of points. Ekblom and Gullman-Strand [10] compared existing theoretical analysis with experimental results of the compressible pipe flow subjected to wall friction and heat addition. Their research focused on some special geometries of experimental setup such as convergence–divergence nozzles. However, the effect of heating in various flow regimes was not widely considered. Landram [11] studied the choking limits and flow variables in a compressible one-dimensional pipe flow under heat addition. He presented generalized graphical results to readily permit passage design for monatomic gases including accommodation of any specified friction factor and a uniform wall heat flux. His work was mainly motivated by reason of determining choking limits in the design of high energy systems.

Mekebel and Loraud [12] studied experimentally an unsteady compressible flow in a long gas transmission pipeline. They concluded that inclusion of heat transfer is necessary in the theoretical analyses, contrary to common assumptions in the isothermal or

* Corresponding author. Tel.: +98 21 66165524; fax: +98 21 66000021.
*E-mail address:* Ziaeirad@mech.sharif.edu (M. Ziaei-Rad).

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**Nomenclature**

| | |
|---|---|
| $Bi$ | Biot number, $h_\infty(D/2+\delta)/k_S$ |
| $C_P$ | specific heat capacity |
| $D$ | pipe inside diameter |
| $f$ | friction factor |
| $G$ | mass velocity, $\rho V$ |
| $g$ | function |
| $H$ | depth of buried pipe center |
| $h$ | heat transfer coefficient, enthalpy, spatial step |
| $k$ | thermal conductivity, Runge–Kutta coefficients |
| $L$ | pipe length |
| $M$ | Mach number, $V/\sqrt{\gamma RT}$ |
| $m$ | number of discretization points |
| $\dot{m}$ | mass flow rate, $\pi D^2\rho V/4$ |
| $n$ | Haaland equation coefficient |
| $Pr$ | Prandtl number, $\nu/\alpha$ |
| $p$ | static pressure |
| $\dot{q}''$ | heat flux |
| $R$ | gas constant |
| $Re$ | Reynolds number, $\rho VD/\mu$ |
| $S$ | Sutherland constant temperature |
| $St$ | Stanton number, $U/GC_P$ |
| $T$ | static temperature |
| $U$ | overall heat transfer coefficient |
| $V$ | velocity |
| $x$ | spatial position |

*Greek letters*

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\beta$ | clustering parameter |
| $\gamma$ | specific heat ratio |
| $\Delta$ | difference |
| $\delta$ | wall thickness |
| $\varepsilon$ | roughness |
| $\eta$ | panel efficiency |
| $\mu$ | dynamic viscosity |
| $\rho$ | density |

*Subscripts*

| | |
|---|---|
| 1, 2 | inlet, outlet |
| 0 | stagnation |
| D | Darcy |
| g | gas |
| i | inside |
| j | node index |
| s | soil |
| w | pipe wall |
| wa | adiabatic wall |
| $\infty$ | ambient |

*Superscripts*

| | |
|---|---|
| $*$ | dimensionless |
| $**$ | choking condition |

adiabatic flows. The Experimental results of external flows [13] show that heating gas flow can reduce the skin friction and even a very significant reduction will be achieved if the surface near the leading edge is heated. In a numerical and experimental work by Garcia et al. [14], the internal compressible flow was studied at T-type junctions without considering the heat transfer effects. In another experimental and numerical investigation of compressible subsonic flows in long micro conduits by Harley et al. [15], it was suggested that the locally fully developed approximation could be used to interpret the experimental data for low and moderate Mach numbers.

In all of the previous works, no research has been conducted on wall heat transfer from a buried pipe with convection on the exposed ground surface or inflow temperature effect on both pressure loss and mass flow rate of gas flow which is important in gas transmission efficiency. Therefore, the objective of this work is to examine numerically the effect of heat transfer and wall friction on turbulent subsonic pipe flow to obtain a maximum efficiency of the gas transmission pipelines.

## 2. Governing equations

Fig. 1 shows a schematic of buried gas transmission pipeline between two gas stations and the related geometrical parameters used in this study. The conservation of mass, momentum and the equation of state for compressible one-dimensional steady-state gas flow through a circular pipe are as follows:

$$G = \rho V = \text{constant} \tag{1}$$

$$\frac{\partial}{\partial x}(\rho V^2) = -\frac{\partial p}{\partial x} - \frac{f_D}{2D}\rho V^2 \tag{2}$$

$$P = \rho RT \tag{3}$$

The Darcy friction factor is mostly presented by well-known Colebrook equation, but the implicit character of this correlation has created a need for a simpler explicit correlation. Many such correlations have been presented and one is the Haaland equation [16].

