NUMERICAL STUDY OF HEAT TRANSFER COEFFICIENT IN POROUS MEDIA

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Abstract — In this study, convective heat transfer in a porous flat plate channel flow is simulated by a direct numerical method. The solid materials consist of uniform distributed blocks, which resemble the porous medium within the channel. The solid materials are assumed to be isothermal and the channel walls are under adiabatic conditions. The Navier–Stokes equations are solved directly in the fluid region without the assumption of volume averaging. The two energy transport equations are solved for the solid and fluid flow separately. The results indicate that the mean bulk temperature across the channel develops faster if the channel aspect ratio gets smaller. On the other hand, the Nusselt number has the highest value at the channel inlet and gradually approaches to a minimum or developing condition at a distance which depends on the value of the aspect ratio.

Keywords — Porous media, heat transfer, numerical method, two energy equations

INTRODUCTION

Porous media are an effective heat transfer enhancement technique because the porous structures intensify mixing of the fluid flow, increase the contact surface area and convection heat transfer. Augmentation of forced convection heat transfer plays a very important role in thermal control systems used in transpiration cooling, solid matrix heat exchanger, microelectronic chips cooling, packed bed reactors, industrial furnace, combustors, fixed bed nuclear propulsion systems and many others. Therefore, fluid flow and heat transfer in porous media have received much attention during the past few decades.

Simpkins and Blythe [1], Hickox and Gartling [2] used the Darcy flow modeling approach in convection problems. In order to take into account wall boundary effects and fluid inertia, Krishnan et al. [3] used an improved form of Darcy model that had additional nonlinear terms and empirical constants, i.e. Brinkman and Forchhiemer extension terms. The governing equations are derived based on the volume averaged technique that cannot illustrate the microscopic flow between the solid particles. The boundary conditions do not satisfy the no-slip condition directly on any solid particle because of the averaging property. Kuwahara et al. [4] and Kuwahara and Nakayama [5] analyzed the flow field through four neighboring square cylinders, which were part of a lattice of square cylinders placed regularly in an infinite space. Coulaud et al. [6] computed the fluid motion through the path between two adjacent rows of periodic cylinders while supposing that the porous domain was formed of many rows of cylinders with a regular structure. Idris et al. [7] studied the flow of power law fluids through anisotropic fibrous media using two dimensional periodic arrays of circular solid inclusions. However, the approach of solving the flow between limited numbers of solid blocks cannot exactly consider the effects of both thermal and velocity boundary conditions of the main boundaries of the problem geometry (for example channel walls). The effect of the other particles on the velocity field of the considered block is also neglected in this approach because a uniform or polynomial velocity distribution is applied at the inlet of a considered block while the arriving flow has passed through other particles and its velocity distribution is not known. Pourshaghaghi et al. [8] solved the problem of natural convection in a porous cavity by using the direct numerical solution. The objective of this work is to simulate laminar forced convection heat transfer through a channel flow filled with square cylinders and placed regularly in the space between two plates. The Navier-Stokes equations along with energy equations of the fluid and solid are solved numerically. Here, emphasis is placed on the presentation of heat transfer coefficient and calculating the Nusselt number.

PHYSICAL MATHEMATICAL MODEL

The configuration of the flow passage considered in this work is a parallel plates channel separated a distance H apart filled with a square lattice of solid cylinders with size a, (Figure 1). The length of the channel is taken long enough to make sure that the fluid flow reaches a fully developed condition far downstream. It is assumed that a forced viscous fluid at temperature \( T_{in} \) enters an insulated porous channel and the square solid blocks are at temperature \( T_S \).

The non-thermal equilibrium governing equations of a two-dimensional laminar flow in a non-dimensional form with assumption of constant physical properties for a general variable \( \phi \) in the conservative form can be written as:

\[
\nabla \cdot (\phi V) = \Gamma \nabla^2 \phi + S_\phi
\]
The values of \( \phi \), \( \Gamma \) and \( S_\phi \) for the conservation equations of mass, momentum and energy are given in the following Table.

<table>
<thead>
<tr>
<th>Equation</th>
<th>( \phi )</th>
<th>( \Gamma )</th>
<th>( S_\phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( x )-momentum</td>
<td>( u^* )</td>
<td>( 1/Re )</td>
<td>( -\partial p^<em>/\partial x^</em> )</td>
</tr>
<tr>
<td>( y )-momentum</td>
<td>( v^* )</td>
<td>( 1/Re )</td>
<td>( -\partial p^<em>/\partial y^</em> )</td>
</tr>
<tr>
<td>Fluid energy</td>
<td>( T_f^* )</td>
<td>( 1/Pe )</td>
<td>0</td>
</tr>
</tbody>
</table>

The non-dimensional variables have been defined as:

\[
u = \frac{u}{u_{in}}, \quad \nu' = \frac{v}{u_u}, \quad x^* = \frac{x}{H}, \quad y^* = \frac{y}{H},
\]

\[
\text{Re} = \frac{\rho u_{in} H}{\mu}, \quad \text{Pe} = \frac{u_{in} H}{a_f}, \quad T^* = \frac{T - T_{in}}{T_S - T_{in}}, \quad P^* = \frac{P}{\rho u_{in}^2}
\]

The proper boundary conditions are:

\[
\phi(0, y^*) = 1 \text{ or } 0, \quad \phi = u^* \text{ or } v^*, T_f^*
\]

\[
\frac{\partial \phi}{\partial x^*}(x^*, 0) = 0, \quad \phi = u^*, T_f^*
\]

\[
\phi(x^*, 0.5) = 0, \quad \phi = u^*, v^*
\]

\[
\frac{\partial \phi}{\partial y^*}(x^*, 0.5) = 0, \quad \phi = T_f^*
\]

\[
\frac{\partial \phi}{\partial y^*}(x^*, 0) = 0, \quad \phi = u^*, T_f^*
\]

\[
\phi(x^*, 0) = 0, \quad \phi = v^*
\]

The total heat leaves from all solid blocks located in column \( j \) per unit length and enter the fluid is obtained by summation of heat transfer from each block as:

\[
 q_j' = m'C_p \left( T_j - T_{in} \right) = \frac{h_j}{N_j} \sum_{i=1}^{N_j} \int \frac{\partial T}{\partial n} dl_i
\]

where \( N_j \) is the number of blocks in the \( j \)th column.

