ABSTRACT
In this paper, the two-phase flow characteristics of liquid-vapor are investigated numerically by the drift flux model inside a short tube orifice. The conservation governing equations are solved numerical by the fourth order of the Runge-Kutta method. As a result of the entrance of the subcooled liquid, the fluid flow inside the short tube orifice includes two regions, single and two-phase flow. The results indicate that the reduction of the pressure and mixture density are respectively 3.27 kPa and 4% for refrigerant R12 and 0.79 kPa and 1% for R134a. On the other hand, the increase of their void fractions is 4% and 0.09% respectively.

Keywords: two-phase flow, drift flux model, numerical method, short tube orifices

INTRODUCTION
Although short tube orifices and small diameter capillary tubes have very simple geometries, but the two-phase flow through these expansion devices are rather complicated. They are very popular in refrigeration systems mostly because of simplicity of design, reliability, ease of replacement, and cost effectiveness. The typical diameter of orifice is between 0.5-2 mm with L/D=3-20, while the typical length of capillary tube 1-6 m [1]. Many experimental and numerical studies show that the flow within the capillary tubes can be divided into two regions, a liquid length, in which only liquid flows and a two-phase flow length in which liquid and vapor flow simultaneously. The transition point is called the bubble or the flash point. The liquid length is characterized by a constant pressure gradient and in the two-phase flow region the results show that a large pressure gradient, which increases sharply in the direction of the flow.

In general, two-phase flow studies the homogeneous flow, the separated flow and the drift flux models for the prediction of the two-phase pressure drop [2]. In the homogeneous flow model, the two-phase mixture can be simulated as a single-phase fluid possessing mean fluid properties. This approach has been widely used in the modeling of the flow in capillary tubes. In the separated flow model, as slip exists between the phases and when the conservation equations are applied to the mixture, an extra variable called void fraction is introduced. Unlike the homogeneous flow model, which only requires experimental information in the friction effect, the separated flow model requires information of the void fraction and the friction effects [3]. In the drift flux model, the conservation equation is formulated by considering the entire mixture. Therefore, the formulation is expressed in terms of four field separations: three for the mixture (continuity, momentum and energy) plus the drift velocity for one of the phases. The interactions between the two phases are specified by appropriate constituting equations: for example, an appropriate drift velocity is specified for a particular flow regime. The drift velocity in turn specifies a drift stress in the momentum equation for the mixture as well as an energy transport term in the energy equation for the mixture. Though this drift flux model has been well documented [4], the model has not been widely applied to the field of flow within capillary tube study. The main reason for this may be due to the lack of experimental information on the drift velocity correlation for the flow within small diameter capillary tubes.

In short tube orifice, flashing flow is believed to have obvious non-homogeneous and non-equilibrium characteristics and be choked at the outlet, so the critical flow rate is independent of downstream conditions. It is imperative to quantify the critical flow rate in short tube for obtaining the optimum system performance. [5]. The existing theoretical models of two-phase critical flow through short tube can be mainly classified into four general groups approximately: homogeneous equilibrium model (HEM), homogeneous frozen model (HFM), non-homogeneous equilibrium model (NEM) and two-fluid model (TFM), [6]. In the HEM, the two-phase mixture can be simulated as a single-phase fluid possessing mean fluid properties. Hence, the mixture is homogeneous in phase composition, vapor and liquid velocities are equal and two-phase mixture is in thermodynamic equilibrium, vapor and liquid with equal temperature. HFM applies to the case where the flow is homogeneous and interfacial mass transfer is restricted due to insufficient time. The average velocities of the phases are equal. NEM is quite complicated, owing to the consideration of non-homogeneous characteristics in that they assume unequal phase velocities. TFM has thus been presented in literature [7, 8], in which the hydrodynamic as well as thermal non-equilibrium effects are considered. TFM can give good predictions for water in long or short tube. Recently, Choi et al. [9] developed a generalized correlation for refrigerants through short tube on the basis of dimensional analysis. However, the empirical correction factors may not have any physical significance and strongly depend on the operating conditions and the refrigerant used.

This study attempts to exploit the possibility of applying the equilibrium two-phase drift flux model to simulate the flow of refrigerant in the orifice and capillary tube expansion devices. Attempts have been made to compare predictions with experimental results. The details flow characteristics of R12 and R134a in a short tube orifice, such as distribution of pressure, void fraction, phase's velocities.