$$\frac{1}{\sqrt{f_D}} = -\frac{1.8}{n}\log\left[\left(\frac{6.9}{Re}\right)^n + \left(\frac{\varepsilon}{3.75D}\right)^{1.11n}\right] \tag{4}$$

Haaland [16] claimed that this equation is especially suited for gas pipelines if $n = 3$, giving a more abrupt transition between smooth and rough flow. This explicit friction factor relation is based on the Colebrook equation and the accuracy is expected to be very similar. In this relation, $\varepsilon$ is the pipe roughness and $Re = \rho VD/\mu$ is Reynolds number. The gas viscosity temperature dependent by Sutherland law is:

$$\mu = \mu_1\left(\frac{T}{T_1}\right)^{3/2}\left(\frac{T_1+S}{T+S}\right) \tag{5}$$

where $\mu_1$ is measured at the inflow temperature $T_1$ and $S$ is the Sutherland constant.

Introducing Eqs. (1), (3) and the Mach number of a perfect gas as $V = M\sqrt{\gamma RT}$ into Eq. (2), the governing Eqs. (1)–(3) can be reformed in two new differential equations as

$$\frac{dp}{d\left(\frac{f_D x}{D}\right)} = -\frac{\gamma p}{1+\gamma M^2}\left[\frac{1}{2}M^2 + \frac{dM^2}{d\left(\frac{f_D x}{D}\right)}\right] \tag{6}$$

$$\frac{dM^2}{M^2} = \frac{dT}{T} - 2\frac{dp}{p} \tag{7}$$

When the gas is brought to rest at the wall so even if there is no heat transfer at the wall, the wall temperature will be higher than the gas temperature. The heat transfer can only be from the wall to the gas if $T_\infty > T_{Wa}$. From this discussion, it follows that the most appropriate temperature to take as the gas temperature is $T_{Wa}$. In this case, the energy equation of the gas flow with overall heat transfer coefficient between the ambient and the gas adiabatic temperature is:

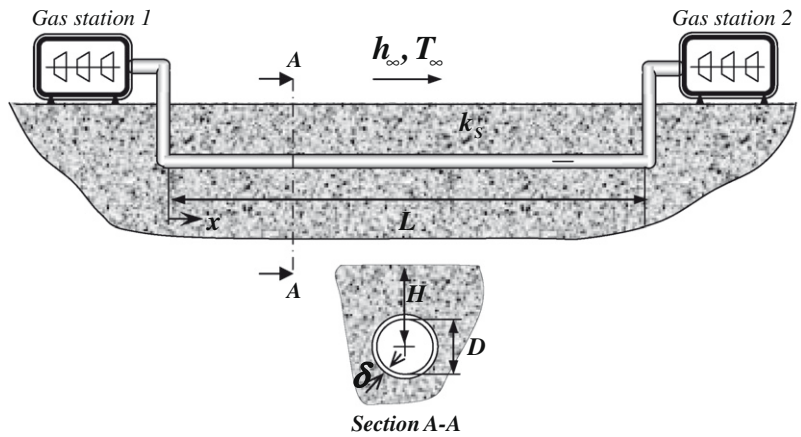


**Fig. 1.** A schematic of buried gas pipeline between two pumping gas stations.

$$\pi DU(T_\infty - T_{wa})\,dx = \dot{m}d\left(h + \frac{1}{2}V^2\right) \tag{8}$$

where the adiabatic wall temperature based on the static and stagnation gas temperatures is:

$$\frac{T_{wa} - T}{T_0 - T} = Pr^{\frac{1}{3}} \tag{9}$$

with $T_0 = T\left(1 + \frac{\gamma-1}{2}M^2\right)$. Using $\dot{m} = \pi D^2 G/4$, $h = C_p T$ and $V = M\sqrt{\gamma RT}$ into Eq. (8), the final result will be:

$$\frac{dT}{d(f_D x/D)} = \frac{1}{1 + \frac{\gamma-1}{2}M^2}\left[\frac{4St}{f_D}(T_\infty - T_{wa}) - \frac{\gamma-1}{2}T\frac{dM^2}{d(f_D x/D)}\right] \tag{10}$$

where $St = U/GC_P$ is termed the Stanton number and is defined based on the ambient and adiabatic wall temperatures difference. This equation represents the gas temperature along the pipe in differential form. Inserting Eqs. (6) and (10) into Eq. (7), the following result can be obtained for Mach number along the pipe.

$$\frac{dM^2}{d(f_D x/D)} = \frac{M^2}{1 - M^2} \times \left[\frac{4St}{f_D}(1 + \gamma M^2)\frac{T_\infty - T_{wa}}{T} + \gamma M^2\left(1 + \frac{\gamma-1}{2}M^2\right)\right] \tag{11}$$