The mean bulk temperature of the fluid and the corresponding the mean Nusselt number of the \( j \)th column are computed as follows.

\[
T_j = T_{j-1} + \frac{q_j'}{\rho u_{in} C_p H u_{in}}
\]

\[
(\text{Nu}_a)_j = \frac{h_j}{\rho u_{in} C_p} = \frac{1}{4N_j} \times \frac{T_j - T_{in}}{T_S - T_{in}}
\]

Accordingly, the mean Nusselt numbers for all blocks within the channel between the inlet and the \( j \)th column will be:

\[
(\text{Nu}_a)_j = \frac{1}{4N_j} \sum_{j=1}^{N_j} \frac{T_j - T_{in}}{T_S - T_{in}}
\]

\( N_j \) is the number of the columns from the channel inlet up to the \( j \)th column.

\[
\text{FIGURE 1}
\]

**SCHEMATIC OF A CHANNEL WITH A UNIFORM DISTRIBUTION OF SOLID SQUARE BLOCKS**

**SOLUTION METHOD**

Assume that the solid grids are distinguished from the fluid grids within the channel flow by a void fraction parameter \( \lambda \). The value of this parameter is assumed to be zero for solid cells and one for fluid cells. A computer code is utilized to fill a proportion of the domain based on the desired porosity.

After generating the porous domain, 200 \( \times \) 440 grid points are used in \( y \) - and \( x \) -directions respectively to produce a simple structured grid in the computational domain with uniform mesh sizes in both directions. In this mesh size, each square block contains 18 \( \times \) 18 grid points. The
governing equations are discretized in an implicit scheme by using the finite volume method based on the SIMPLE algorithm in the void regions. The main advantage of the finite volume method is its flexibility in treating arbitrary geometries efficiency. The algebraic discretized continuity and momentum equations are solved by ADI iterative technique. To obtain the temperature distribution of the solid particles and the fluid, the fluid and solid energy equations are solved simultaneously after solving the momentum equations.

The code validation was checked by comparison of the results in an empty channel without any solid block with the results of Sparrow [9] and good adjustment was achieved.

RESULTS AND DISCUSSION

Fig. 2 shows the mean bulk temperature of the fluid along the channel for three different values of the aspect ratio, \( a/H \). The figure indicates that the mean bulk temperature approaches the solid block temperature and thermal equilibrium condition is accomplished when the aspect ratio, \( a/H \), becomes small. The entrance length to reach this situation relative to the channel width is about 1, 2.7 and 6.7 respectively, where \( x \) measures from the inlet of the channel.

Figure 3 reports the dimensionless wall temperature of the channel as a function of the aspect ratio \( a/H \) for the Reynolds number 500 and porosity 0.75. The aspect ratio serves as a parameter in the figure. The wall temperature pattern is similar to the mean bulk temperature of the fluid except for the location where the wall temperature reaches the temperature of block cylinders. This difference is due to the heat resistance between the fluid and the wall.

Figure 4 illustrates the mean transverse Nusselt number of the blocks against the channel length for different aspect ratios. To generalize the chart, the ordinate of the figure is divided by the Peclet number. The characteristic length of the Nusselt number and Peclet number are based on the side of the block and of the channel width respectively. The results show that the mean Nusselt number of the first column is a maximum and then it falls like an exponential function and finally goes to zero. The reason for this behavior is due to the heat transfer coefficient which is defined based on the temperature difference between the blocks and the mean bulk of the fluid. Therefore, when the mean bulk temperature approaches the solid blocks temperature, the Nusselt number gets zero.

Figure 5 states that the overall Nusselt number of all blocks within the channel between the inlet and the \( j \)th column. In this figure also, the overall Nusselt number is divided by the Peclet number. In fact, the overall Nusselt number is the sum of area-weighted Nusselt number over all blocks cylinder. The trend is more or less similar to the trend of the previous plot, i.e., the mean traverse Nusselt number. Depending on the value of the aspect ratio, the overall Nusselt number decreases gradually from a maximum at the inlet to a constant value at a specified distance. This distance can be interpreted as an entrance length where the overall Nusselt number reaches a minimum value. This value becomes small as the aspect ratio is small. In other word, the fluid flow will soon be developed for smaller aspect ratio.
CONCLUSIONS

In this paper, a direct numerical simulation of forced convection is discussed. A uniform distribution of square solid cylinders is employed between two parallel plates separated a distance H apart. The effect of each block on the velocity field is taken into account. The mean bulk temperature of the fluid approaches the block cylinder temperature or indeed thermal equilibrium condition is accomplished sooner when the aspect ratio, \( a/H \), becomes small. The results show that the mean Nusselt number of the first column is a maximum and then it becomes smaller and smaller. The reason is due to the heat transfer coefficient which defined based on the temperature difference between the blocks and the mean bulk of the fluid. Depending on the aspect ratio, the overall Nusselt number of all blocks within the channel between the inlet and the jth column is approached to a specific value. This value becomes small in a short distance from the inlet when the aspect ratio is small. In other word, the fluid flow being developed sooner for the smaller aspect ratios.

REFERENCES