MATHEMATICAL MODELING
The flow within the orifice or capillary tube is divided into a subcooled single liquid and a saturated two-phase flow region. However, because of short length of the orifice, its single liquid region is neglected.
Subcooled Single Liquid Region

In this case, the friction pressure drop based on the one-dimensional, fully developed turbulent flow assumption is as follows.

\[
\left( \frac{dp}{dz} \right)_f = \frac{f_f G^2}{2 \rho_f D}
\]  

(1)

In small diameter tubes, Mikol [10] reported that the friction factor is about 20% higher than that of smooth tubes. This is because the relative roughness has a marked effect on the wall shear stress. So, the friction factor for turbulent flow should be calculated from the Colebrook’s correlation.

\[
\frac{1}{\sqrt{f_D}} = 1.14 - 2 \log \left( \frac{e}{D} + \frac{9.34}{Re \sqrt{f_D}} \right)
\]  

(2)

Two-Phase Flow Region

In this region, liquid flashes into vapor purely because of the reducing pressure. The one-dimensional, steady state of mass, momentum and energy equations for a two-phase flow of vapor-liquid phases through an adiabatic short tube orifice are given respectively by:

\[
G = \rho_m u_m = \alpha \rho_g u_g + (1-\alpha) \rho_f u_f
\]  

(3)

where

\[
\rho_m = \alpha \rho_g + (1-\alpha) \rho_f
\]  

(4)

\[
\frac{d}{dz} G \left[ \left( 1-x \right) u_f + x u_g \right] - \frac{dp}{dz} = -\left( f_{fs} \right)_D \frac{G^2}{2D \rho_f} \phi_{fs}
\]  

(5)

\[
\frac{d}{dz} G \left[ \left( 1-x \right) \left( h_f + \frac{u_f^2}{2} \right) + x \left( h_g + \frac{u_g^2}{2} \right) \right] = 0
\]  

(6)

where the vapor void fraction in term of the quality will be

\[
\alpha = \frac{G x}{\rho_g u_g}
\]  

(7)

Drift velocities of vapor relative to the centers of the mass, \( u_{gm} \), and volume, \( u_{gj} \), of the mixture are respectively.

\[
u_{gm} = u_g - u_m = u_g - \frac{G}{\rho_m}
\]  

(8)

\[
u_{gj} = u_g - \left( j_g + j_f \right)
\]  

(9)

Combing Eqs. (3, 4) and (8, 9), the mass flux of the mixture is obtained as follows.

\[
G = \rho_m u_m - \rho_f u_f
\]  

(10)

Using a correlation for the vapor drift velocity relative to the center of the volume of the mixture presented by Zuber and Findlay [11] as:

\[
u_{gj} = 1.48 \left( \frac{\rho_f - \rho_g}{\rho_f^2} \right) g^0.25 \sigma
\]  

(11)

After inserting Eq. (7) into Eq. (3), and differentiating Eqs. (3, 5, 6, 10), the general form of the differential equation for four variables \( x, u_f, u_g \) and \( p \) is given by:

\[
a_i \frac{dx}{dz} + b_i \frac{du_f}{dz} + c_i \frac{du_g}{dz} + d_i \frac{dp}{dz} = f_i
\]  

(12)

The coefficients \( a_i, b_i, c_i, d_i \), and \( f_i \) are given in the Appendix A for \( i = 1, 2, 3 \) and 4. They are functions of the physical properties of the liquid-vapor phases and their derivatives, \( d \phi dp \), where \( \phi \) is a general variable such as \( T, h, \mu, \).

SOLUTION METHOS

In the above analysis, if a subcooled single-phase liquid enters the short tube orifice, Eq. (1) is first applied to the subcooled liquid region with the known initial conditions at the orifice inlet, for instance, pressure and mass flow rate. Once the saturated condition is reached, the two-phase flow calculations are started until the mass flow rate reaches the critical condition. When this happens, the critical flow rate is almost independent of the downstream pressure. In this case Eqs. (12) for \( i = 1, 2, 3 \) and 4 are solved simultaneously by the fourth order Rung-Kutta method for unknown variables \( x, u_f, u_g \) and \( p \) under their specified initial conditions.

The existing derivatives of \( d \phi dp \) in Eq. (12) can be obtained by fitting the fluid properties data versus pressure from the table of the thermophysical properties of matter.