The overall heat transfer coefficient appears in the definition of the Stanton number considering the convection on the exposed ground surface is expressed as:

$$\frac{1}{U} = \frac{1}{h_i} + \frac{D\ln(1 + 2\delta/D)}{2k_w} + \frac{D\cosh^{-1}\left(\frac{2H}{D+2\delta}\right)}{2\eta k_s} \tag{12}$$

where $\delta$ and $H$ are the wall thickness and depth of the buried pipe center to the ground surface. Parameters $k_w$ and $k_S$ are the thermal conductivity of the pipe and soil, respectively. The panel efficiency, $\eta$, is used to take into account the effects of finite convection heat loss from the exposed ground surface. It is a strong function of both the Biot number and $H/D$ and asymptotically approach to unity as the Biot number becomes large regardless of the value of $H/D$. For $H/D = 1.5$ it can be correlated by the numerical values of Chung et al. [17] as:

$$\begin{aligned} \eta &= 0.8882Bi^{0.2379}, \quad \text{if } 0.001 < Bi \le 0.5 \\ \eta &= 0.9025Bi^{0.0276}, \quad \text{if } 0.5 < Bi \le 100 \end{aligned} \tag{13}$$

where $Bi = h_\infty(D/2+\delta)/k_s$.

The inside heat transfer coefficient based on Dittus–Boelter equation for fully developed pipe flow is as follows:

$$\frac{h_i D}{k_g} = 0.023Re^{0.8}Pr^{\frac{1}{3}} \tag{14}$$

where $k_g$ is the thermal conductivity of the gas. By simultaneous solution of Eqs. (10) and (11) numerically for temperature and Mach number and using Eq. (6) for the calculation of the pressure, all of the flow parameters can be determined. When the outflow Mach number reaches one, the flow will be choked and the value of the mass flow rate approaches to a maximum.

## 3. Numerical procedure

The set of ordinary differential equations (10) and (11) can be re-written as:

$$\begin{aligned} \frac{dM}{dx^*} &= g_1(x^*, M, T^*) \\ \frac{dT}{dx^*} &= g_2(x^*, M, T^*) \end{aligned} \tag{15}$$

with the following initial conditions:

$$M(x_1^* = 0) = M_1, \quad T^*(x_1^* = 0) = 1 \tag{16}$$

in which $g_1$ and $g_2$ are dimensionless functions. All of the parameters have been made dimensionless according to the following parameters:

$$\begin{aligned} &x^* = f_D x/D, \quad p^* = p/p_1, \\ &\dot{m}^* = \dot{m}/\dot{m}_1, \quad T^* = T/T_1, \quad V^* = V/V_1 \end{aligned} \tag{17}$$

where subscript '1' denotes values at the inflow. $\dot{m}_1$ is the mass flow rate computed at $T = T_1$.

Since the governing equations are highly non-linear and the flow variables change very rapidly at the outflow where the choked condition occurs, it is necessary to pack the computational nodes near the outflow such that there are enough nodes around the choked point. Hence, for the location of the grid nodes in flow direction, the following algebraic relation is used to locate the nodal points.

$$x^* = \frac{f_D L}{D}\left[1 + (\beta + 1)\frac{\left(\frac{\beta+1}{\beta-1}\right)^{\xi-1} - 1}{\left(\frac{\beta+1}{\beta-1}\right)^{\xi} + 1}\right] \tag{18}$$

where $\beta = 1.006$ and $0 \leqslant \xi < 1$.

Using a fourth-order two-step Runge–Kutta method, the values of Mach number and temperature at node $j + 1$ in terms of node $j$ are:

$$\begin{aligned} M_{j+1} &= M_j + \tfrac{1}{6}(k_{1,M} + 2k_{2,M} + 2k_{3,M} + k_{4,M}) \\ T^*_{j+1} &= T^*_j + \tfrac{1}{6}(k_{1,T^*} + 2k_{2,T^*} + 2k_{3,T^*} + k_{4,T^*}) \end{aligned}, \quad j = 1, 2, \ldots, m-1 \tag{19}$$

The Runge–Kutta method coefficients in this equation for each $j$ index are also defined by:

$$\begin{aligned} k_{1,M} &= h_j g_1(x^*_j, M_j, T^*_j) \\ k_{2,M} &= h_j g_1\left(x^v_j + \frac{h_j}{2}, M_j + \frac{k_{1,M}}{2}, T^*_j\right) \\ k_{3,M} &= h_j g_1\left(x^*_j + \frac{h_j}{2}, M_j + \frac{k_{2,M}}{2}, T^*_j\right) \\ k_{4,M} &= h_j g_1(x^*_j + h_j, M_j + k_{3,M}, T^*_j) \end{aligned} \tag{20}$$

and

$$\begin{aligned} k_{1,T^*} &= h_j g_2(x^*_j, M_j, T^*_j) \\ k_{2,T^*} &= h_j g_2\left(x^*_j + \frac{h_j}{2}, M_j, T^*_j + \frac{k_{1,T^*}}{2}\right) \\ k_{3,T^*} &= h_j g_2\left(x^*_j + \frac{h_j}{2}, M_j, T^*_j + \frac{k_{2,T^*}}{2}\right) \\ k_{4,T^*} &= h_j g_2(x^*_j + h_j, M_j, T^*_j + k_{3,T^*}) \end{aligned} \tag{21}$$

where

$$h_j = x^*_{j+1} - x^*_j, \quad j = 1, 2, \ldots, m-1 \tag{22}$$

A Fortran code was developed to apply the method for solving the derived governing equations. The code computes the Mach number and temperature at each nodal point towards the outflow. In each step, the other flow variables such as velocity, pressure and density are determined by combining Eqs. (1) and (3), as:

$$\begin{aligned} V^*_{j+1} &= \frac{M_{j+1}}{M_1}\sqrt{T^*_{j+1}}, \\ p^*_{j+1} &= \rho^*_{j+1} T^*_{j+1}, \end{aligned} \quad \rho^*_{j+1} = \frac{1}{V^*_{j+1}}, \quad j = 1, 2, \ldots m-1 \tag{23}$$

In order to obtain the effects of heat transfer on the gas flow under the choked condition, the solution is started with a very small initial value for inlet Mach number, $M_1$, then the flow variables such as outlet Mach number, $M_2$, being calculated along the pipe towards the outlet. In the second step, an increment of $\Delta M_1$ is added to $M_1$, and $M_2$ is calculated again at the outlet. This procedure is repeated until the outlet Mach number reaches one ($M_2 = 1$). The value of $\Delta M_1$ in each step should be selected very carefully, because the outflow Mach number is very sensitive to $\Delta M_1$ for long pipes. This can cause a major error in the computational calculations. The change of the outlet Mach number ($\Delta M_2$) between two steps is calculated and controlled by the change of $\Delta M_1$ in an automatic way. The value of $\Delta M_1$ is selected in the range of $10^{-10} < \Delta M_1 < 10^{-3}$ such that $\Delta M_2$ is limited to 0.01. For non-choked flow, the computations are continued until the outflow pressure reaches a specific pressure applied at this section.

## 4. Validation of the numerical scheme

To show the sensitivity of important dependent variables on spatial grids, the spatial discretization error was checked by the global conservation of mass-balance at different cross sections and it was found that the error was less than one percent in most cases.

The numerical solution accuracy of the assumed model has also been verified by comparison between the results of the present work with experimental and numerical data of the previous researches. Two different test cases are used to validate the steady state numerical solution. The first one is the pressure change in pipe flow with $L/D = 500$ under the friction and adiabatic wall conditions, which is plotted in Fig. 2. The results are compared with the experimental data of Ekblom and Gullman-Strand [10]. The fluid is the air passing through a pipe with a diameter of $D = 0.5$ m. The inflow Mach number, pressure and temperature, respectively, are $M_1 = 0.23$, $p_1 = 392$ *kPa* and $T_1 = 25$ °C.

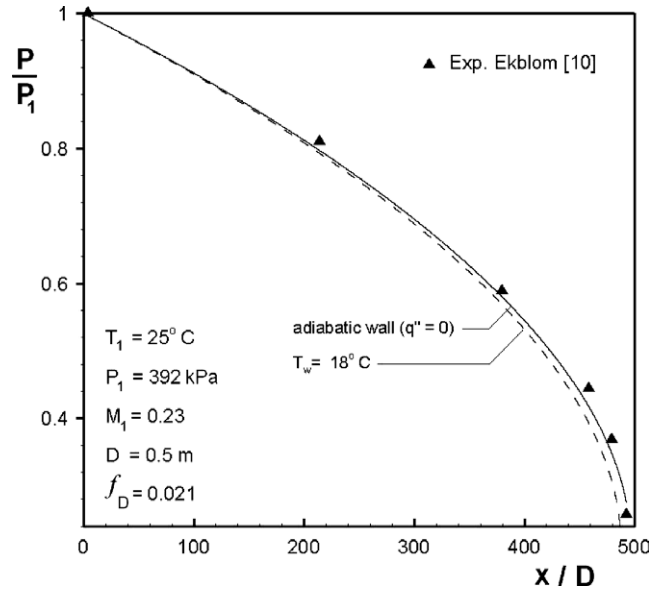


**Fig. 2.** Effect of wall conditions on gas pressure along the pipe.

The friction factor was assumed to be constant and equal to $f_D = 0.021$. A good agreement between this experimental data and the present numerical work is observable in this graph. This figure, also, includes the numerical results of the case in which the wall temperature is constant ($T_w = 18$ °C). In this case, the gas pressure decreases a little more than the adiabatic wall condition. This means that for this case the heat gain is more dominant than the gas expansion.