RESULTS AND DISCUSSION

To validate the presented drift flux model, a comparison has been made between the present numerical results with the experimental results of Mikol [10] and the numerical results of Liang et al. [12] using the refrigerant R12 as a working fluid. The subcooled liquid enters a sharped-edge orifice with diameter 1.41 mm as a subcooled single-phase liquid under conditions \( p_{in} = 8.58 \) bar, \( T_{in} = 32.8^\circ \) C and \( G = 2241.4 \) kg/s.m². Fig. 1 shows the distribution pressure along the orifice for the refrigerants R12. A good agreement is observed among the results. In addition, the pressure distribution of the refrigerant R134a is also obtained within the orifice with the same diameter and inlet conditions.

Fig. 2 illustrates the velocity of each phase in the two-phase region. The velocity gradient of each phase of the
refrigerant R12 is higher than that of the R134a in the two-phase region. The reason is that the latent heat of the former refrigerant is lower than the latent heat of the latter one. In other words, the flashing rate within the orifice is higher for the R12.

Fig. 1 Pressure distribution along the orifice for two different refrigerants at the same inlet conditions

Fig. 2 Velocities of liquid and vapor phases in the two-phase region along the orifice

Fig. 3 Void fraction in two-phase region along the orifice

CONCLUSIONS

The two-phase flow characteristics of liquid-vapor are investigated numerically by the drift flux model within a short tube orifice. As a result of the entrance of the subcooled liquid, the fluid flow within the short tube orifice includes two regions, single and two-phase flow. The results indicate that the reduction of the pressure and mixture density are respectively 3.27 kPa and 4% for R12 and 0.79 kPa and 1% for R134a. On the other hand, the increase of their void fractions is 4% and 0.09% respectively.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>orifice diameter, m</td>
</tr>
<tr>
<td>$f$</td>
<td>friction factor, function</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity, m/s²</td>
</tr>
<tr>
<td>$G$</td>
<td>Mixture mass flux, kg/s.m²</td>
</tr>
<tr>
<td>$h$</td>
<td>enthalpy, J/kg</td>
</tr>
<tr>
<td>$j$</td>
<td>superficial velocity, m/s</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure, mPa</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number, $\frac{Du\mu}{\rho}$</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity, m/s</td>
</tr>
<tr>
<td>$x$</td>
<td>vapor quality</td>
</tr>
<tr>
<td>$z$</td>
<td>z-coordinate</td>
</tr>
</tbody>
</table>

Greek Letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>void fraction</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>roughness, m</td>
</tr>
<tr>
<td>$\phi$</td>
<td>general variable</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity, kg/s.m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density, kg/m³</td>
</tr>
</tbody>
</table>

Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Darcy</td>
</tr>
<tr>
<td>$f$</td>
<td>liquid</td>
</tr>
<tr>
<td>$F$</td>
<td>friction</td>
</tr>
<tr>
<td>$g$</td>
<td>vapor</td>
</tr>
</tbody>
</table>
The drift velocity of vapor relative to the centre of the volume of the mixture is denoted as \( g_j \) and the mass of the mixture as \( g_m \).

**REFERENCES**


**APPENDIX A**

\[
a_2 = u_g - u_f, \quad b_2 = 1 - x, \quad c_2 = x, \quad d_2 = \frac{1}{G}
\]

\[
f_2 = -\frac{f_{10} G}{2D \rho_f} \varphi_f^2
\]

\[
a_3 = h_g - h_f + \frac{u_g^2 - u_f^2}{2}, \quad b_3 = (1-x) u_f, \quad c_3 = x u_g
\]

\[
d_3 = (1-x) \frac{dh_f}{dP} + x \frac{dh_g}{dP}, \quad f_3 = 0
\]

\[
a_4 = \frac{u_g}{u_f} - 1, \quad b_4 = (1-x) \frac{u_g}{u_f}, \quad c_4 = -\frac{1-x}{u_f}
\]

\[
d_4 = \frac{0.37 \left( g \sigma \right)^{0.25}}{G \rho_f^{0.5} \left( \rho_f - \rho_g \right)^{0.75}} \left[ (3 \rho_f - 2 \rho_g) \frac{d \rho_f}{dP} - \rho_f \frac{d \rho_g}{dP} \right]
\]

\[
f_4 = 0
\]

\[
a_1 = 1 - \frac{\rho_f u_f}{\rho_g u_g}, \quad b_1 = \rho_f \left( \frac{1}{G} + \frac{x}{\rho_g u_g} \right), \quad c_1 = \frac{x \rho_f u_f}{\rho_g u_g^2}
\]

\[
d_1 = u_f \left[ \frac{1 - x}{G \rho_g u_g} \right] \frac{d \rho_f}{dP} + x \frac{\rho_f u_f}{\rho_g u_g} \frac{d \rho_g}{dP}
\]

\[
f_1 = 0
\]