In the second case, Fig. 3 represents the effect of constant wall heat flux on the inflow Mach number for the choked flow condition. The abscissa axis in this figure shows the required pipe length

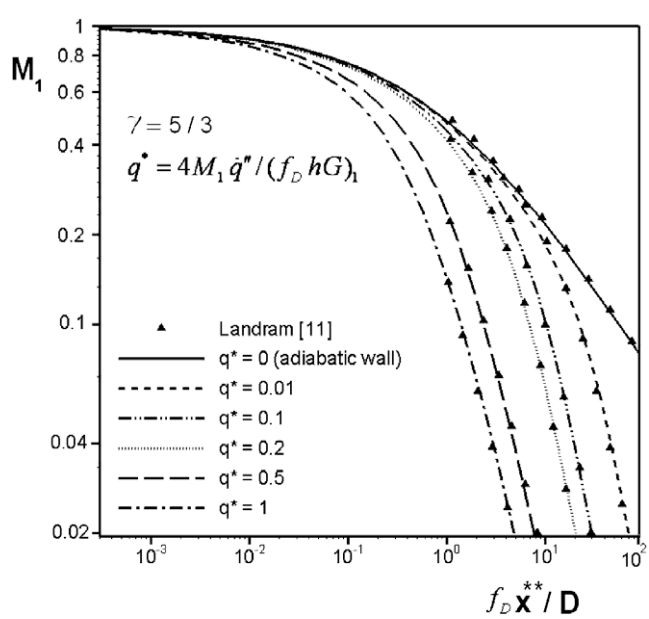


**Fig. 3.** Effect of wall heat flux on inflow Mach number for choked flow.

corresponding to $M_2 = 1$ at the outflow. The dimensionless wall heat flux has been defined by $q^* = 4M_1\dot{q}''/(f_D hG)_1$ in which $h$, $M$ and $G$ are the gas enthalpy, Mach number and mass flow rate per unit area all at the inlet conditions respectively. The plot also includes the results of Landram [11]. It can be seen that heating the gas flow can reduce the choking length. This means that the pipe flow with heating needs a shorter length than that without heating.

## 5. Results and discussion

The capacity of a transmission network depends on static and dynamic elements, as well as on operational conditions. The static elements are the technical characteristics of the network itself. These elements include the network architecture and the specific properties of the equipments. In a gas transmission network, these properties include: the diameter of the pipelines, the roughness of the pipeline material or the friction factor, which has an influence on the pressure losses and the technical characteristics of other equipments, such as valves or heating facilities.

The dynamic elements refer to the way the network is being utilized and operated. For a gas transmission network, these variables are: the properties of the gas injected/delivered at the entry/exit points such as pressure, temperature and gases chemical composition, the distribution of the nominations between the various entry points of the network and the gas demand or mass flow rate at the exit point.

The operational constraints are the boundaries set on each variable by the different parties. In particular: the minimum/maximum pressure guaranteed at the interconnection points, the gas supply (at the entry points) and off-take (at the exit points) are required to be the same within certain margins, a minimum gas pressure is required at each exit point and the operating limits of the ancillary equipments have to be respected, typically on the volume flow and thermodynamic properties.

By obtaining the behavior of flow variables under the heat transfer and friction factor effects for different static elements, it is possible to design an economical pipeline for transferring specific amount of material between two points according to the effects of these parameters on flow losses. The effects of heat transfer is considered by different Biot numbers, while the effects of pipe diameter, pipe length and the friction factor all are summarized in a dimensionless parameter $f_D L/D$. It is notable that for a specific pipe length between two pumping stations, increasing this parameter is regarded as increasing the pipe wall roughness or reducing the pipe diameter.

With respect to all these parameters, the design of the transmission pipeline would be possible by simultaneously solution of presented differential equations (10) and (11) numerically and obtaining all other flow parameters such as pressure, temperature and Mach number distributions, as well as the rate of mass flow carried under different conditions.

Before presenting the results, it is noteworthy to mentioned that the physical properties of the common soil are taken as $1 \times 10^{-7} < \alpha_s < 2.4 \times 10^{-7}$ m$^2$/s and $k_s = 0.52$ W/m K. Also the physical properties of methane as a natural gas consist of $k_g = 0.035$ W/m K, $Pr = 0.71$, molecular weight of 16.04 kg/kmol and the specific heat ratio of $\gamma = 1.299$. The wall thickness and roughness of the steel pipe are assumed to be $\delta = 0.02$ m and $\varepsilon = 4.6 \times 10$–5 m with thermal conductivity of $k_w = 30$ W/m K. It is also assumed that the pipeline has a diameter of $D = 1.4$ m and is buried under the ground at a depth of $H/D = 1.5$. The pressure and temperature of the surrounding air are assumed to be $P_\infty = 1.01 \times 10^5$ Pa and $T_\infty = 283$ K respectively. The reference mass flow rate passing through the pipe at $T_1 = 303$ K and $p_1/p_2 = 2$ is equal to $\dot{m}_1 = 272.85$ kg/s. The choked mass flow rate is also defined at the ambient temperature, and is equal to $\dot{m}^{**} = 2415.75$ kg/s. In this case, the pressure ratio will be $p_1/p_2 = 8.9$.

Fig. 4 illustrates the effect of heat transfer on the inlet Mach number against the pressure ratio for different values of $f_D L/D$. At a specific value of $f_D L/D$, the inlet Mach number reaches its maximum value and after that remains constant. This means that after this point the inflow Mach number would be independent of the pressure ratio in a choked flow. The results, also, show that a bigger

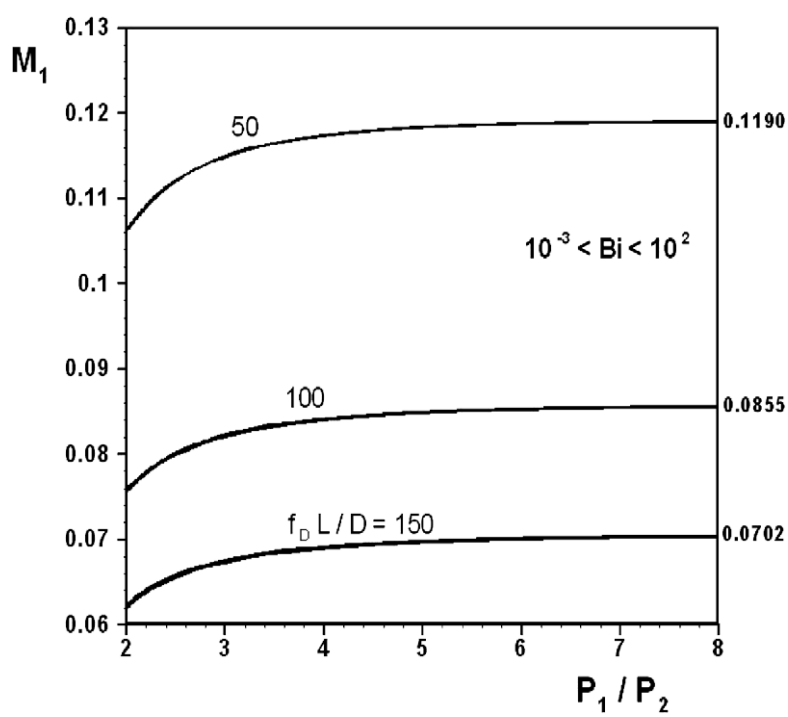


**Fig. 4.** Effect of heat transfer on inflow Mach number for different pressure ratios and various $f_D L/D$.

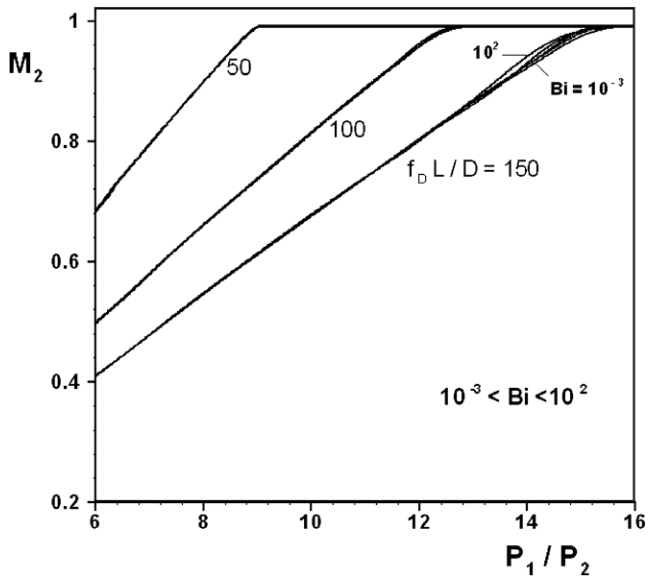


**Fig. 5.** Effect of heat transfer on outflow Mach number for different pressure ratios and various $f_DL/D$.

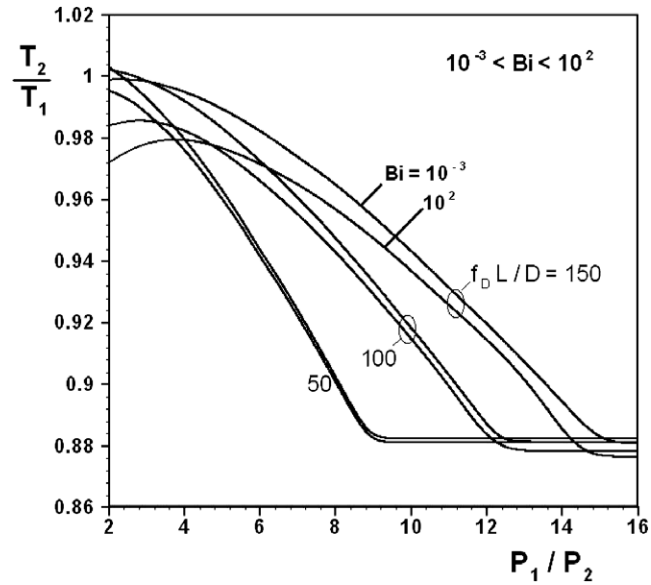


**Fig. 6.** Effect of heat transfer on temperature loss for different pressure ratios and various $f_DL/D$.

inflow Mach number needs a shorter pipe length to reach a maximum value or in fact a choked flow needs a shorter pipe length for a higher inflow Mach number.

The heat transfer effect was also studied here by changing the Biot number. Generally speaking, because of very low heat addition to the gas flow per unit mass, the change in Mach number is very small for all $f_DL/D$. The results were obtained for a wide range of Biot number ($0.001 < Bi < 100$) and it can be seen that there is no considerable effect of heat transfer regardless of wide variation in the inflow Mach number.

Fig. 5 shows the outflow Mach number versus the pressure ratio for different $f_DL/D$ and Biot numbers. This figure exhibits that a longer pipeline requires more inflow pressure to have a choked flow condition. In addition, comparison between Figs. 4 and 5 indicates that the change of the outflow Mach number is more sensitive to the inflow pressure than the inflow Mach number. The effect of heat transfer on outflow Mach number can only be observed for large $f_DL/D$ values and large pressure ratios, i.e. near the choked condition. In a specific pressure ratio, it was found that increasing Biot number could increase the outflow Mach number and consequently reduce the choking length.

Fig. 6 indicates the effect of Biot number on the gas temperature loss along the pipeline. Although, there is a small distance between these lines, it can be investigated that for large $f_DL/D$ values, the

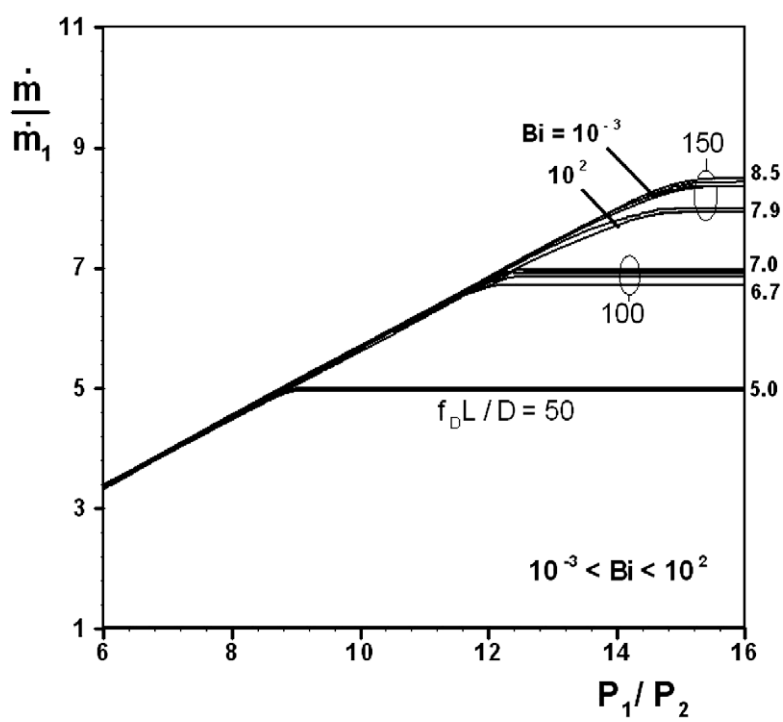


**Fig. 7.** Effect of heat transfer on mass flow rate for different pressure ratios and various $f_DL/D$.

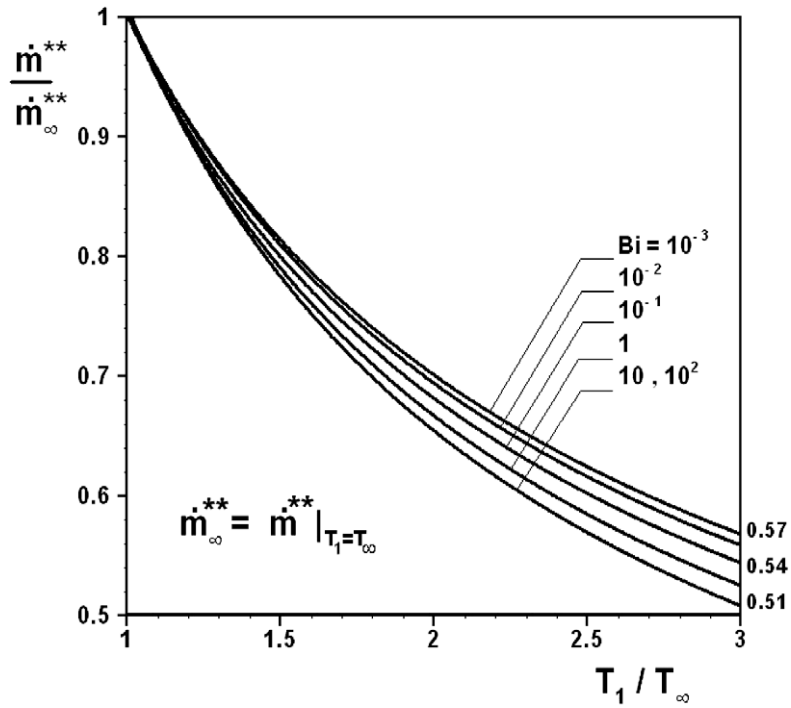


**Fig. 8.** Effect of heat transfer and inflow gas temperature on the mass flow rate for choked flow.

outflow gas temperature drops more than the other cases, especially for larger Biot numbers. The temperature drop is also more obvious for lower pressure ratios. It means that for large $f_D L/D$ values, the temperature loss will increase, if lower pressure drop is desired along the pipe.

The mass flow rates of the gas in the pipe for different $f_D L/D$, pressure ratios and Biot numbers were presented in Fig. 7. The reference mass flow rate in the denominator of the ordinate axis is obtained at the inlet temperature and the pressure ratio of $p_1/p_2 = 2$. From this figure, one can observe that though the rate of mass carried by the pipeline will be increased by $f_D L/D$, the heat transfer has more effect on the reduction of mass flow rate in large $f_D L/D$ values. In other words, in order to increase the rate of mass flow rate from the pipeline with higher $f_D L/D$ values, one should reduce the rate of heat transfer from the pipeline, while for small $f_D L/D$ values, the heat transfer has no considerable effect on the mass flow rate.

Fig. 8 reports the effect of inflow gas temperature on the mass flow rate for choked flow. The graph was plotted for different Biot numbers. It is clear that the mass flow rate can be increased if the inflow gas temperature is cooled before entering to the pipe. However, it is also interesting to note that the curves corresponding to higher Biot numbers change faster than the lines with lower ones. This means that increasing the Biot number together with inflow gas temperature both leads to a lower choked mass flow rate passing through the pipe, however, the effect of inflow temperature is more significant than the pipe wall heat transfer.

## 6. Concluding remarks

Transmission of natural gas in pipelines with friction and heat transfer was studied with one-dimensional compressible flow model. The results were obtained here are applicable in design and optimization of gas pipelines to carry high amounts of mass flow rate with low pressure loss between two specific stations. The effects of various parameters on flow variables were studied in this paper. The results show that at a specific value of inflow to outflow pressure ratio, inlet Mach number reaches its maximum value and after that remains constant. This means that after this point in choked flow, the inflow Mach number is independent of the pressure ratio. In addition, the change of the outflow Mach number is more sensitive than the inflow Mach number relative to the inflow pressure. Even the rate of heat transfer has no considerable effect on inflow Mach number in different $f_D L/D$ ranges, it has reduced the choking length in higher $f_D L/D$ values. Also, the temperature reduction increases in this case for small pressure ratios.

The results also indicate that for large $f_D L/D$ values, which may be translated to high roughness pipe or small pipes in diameter, decreasing the rate of heat transfer from the pipe wall indicated here by Biot number, from 100 to 0.001 will cause in increase of about 7% in the rate of mass flow carried by the pipeline, while for smaller $f_D L/D$ values, the change in the rate of mass flow has not a considerable effect. Furthermore, from the results it is clear that the mass flow rate of choked flow is increased whether the gas flow is cooled before entrance to the pipe or the rate of heat transfer to the pipe is reduced, however the former influence is higher.

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